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MHD FLOWS IN CORRUGATED CHANNEL

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Abstract: We study the free convection flow of a viscous incompressible fluid between two finitely long vertical walls, one of which is corrugated, in the presence of an external transverse magnetic field which is applied uniformly. The walls are maintained at constant but different temperatures. We look at the case when heat source or sink is presented in the fluid. The governing equations are solved by a perturbation technique. Expressions for the zeroth-order, the first-order and the total solutions are obtained and evaluated numerically. A parametric study of some of the physical parameters involved in the problem is performed to illustrate the influence of these parameters on the flow characteristics.

AMS Subject Classification: 76D05, 76M25
Key Words: MHD flows, free convection, corrugated, constricted and dilated channels

1. Introduction

The flow of viscous incompressible and electrically conducting fluid in the presence of free convection has been studied by many authors. This is well documented in [5].

On the other hand, over a roughened (corrugated) wall has revealed a successful method in enhancing heat transfer and, therefore, in reducing the heat exchanger size. Although most prior studies have focused on viscous fluid flow

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over smooth flat walls, while some small amount of wall roughness must occur in any real experiment. Thus the effects of wall roughness in physical problems such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces, film vaporization in combustion chambers, nuclear engineering, and other industrial areas are very significant.

Shanka and Sinha [6] have made a detailed study of the Rayleigh problem for a wavy wall and arrived at certain interesting conclusions; namely, that at low Reynolds numbers the roughness/waviness of the wall quickly ceases to be of importance as the liquid is dragged along by the wall, and the known potential solution emerges in time. Lessen and Gangwani [4] have made a very interesting analysis of the effects of small amplitude wall waviness upon the stability of the laminar boundary layers. Lekoudis et al [3] have made a linear analysis of compressible boundary layer flows over a wavy wall. They have determined increased heat transfer due to wall roughness and provided the mean flow for stability analysis. In all these studies, the authors have taken the wavy wall to be oriented in a horizontal direction and studied the effect of the waviness on the flow field.

Vajravelu and Sastri [7] investigated the problem of free convection in an incompressible viscous fluid bounded by a long (compared to width of the channel) vertical wavy wall and a parallel flat wall. They have given special attention to the effects of wall waviness on the flow and heat transfer characteristics. The main objective of this paper is to study the specific problem under the influence of an externally applied magnetic field.

Ram [5], in his reviewed article, presented an account of several investigations of heat and mass transfer carried out by several authors in the field of magnetohydrodynamics (MHD). An application of decomposition method to a steady two-dimensional blood flow through a constricted artery in the presence of a uniform transverse magnetic field has been presensted recently by Halder [2]. Most recently, Ajadi [1] obtain closed form solutions for the velocity profiles and the skin friction of the particulate flow of a dusty viscous incompressible conducting fluid between two types of boundary motions – oscillatory and non-oscillatory, under the influence of gravitational force.

The present investigation gives an analytical and numerical solutions to a steady two-dimensional free convection flow of a viscous incompressible and electrically conducting fluid in constricted, and dilated channels in the presence of a uniform transverse magnetic field. The contributions of the magnetic field parameter H, in particular, and those of the other parameters wall-temperature ratio m, heat source/sink parameter α and Prandtl number P_r, in general, to the flow characteristics are found to be quite significant.

