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TOPOLOGICAL VECTOR SPACES WITHOUT REAL ANALYTIC "F-NORMS"

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Abstract: Here we point out the following consequences of a previous result of mine. Let V be a Hausdorff and locally convex topological vector space without any continuous norm and $| \ |_F$ an F-norm defining the topology of V. Then there is no pair (U, f) with U an open neighborhood of $0 \in V$ and $f : U \to \mathbb{R}$ a real analytic function such that $f(0) = 0$ and the sets $\{f < c\}_{c>0}$ form a fundamental system of open neighborhoods of 0 in V. Furthermore, there is no strictly increasing function $g : [0, +\infty) \to [0, +\infty)$ such that $g \circ | \ |_F : V \to [0, +\infty)$ is real analytic.

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Here we point out the following consequences of a previous result of mine ([1], Theorem 1).

Theorem 1. *Let V be a Hausdorff and locally convex topological vector space without any continuous norm. Then there is no pair (U, f) with U an open neighborhood of $0 \in V$ and $f : U \to \mathbb{R}$ a real analytic function such that $f(0) = 0$ and the sets $\{f < c\}_{c>0}$ form a fundamental system of open neighborhoods of* 0 *in V.*

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Theorem 2. *Let V be a Hausdorff, locally convex and metrizable topological vector space without any continuous norm. Let $|\ |_F$ be any F-norm on V defining the topology of V (see [4] or the introduction of [5]). Then there is no strictly increasing function $g : [0, +\infty) \to [0, +\infty)$ such that $g \circ |\ |_F : V \to [0, +\infty)$ is real analytic.*

Proof of Theorem 1. By [3], Theorem 2.6.13, V has a subspace E isomorphic to $\mathbb{R}^{\mathbb{N}}$. Set $D := E \cap U$. It is sufficient to prove the result for D. By [2], Corollary 1.1, every real analytic function on D depends locally only from finitely many variables and hence for any $f : D \to \mathbb{R}$ a real analytic function such that $f(0) = 0$ the set $\bigcap_{c>0}\{f < c\}$ contains the intersection with D of a finite-codimensional linear subspace of E, concluding the proof. □

Proof of Theorem 2. This is a particular case of [1], Theorem 1, but it also follows from the proof of Theorem 1. □

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