

NEUTROSOPHIC SET – A GENERALIZATION OF
THE INTUITIONISTIC FUZZY SET

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Abstract: In this paper we generalize the intuitionistic fuzzy set (IFS), paraconsistent set, and intuitionistic set to the neutrosophic set (NS). Many examples are presented. Distinctions between NS and IFS are underlined.

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1. Introduction

One first presents the evolution of sets from fuzzy set to neutrosophic set. Then one introduces the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $]^{-}0, 1^{+}[$ is the non-standard unit interval, and thus one defines the neutrosophic set. One gives examples from mathematics, physics, philosophy, and applications of the neutrosophic set. Afterwards, one introduces the neutrosophic set operations (complement, intersection, union, difference, Cartesian product, inclusion, and n-ary relationship), some generalizations and comments on them, and finally the distinctions between the neutrosophic set and the intuitionistic fuzzy set.

2. Short History

The *fuzzy set* (FS) was introduced by L. Zadeh in 1965, where each element had a degree of membership.

The *intuitionistic fuzzy set* (IFS) on a universe X was introduced by K. Atanassov in 1983 as a generalization of FS, where besides the degree of membership $\mu_A(x) \in [0,1]$ of each element $x \in X$ to a set A there was considered a degree of non-membership $\nu_A(x) \in [0,1]$, but such that

$$x \in X \mu_A(x) + \nu_A(x) \leq 1. \quad (2.1)$$

According to Deschrijver and Kerre [6] the *vague set* defined by Gau and Buehrer [6] was proven by Bustine and Burillo [3] to be the same as IFS.

Goguen [9] defined the *L-fuzzy set* in X as a mapping $X \rightarrow L$ such that (L^*, \leq_{L^*}) is a complete lattice, where $L^* = \{(x_1, x_2) \in [0,1]^2, x_1 + x_2 \leq 1\}$ and $(x_1, x_2) \leq_{L^*} (y_1, y_2) \iff x_1 \leq y_1 \text{ and } x_2 \geq y_2$. The *interval-valued fuzzy set* (IVFS) apparently first studied by Sambuc [10], which were called by Deng [5] *grey sets*, and IFS are specific kinds of L-fuzzy sets.

According to Cornelis et al [4], Gehrke et al [7] stated that: "Many people believe that assigning an exact number to an expert's opinion is too restrictive, and the assignment of an interval of values is more realistic", which is somehow similar with the imprecise probability theory where instead of a crisp probability one has an interval (upper and lower) probabilities as in Walley [15].

Atanassov [1] defined the *interval-valued intuitionistic fuzzy set* (IVIFS) on a universe X as an object A such that:

$$A = \{(x, M_A(x), N_A(x)), x \in X\}, \quad (2.2)$$

with

$$M_A : X \rightarrow \text{Int}([0,1]) \text{ and } N_A : X \rightarrow \text{Int}([0,1]), \quad (2.3)$$

and

$$x \in X \sup M_A(x) + \sup N_A(x) \leq 1. \quad (2.4)$$

Belnap [2] defined a four-valued logic, with truth (T), false (F), unknown (U), and contradiction (C). He used a lattice where the four components were inter-related.

In 1995, starting from philosophy (when I fretted to distinguish between *absolute truth* and *relative truth* or between *absolute falsehood* and *relative falsehood* in logics, and respectively between *absolute membership* and *relative membership* or *absolute non-membership* and *relative non-membership* in set

theory) I began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tight scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, I combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy (I was excited by paradoxism in science and arts and letters, as well as by paraconsistency and incompleteness in knowledge). How to deal with all of them at once, is it possible to unify them?

I proposed the term “neutrosophic” because “neutrosophic” etymologically comes from “neutro-sophy” (French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill/wisdom) which means knowledge of neutral thought, and this third/neutral represents the main distinction between “fuzzy” and “intuitionistic fuzzy” logic/set, i.e. the *included middle* component (Lupasco-Nicolescu’s logic in philosophy), i.e. the neutral/indeterminate/unknown part (besides the “truth”/“membership” and “falsehood”/“non-membership” components that both appear in fuzzy logic/set). See *The Proceedings of the First International Conference on Neutrosophic Logic*, The University of New Mexico, Gallup Campus, 1-3 December 2001, at <http://www.gallup.unm.edu/~smarandache/FirstNeutConf.htm>.