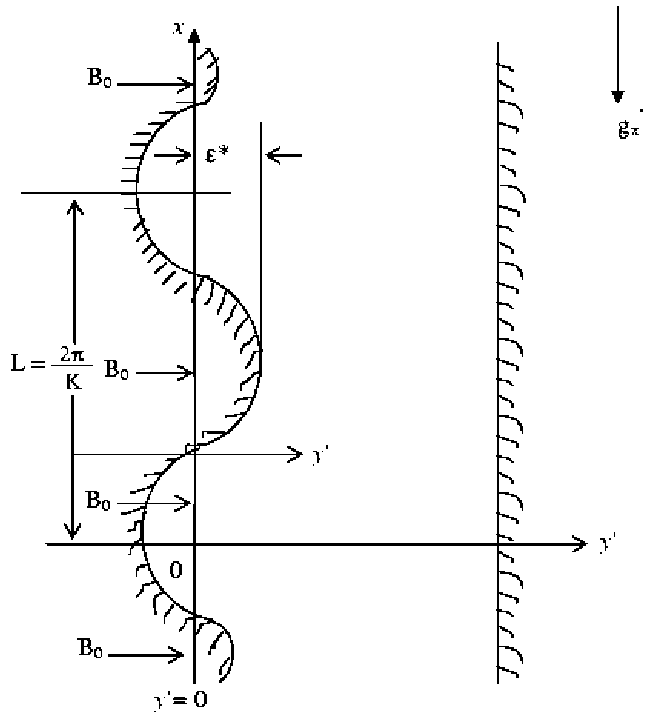

Figure 1:

2. Mathematical Formulation of the Problem

We consider the corrugated wall in which x'-axis is taken vertically upward, and parallel to the direction of buoyancy and the y'-axis is normal to it. The corrugated wall is represented by $y' = \in^* \cos Kx' (| \in^* | < 1)$ and $y' = d$ (flat wall). We study the free convection flow of an incompressible electrically conducting viscous fluid confined to the vertical corrugated wall and a parallel flat wall of finite length (Figure 1).

We make the following assumptions:

(i) the fluid properties are assumed to be constant and the Boussinesq approximation will be used so that the density variation is retained only in the buoyancy term;

(ii) the flow is laminar and two-dimensional;

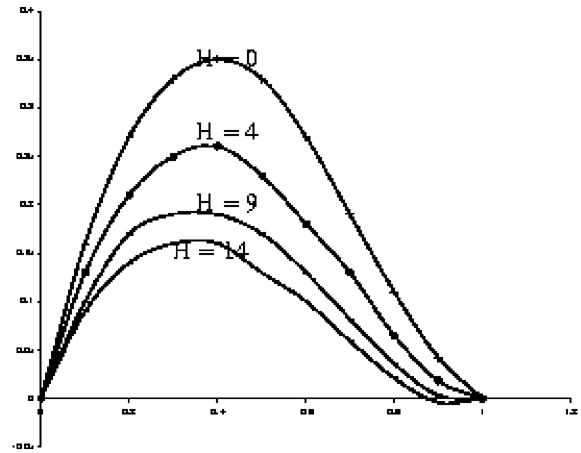

Figure 2:

(iii) the viscous and magnetic dissipative effects are neglected in the energy equation;

(iv) the volumetric heat source/sink term in the energy equation is constant;

(v) the wavelength L of the corrugated wall is large such that K, the wave number is small;

(vi) the electric field is zero; and the induced magnetic field is negligble compared to the applied magnetic field.

With these assumptions, the steady flow and heat transfer in a viscous incompressible conducting fluid are governed by the momentum equations, continuity equation and the energy equation in the form:

$$\rho\left(u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial P'}{\partial x'}+\mu\left(\frac{\partial^2 u'}{\partial x'^2}+\frac{\partial^2 u'}{\partial y'^2}\right)-\sigma B_0^2 u'-\rho g_{x'}, \tag{1}$$

$$\rho\left(u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}\right)=-\frac{\partial P'}{\partial y'}+\mu\left(\frac{\partial^2 v'}{\partial x'^2}+\frac{\partial^2 v'}{\partial y'^2}\right), \tag{2}$$

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0, \tag{3}$$

$$\rho C_p\left(u'\frac{\partial T}{\partial x'}+v'\frac{\partial T}{\partial y'}\right)=k\left(\frac{\partial^2 T}{\partial x'^2}+\frac{\partial^2 T}{\partial y'^2}\right)+Q, \tag{4}$$

where u', v' are the velocity components, T is the temperature, P' is the pressure, B_0 is the transverse magnetic field, σ is the coefficient of electric conductivity, Q is the constant heat addition/absorption, and the other symbols have

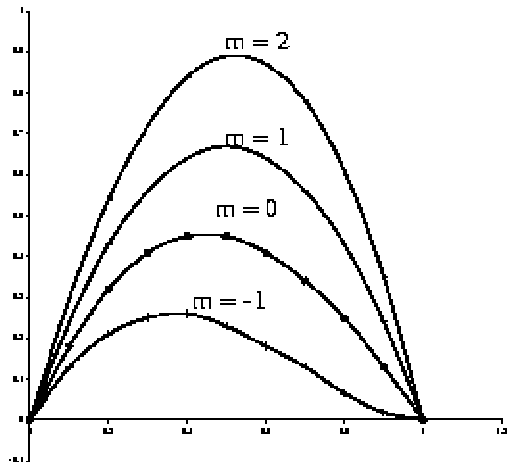

Figure 3:

their usual meanings. The relevant boundary conditions of the problem are:

$$\left.\begin{array}{ll} u' = v' = 0, \quad T = T_c \text{ at } y' = \epsilon^* \cos Kx', \\ u' = v' = 0, \quad T = T_1 \text{ at } y' = d. \end{array}\right\} \tag{5}$$

We next introduce the nondimensional flow and heat transfer variables as

$$(x, y) = \frac{1}{d}(x', y'), \quad (u, v) = \frac{d}{\nu}(u', v'),$$

$$L = Kd, \overline{P} = \frac{P'd^2}{\rho\nu^2}, \quad \epsilon = \frac{\epsilon^*}{d}, \quad \overline{P_s} = \frac{P_s' d^2}{\rho\nu^2},$$

and

$$\theta = \frac{T - T_s}{T_c - T_s}, \quad T_c - T_s \neq 0,$$

where the subscript s denotes quantities in the static fluid condition and subscript c denotes corrugated wall condition, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity.

Then on using the well known Boussinesq approximation

$$\rho_s = \rho + \rho\beta(T_c - T_s)\theta,$$

equation (1) in the static fluid condition and by the method of perturbation

$$\left.\begin{array}{l} u(x, y) = u_0(y) + u_1(x, y), \ v(x, y) = v_1(x, y), \\ \overline{P}(x, y) = \overline{P_0}(x) + \overline{P_1}(x, y), \\ \theta(x, y) = \theta_0(y) + \theta_1(x, y), \end{array}\right\} \tag{6}$$

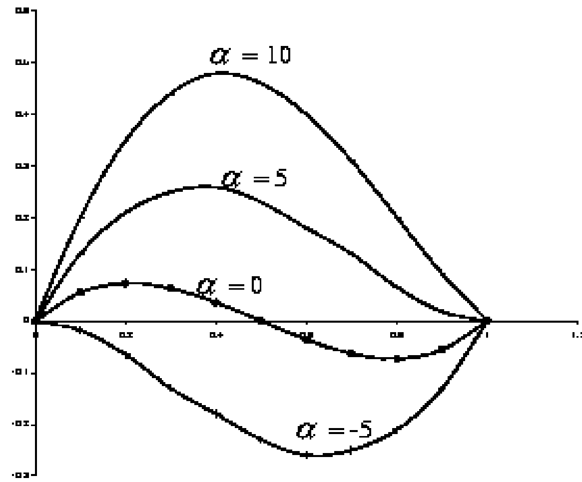

Figure 4:

equations (1) to (4) become

$$\frac{d^2u_0}{dy^2} - Hu_0 + G\theta_0 = C, \quad \frac{d^2\theta_0}{dy^2} = -\alpha, \tag{7}$$

to the zeroth-order and

$$u_0\frac{\partial u_1}{\partial x} + v_1\frac{du_o}{dy} = -\frac{\partial \overline{P_1}}{\partial x} + \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + G\theta_1 - Hu_1, \tag{8}$$

$$u_0\frac{\partial v_1}{\partial x} = -\frac{\partial \overline{P_1}}{\partial y} + \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}, \tag{9}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0, \tag{10}$$

$$P_r(u_0\frac{\partial \theta_1}{\partial x} + v_1\frac{d\theta_0}{dy}) = \frac{\partial^2 \theta_1}{\partial x^2} + \frac{\partial^2 \theta_1}{\partial y^2} \tag{11}$$

to the first-order, where $C = \frac{\partial}{\partial x}(P_0 - P_s)$ is taken to be zero (cf. [7]),