3. Definition of Neutrosophic Set

Let T, I, F be real standard or non-standard subsets of $]0, 1[$, with $\sup T = t_{\sup}$, $\inf T = t_{\inf}$, $\sup I = i_{\sup}$, $\inf I = i_{\inf}$, $\sup F = f_{\sup}$, $\inf F = f_{\inf}$, and $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$, $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$. Here T, I, F are called *neutrosophic components*.

Let U be a universe of discourse, and M a set included in U . An element x from U is noted with respect to the set M as $x(T, I, F)$ and belongs to M in the following way:

It is $t\%$ true in the set, $i\%$ indeterminate (unknown if it is) in the set, and $f\%$ false, where t varies in T , i varies in I , f varies in F .

4. General Examples

Let A, B , and C be three neutrosophic sets.

One can say, by language abuse, that any element neutrosophically belongs to any set, due to the percentages of truth/indeterminacy/falsity involved, which varies between 0 and 1 or even less than 0 or greater than 1.

Thus: $x(0.5,0.2,0.3)$ belongs to A (which means, with a probability of 50% x is in A, with a probability of 30% x is not in A, and the rest is undecidable); or $y(0,0,1)$ belongs to A (which normally means y is not for sure in A); or $z(0,1,0)$ belongs to A (which means one does know absolutely nothing about z 's affiliation with A); here $0.5+0.2+0.3=1$; thus A is a NS and an IFS too. More general, $y((0.20-0.30), (0.40-0.45), [0.50-0.51], \{0.20, 0.24, 0.28\})$ belongs to the set B, which means:

— with a probability in between 20-30% y is in B (one cannot find an exact approximation because of various sources used).

— with a probability of 20% or 24% or 28% y is not in B.

— the indeterminacy related to the appurtenance of y to B is in between 40-45% or between 50-51% (limits included).

The subsets representing the appurtenance, indeterminacy, and falsity may overlap, and $n_{sup} = 0.30+0.51+0.28 > 1$ in this case; then B is a NS but is not an IFS; we can call it *paraconsistent set* (from *paraconsistent logic*, which deals with paraconsistent information).

Or, another example, say the element $z(0.1, 0.3, 0.4)$ belongs to the set C, and here $0.1 + 0.3 + 0.4 < 1$; then B is a NS but is not an IFS; we can call it *intuitionistic set* (from *intuitionistic logic*, which deals with incomplete information).

Remarkably, in the same NS one can have elements which have paraconsistent information (sum of components > 1), others incomplete information (sum of components < 1), others consistent information (in the case when the sum of components $= 1$), and others interval-valued components (with no restriction on their superior or inferior sums).

5. Physics Examples

a) For example the Schrödinger's Cat Theory says that the quantum state of a photon can basically be in more than one place in the same time, which translated to the neutrosophic set means that an element (quantum state) belongs and does not belong to a set (one place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory.

In Schrödinger's equation on the behavior of electromagnetic waves and “matter waves” in quantum theory, the wave function ψ which describes the superposition of possible states may be simulated by a neutrosophic function, i.e. a function whose values are not unique for each argument from the domain of definition (the vertical line test fails, intersecting the graph in more points).

Do not we better describe, using the attribute “neutrosophic” than “fuzzy” or any others, a quantum particle that neither exists nor non-exists?

b) How to describe a particle \star in the infinite micro-universe that belongs to two distinct places P_1 and P_2 in the same time? $\star \chi P_1$ and $\star \varpi P_1$ as a true contradiction, or $\star \chi P_1$ and $\star \chi \cdot P_1$.

6. Philosophical Examples

Or, how to calculate the truth-value of Zen (in Japanese) / Chan (in Chinese) doctrine philosophical proposition: the present is eternal and comprises in itself the past and the future?

In Eastern philosophy the contradictory utterances form the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.