$\alpha = \dfrac{Qd^2}{k(T_c - T_s)}$ - the nondimensional heat source/sink parameter,

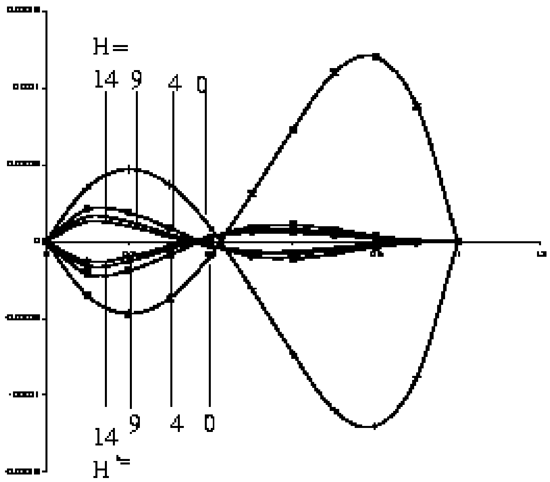

Figure 5:

$$P_r = \frac{\mu C_p}{k} \quad \text{- the Prandtl number,}$$

$$\epsilon = \frac{\epsilon^*}{d} \quad \text{- the dimensionless amplitude,}$$

$$m = \frac{T_1 - T_s}{T_c - T_s} \quad \text{- the wall-temperature ratio,}$$

$$H = \frac{\sigma B_0^2}{\rho \nu} \quad \text{- the magnetic field parameter,}$$

$$G = \frac{d^2 g_{x'} \beta (T_c - T_s)}{\nu^2} \text{ - the Grashof number,}$$

and β is the volumetric coefficient of the thermal expansion. With the help of (6) the boundary conditions (5) can be easily simplified to

$$\left.\begin{array}{l} u_0 = 0, \quad \theta_0 = 1 \quad \text{on} \quad y = 0, \\ u_0 = 0, \quad \theta_0 = m \quad \text{on} \quad y = 1, \end{array}\right\} \tag{12}$$

$$\left.\begin{array}{l} u_1 = -u_0', \quad v_1 = 0, \quad \theta_1 = -\theta_0' \quad \text{on} \quad y = 0, \\ u_1 = 0, \quad v_1 = 0, \quad \theta_1 = 0 \quad \text{on} \quad y = 1, \end{array}\right\} \tag{13}$$

where a prime denotes differentiation with respect to y.

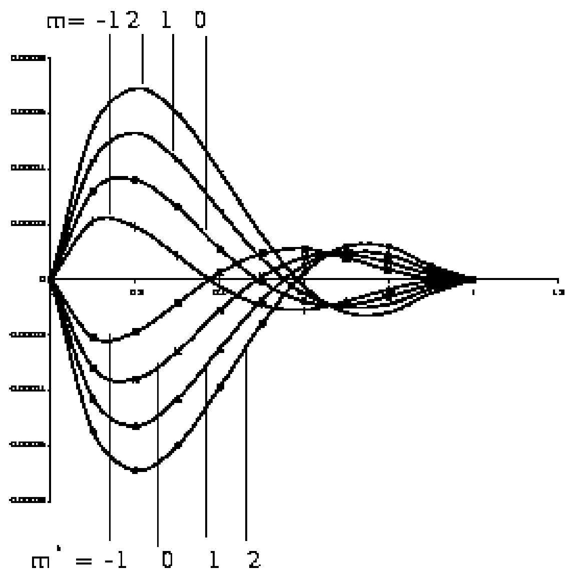

Figure 6:

Introducing the stream function $\bar{\psi}_1$ in the usual way

$$u_1 = -\frac{\partial \bar{\psi}_1}{\partial y}, \quad v_1 = \frac{\partial \bar{\psi}_1}{\partial x},$$

the equation (10) is satisfied identically. On eleminating $\overline{P_1}$ from (8) and (9), and assuming

$$\bar{\psi}_1(x, y) = \in e^{iLx}\psi(L, y), \quad \theta_1(x, y) = \in e^{iLx}\phi(L, y), \tag{14}$$