How to judge the truth-value of a metaphor, or of an ambiguous statement, or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

There are many ways to construct them, in terms of the practical problem we need to simulate or approach. Below there are mentioned the easiest ones.

7. Application

A cloud is a neutrosophic set, because its borders are ambiguous, and each element (water drop) belongs with a neutrosophic probability to the set (e.g. there are a kind of separated water drops, around a compact mass of water drops, that we do not know how to consider them: in or out of the cloud).

Also, we are not sure where the cloud ends nor where it begins, neither if some elements are or are not in the set. That is why the percent of indeterminacy is required and the neutrosophic probability (using subsets – not numbers – as components) should be used for better modeling: it is a more organic, smooth, and especially accurate estimation. Indeterminacy is the zone of ignorance of a proposition's value, between truth and falsehood.

8. Operations with Classical Sets

We need to present these set operations in order to be able to introduce the neutrosophic connectors. Let S_1 and S_2 be two (unidimensional) real standard or non-standard subsets included in the non-standard interval $]^{-}0, \infty)$ then one defines the following sets.

8.1. Addition of Classical Sets

$S_1/S_2 = \{x \in x=s_1+s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$,
with $\inf S_1/S_2 = \inf S_1 + \inf S_2$, $\sup S_1/S_2 = \sup S_1 + \sup S_2$; and, as some particular cases, we have

$\{a\}/S_2 = \{x \in x=a+s_2, \text{ where } s_2 \in S_2\}$
with $\inf \{a\}/S_2 = a + \inf S_2$, $\sup \{a\}/S_2 = a + \sup S_2$.

8.2. Subtraction of Classical Sets

$S_1 \ominus S_2 = \{x \in x=s_1-s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$,
with $\inf S_1 \ominus S_2 = \inf S_1 - \sup S_2$, $\sup S_1 \ominus S_2 = \sup S_1 - \inf S_2$; and, as some particular cases, we have

$\{a\} \ominus S_2 = \{x \in x=a-s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{a\} \ominus S_2 = a - \sup S_2$, $\sup \{a\} \ominus S_2 = a - \inf S_2$; also $\{1^+\} \ominus S_2 = \{x \in x=1^+-s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{1^+\} \ominus S_2 = 1^+ - \sup S_2$, $\sup \{1^+\} \ominus S_2 = 100 - \inf S_2$.

8.3. Multiplication of Classical Sets

$S_1 \odot S_2 = \{x \in x=s_1 \odot s_2, \text{ where } s_1 \in S_1 \text{ and } s_2 \in S_2\}$.
with $\inf S_1 \odot S_2 = \inf S_1 \odot \inf S_2$, $\sup S_1 \odot S_2 = \sup S_1 \odot \sup S_2$;
and, as some particular cases, we have

$\{a\} \odot S_2 = \{x \in x=a \odot s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{a\} \odot S_2 = a \odot \inf S_2$, $\sup \{a\} \odot S_2 = a \odot \sup S_2$;
also $\{1^+\} \odot S_2 = \{x \in x=1^+ \odot s_2, \text{ where } s_2 \in S_2\}$,
with $\inf \{1^+\} \odot S_2 = 1^+ \odot \inf S_2$, $\sup \{1^+\} \odot S_2 = 1^+ \odot \sup S_2$.

8.4. Division of a Classical Set by a Number

Let $k \in \mathbb{R}^*$, then $S_1 \oslash k = \{x \in x=s_1/k, \text{ where } s_1 \in S_1\}$.

8.5. Neutrosophic Set Operations

One notes, with respect to the sets A and B over the universe U , $x = x(T_1, I_1, F_1) \chi A$ and $x = x(T_2, I_2, F_2) \chi B$, by mentioning x 's *neutrosophic membership, indeterminacy, and non-membership respectively appurtenance*.

And, similarly, $y = y(T', I', F') \chi B$.

If, after calculations, in the below operations one obtains values < 0 or > 1 , then one replaces them with $^-0$ or 1^+ respectively.

Complement of A. If $x(T_1, I_1, F_1) \chi A$, then $x(\{1^+\}0T_1, \{1^+\}0I_1, \{1^+\}0F_1) \chi C(A)$.