(perturbation series expansion for small wavelength L in which terms of exponential order arise) from which we infer

$$u_1(x, y) = - \in e^{iLx}\psi'(L, y), \quad v_1 = \in iLe^{iLx}\psi(L, y), \tag{15}$$

equations (8), (9) and (11) yield

$$\psi^{iv} - iLu_0\psi'' + iL^3u_0\psi - iLu_0'' - 2L^2\psi'' + L^4\psi - H\psi'' = G\phi', \tag{16}$$

and

$$\phi'' - L^2\phi = P_r iL(u_0\phi + \psi\theta'), \tag{17}$$

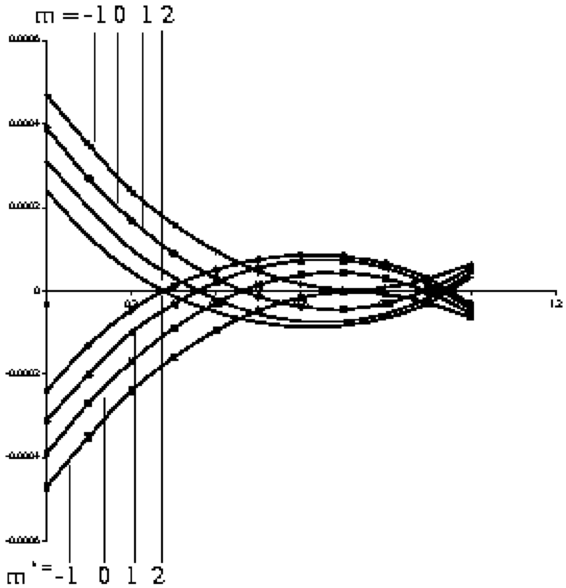

Figure 7:

subject to the boundary conditions (in terms of Ψ_1):

$$\left.\begin{array}{l} \dfrac{\partial \Psi_1}{\partial y} = -u_0^{'}, \quad \dfrac{\partial \Psi_1}{\partial x} = 0, \quad \theta_1 = -\theta_0^{'} \quad \text{on} \quad y = 0, \\ \dfrac{\partial \Psi_1}{\partial y} = 0, \quad \dfrac{\partial \Psi_1}{\partial x} = 0, \quad \theta_1 = 0 \quad \text{on} \quad y = 1. \end{array}\right\} \tag{18}$$

If we consider small values of L (or $K \ll 1$), then substituting

$$\psi(L, y) = \sum_{j=0}^{\infty} L^j \psi_j, \quad \phi(L, y) = \sum_{j=0}^{\infty} L^j \phi_j \qquad (j = 0, 1, 2...,),$$

into (16), (17) and (18) gives, restricting to real part and taken to order of L^2, the following sets of ordinary differential equations and corresponding boundary conditions:

$$\psi_0^{iv} - H\psi_0'' = G\phi_0', \qquad \phi_0'' = 0, \tag{19}$$

$$\left.\begin{array}{ll} \psi_1^{iv} & -i(u_0\psi_0'' - u_0''\psi_0) - G\phi_1' - H\psi_1'' = 0, \\ & \phi_1'' = P_r i(u_0\phi_0 + \psi_0\theta_0'), \end{array}\right\} \tag{20}$$

$$\left.\begin{array}{ll} \psi_2^{iv} & +i(\psi_1 u_0'' - u_0\psi_1'') - 2\psi_0'' - G\phi_2' - H\psi_2'' = 0, \\ & \phi_2'' = P_r i(u_0\phi_1 + \psi_1\theta_0') + \phi_0, \end{array}\right\} \tag{21}$$