Intersection. If $x(T_1, I_1, F_1) \chi A$, $x(T_2, I_2, F_2) \chi B$, then $x(T_1?T_2, I_1?I_2, F_1?F_2) \chi A \text{ } 3 \text{ } B$.

Union. If $x(T_1, I_1, F_1) \chi A$, $x(T_2, I_2, F_2) \chi B$, then $x(T_1/T_20T_1?T_2, I_1/I_20I_1?I_2, F_1/F_20F_1?F_2) \chi A \text{ } 4 \text{ } B$.

Difference. If $x(T_1, I_1, F_1) \chi A$, $x(T_2, I_2, F_2) \chi B$, then $x(T_10T_1?T_2, I_10I_1?I_2, F_10F_1?F_2) \chi A \setminus B$, because $A \setminus B = A \text{ } 3 \text{ } C(B)$.

Cartesian Product. If $x(T_1, I_1, F_1) \chi A$, $y(T', I', F') \chi B$, then $(x(T_1, I_1, F_1), y(T', I', F')) \chi A \text{ } \% \text{ } B$.

M is a subset of N. If $x(T_1, I_1, F_1) \chi M \vee x(T_2, I_2, F_2) \chi N$, where $\inf T_1 \leq \inf T_2$, $\sup T_1 \leq \sup T_2$, and $\inf F_1 \leq \inf F_2$, $\sup F_1 \leq \sup F_2$.

Neutrosophic n-ary Relation. Let A_1, A_2, \dots, A_n be arbitrary non-empty sets.

A Neutrosophic n-ary Relation R on $A_1 \% A_2 \% \dots \% A_n$ is defined as a subset of the Cartesian product $A_1 \% A_2 \% \dots \% A_n$, such that for each ordered n-tuple $(x_1, x_2, \dots, x_n)(T, I, F)$, T represents the degree of validity, I the degree of indeterminacy, and F the degree of non-validity respectively of the relation R .

It is related to the definitions for the *Intuitionistic Fuzzy Relation* independently given by Atanassov 1984, 1989, Toader Buhaescu 1989, Darinka Stoyanova 1993, Humberto Bustince Sola and P. Burillo Lopez 1992-1995.

9. Generalizations and Comments

From the intuitionistic logic, paraconsistent logic, dialetheism, faillibilism, paradoxes, pseudoparadoxes, and tautologies we transfer the “adjectives” to the sets, i.e. to intuitionistic set (set incompletely known), paraconsistent set, dialetheist set, faillibilist set (each element has a percentage of indeterminacy), paradoxist set (an element may belong and may not belong in the same time to the set),

pseudoparadoxist set, and tautologic set respectively.

Hence, the neutrosophic set generalizes:

- the *intuitionistic set*, which supports incomplete set theories (for $0 < n < 1$ and $i = 0$, $0 \leq t, i, f \leq 1$) and incomplete known elements belonging to a set;
- the *fuzzy set* (for $n = 1$ and $i = 0$, and $0 \leq t, i, f \leq 1$);
- the *intuitionistic fuzzy set* (for $t+i+f=1$ and $0 \leq i < 1$);
- the *classical set* (for $n = 1$ and $i = 0$, with t, f either 0 or 1);
- the *paraconsistent set* (for $n > 1$ and $i = 0$, with both $t, f < 1$); there is at least one element $x(T,I,F)$ of a paraconsistent set M which belongs at the same time to M and to its complement set $C(M)$;
- the *faillibilist set* ($i > 0$);
- the *dialethist set*, which says that the intersection of some disjoint sets is not empty (for $t = f = 1$ and $i = 0$; some paradoxist sets can be denoted this way too);

every element $x(T,I,F)$ of a dialethist set M belongs at the same time to M and to its complement set $C(M)$;

- the *paradoxist set*, each element has a part of indeterminacy if it is or not in the set ($i > 1$);
- the *pseudoparadoxist set* ($0 < i < 1$, $t + f > 1$);
- the *tautological set* ($i < 0$).

Compared with all other types of sets, in the neutrosophic set each element has three components which are subsets (not numbers as in fuzzy set) and considers a subset, similarly to intuitionistic fuzzy set, of “indeterminacy” – due to unexpected parameters hidden in some sets, and let the superior limits of the components to even boil *over 1* (overflowed) and the inferior limits of the components to even freeze *under 0* (underdried).

For example: an element in some tautological sets may have $t > 1$, called “overincluded”. Similarly, an element in a set may be “overindeterminate” (for $i > 1$, in some paradoxist sets), “overexcluded” (for $f > 1$, in some unconditionally false appurtenances); or “undertrue” (for $t < 0$, in some unconditionally false appurtenances), “underindeterminate” (for $i < 0$, in some unconditionally true or false appurtenances), “underfalse” (for $f < 0$, in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true ($t > 1$, and $f < 0$ or $i < 0$) and conditionally true appurtenances ($t \leq 1$, and $f \leq 1$ or $i \leq 1$).

In a *rough set* RS, an element on its boundary-line cannot be classified neither as a member of RS nor of its complement with certainty. In the neutrosophic set a such element may be characterized by $x(T, I, F)$, with corresponding

set-values for T, I, F ?]⁻⁰, 1⁺[.

Compared to *Belnap’s quadruplet logic*, NS and NL do not use restrictions among the components – and that’s why the NS/NL have a more general form, while the middle component in NS and NL (the indeterminacy) can be split in more subcomponents if necessarily in various applications.

10. Differences between Neutrosophic Set (NS) and Intuitionistic Fuzzy Set (IFS)

a) Neutrosophic Set can distinguish between *absolute membership* (i.e. membership in all possible worlds; we have extended Leibniz’s absolute truth to absolute membership) and *relative membership* (membership in at least one world but not in all), because NS(absolute membership element)=1⁺ while NS(relative membership element)=1. This has application in philosophy (see the neutrosophy). That is why the unitary standard interval [0, 1] used in IFS has been extended to the unitary non-standard interval]⁻⁰, 1⁺[in NS.

Similar distinctions for *absolute or relative non-membership*, and *absolute or relative indeterminate appurtenance* are allowed in NS.

b) In NS there is no restriction on T, I, F other than they are subsets of]⁻⁰, 1⁺[, thus: ⁻⁰[inf T + inf I + inf F [sup T + sup I + sup F [3⁺.

The inequalities (2.1) and (2.4) of IFS are relaxed.

This non-restriction allows paraconsistent, dialetheist, and incomplete information to be characterized in NS {i.e. the sum of all three components if they are defined as points, or sum of superior limits of all three components if they are defined as subsets can be > 1 (for paraconsistent information coming from different sources), or < 1 for incomplete information}, while that information can not be described in IFS because in IFS the components T (membership), I (indeterminacy), F (non-membership) are restricted either to t+i+f=1 or to t² + i² + f² [1, if T, I, F are all reduced to the points t, i, f respectively, or to sup T + sup I + sup F = 1 if T, I, F are subsets of [0, 1].

Of course, there are cases when paraconsistent and incomplete informations can be normalized to 1, but this procedure is not always suitable.

c) Relation (2.3) from interval-valued intuitionistic fuzzy set is relaxed in NS, i.e. the intervals do not necessarily belong to Int[0,1] but to [0,1], even more general to]⁻⁰, 1⁺[.

d) In NS the components T, I, F can also be *non-standard* subsets included in the unitary non-standard interval]⁻⁰, 1⁺[, not only *standard* subsets included in the unitary standard interval [0, 1] as in IFS.

e) NS, like dialetheism, can describe paradoxist elements, $NS(\text{paradoxist element}) = (1, I, 1)$, while IFL can not describe a paradox because the sum of components should be 1 in IFS.

f) The connectors in IFS are defined with respect to T and F, i.e. membership and non-membership only (hence the Indeterminacy is what is left from 1), while in NS they can be defined with respect to any of them (no restriction).

g) Component "T", indeterminacy, can be split into more subcomponents in order to better catch the vague information we work with, and such, for example, one can get more accurate answers to the *Question-Answering Systems* initiated by Zadeh [16] (in Belnap's four-valued logic [2] indeterminacy is split into Uncertainty (U) and Contradiction (C), but they were inter-related).

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