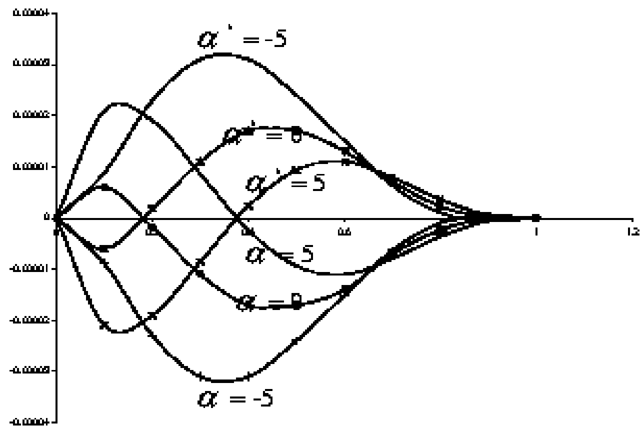

Figure 8:

and

$$\left.\begin{array}{rl}\psi_0' & = u_0', \psi_0 = 0, \phi_0 = -\theta_0' \quad \text{on} \quad y = 0, \\ & \psi_0' = 0, \quad \psi_0 = 0, \quad \phi_0 = 0 \quad \text{on} \quad y = 1,\end{array}\right\} \tag{22}$$

$$\left.\begin{array}{l}\psi_j' = 0, \quad \psi_j = 0, \quad \phi_j = 0 \quad \text{on} \quad y = 0, \\ \psi_j' = 0, \quad \psi_j = 0, \quad \phi_j = 0 \quad \text{on} \quad y = 1,\end{array}\right\} \quad \text{for } j \geq 1\,. \tag{23}$$

3. Results and Discussion

Results for equations (7), (19) to (21) consistent with the boundary conditions (12), (22) and (23) have been obtained analytically but not presented here.

The expressions u_0, θ_0, the zeroth-order solutions, u_1, v_1 and θ_1, the first-order solutions or the perturbed parts arising out of small roughness of a wall of the channel and u and θ, the total solutions, have been evaluated numerically at various values of y for several sets of values of the parameters H, m, α and P_r. Here we took L = 0.01, G = 5, ϵ (= -0.025, 0.025) and P_r (= 0.5, 0.71 and 0.92). H varies with the flow. The other quantities α and m are varied to stimulate physically realistic situations. When it is postulated that the average of the temperatures of the two walls is equal to that of the static fluid it means $m = -1$. $m = 0$ implies that the temperature of the static fluid is equal to that of the flat wall, $m = 1$ means equal wall temperatures while $m = 2$ indicates that wall temperatures are unequal. In the absence of heat source we have $\alpha = 0$ while $\alpha = 5$ corresponds to source and $\alpha = -5$ gives sink.

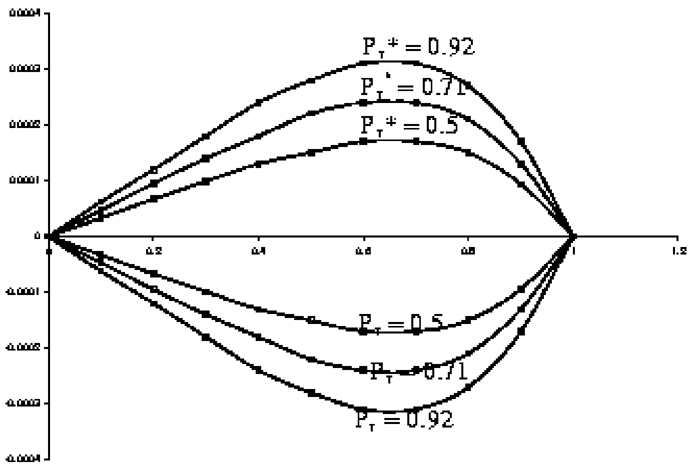

Figure 9:

Figure 2 to Figure 4, the results of which are naturally applicable to the case of a channel both of whose walls are flat, present profiles for the zeroth-order velocity u_0 for several values of the magnetic field parameter H, the wall-temperature ratio m and heat source/sink parameter α respectively. From Figure 2, it is obvious that as H increases, the Lorentz force, which oposes the flow, also increases and leads to enhanced deceleration of the flow. This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free-convection flow. From Figure 3, it is clear that with increase in wall-temperature ratio m, the magnitude of the fluid velocity u_0 increases across the entire channel width. Qualitative similar behaviour of the fluid velocity occurs with an increase in α (Figure 4). We observe from Figure 4 that when $\alpha = 0$ (no heat source or sinks) the fluid velocity u_0 increases steadily in the first half of the channel, while in the other half u_0 is a decreasing function of y. We notice further that in the presence of heat sinks ($\alpha = -5 < 0$) the velocity u_0 decreases from its value at the wall $y = 0$ to a minimum velocity at around $y = 0.6$ and then increasing steadily to its value at $y = 1$. In the presence of heat sources ($\alpha = 5 > 0$) the behaviour of the fluid velocity is the exact opposite of that observed in the case of heat sinks ($\alpha < 0$). It is obvious that from (7b) the (zeroth-order) temperatre θ_0 of the fluid is not affected by the magnetic field parameter H. Therefore, changes in the value of H will cause no changes in the profiles of θ_0. For this reason, no figure for θ_0 is presented herein.

Some profiles of u_1, v_1, and θ_1(calculated at $Lx = \frac{\pi}{2}$) depicting the effects of magnetic field, the wall-temperature ratio, the heat source/sink parameter and Prandtl number have been presented in Figure 5 to Figure 9.

Figure 5 shows the influence of magnetic field parameter H on the fluid velocity v_1 perpendicular to the channel length. We observe that the velocity v_1 decreases when H increases and become positive near the wavy wall while remaining negative in a larger region near the flat wall. The temperature profiles θ $(= \theta_0 + \theta_1)$ are not presented as they are not significantly influenced by the magnetic field.

Figure 6 and Figure 7 show the v_1 and u_1 profiles respectively with changes in the values of the wall-temperature ratio m. It is evident that with increase in the wall-temperature ratio m while u_1 profiles decrease, v_1 profiles increase.

Figure 8 exhibits v_1 profiles with changes in the values of the heat source/sink parameter α. As can be seen from Figure 8, v_1 profiles increase with increasing the heat source parameter α.

It is important to mention that Figure 2, Figure 3 and Figure 4 similarly describe the behaviour of the total fluid velocity u $(= u_0 + u_1)$ when the parameters H, m and α are varied, respectively.

The effects of the Prandtl number P_r on fluid temperature θ_1 is presented in Figure 9. It is observed that the temperature θ_1 decreases with an increase in Prandtl number P_r. No figures for v_1, u_1, u and θ are presented herein since changes in the values of P_r will cause no changes in the profiles of v_1, u_1, u and θ.

From Figure 5 to Figure 9, the non-asterisks are applicable to the case of constricted channel ($\epsilon < 0$) while the asterisks correspond to dilated channel($\epsilon >$ 00). After a keen perusal of Figure 5 to Figure 9, we arrive at the striking conclusion that the variation with each of the parameters H, m, α and P_r in the case of dilated channel ($\epsilon > 0$) the behaviour of the fluid flow characteristics is the exact opposite of that observed in the case of constricted channel ($\epsilon < 0$).

References

[1] S.A. Ajadi, A note on the unsteady flow of dussty viscous fluid between two parallel plates, *J. Appl. Math and Computing*, **18**, No. 1-2 (2005), 393-403.

[2] K. Halder, Application of Adomains approximation to blood flow through arteries in the presence of a magnetic field, *J. Appl. Math and Computing*, **12**, No. 2 (2003), 267-279.

[3] S.G. Lekoudis, A.H. Nayfeh, W.S. Saric, Compressible boundary layers over wavy walls, *Phys. Fluids*, **19**, No. 4 (1976), 514-519.

[4] M. Lessen, S.T. Gangwani, Effect of small amplitude wall waviness upon the stability of laminar boundary layer, *Phys. Fluids*, **19**, No. 4 (1976), 510-513.

[5] P.C. Ram, Recent development of heat and mass transfer in hydromagnetic flows, *Int. J. Energy Research*, **15** (1991), 691-713.

[6] P.N. Shankar, U.N. Sinha, The Rayleigh problem for a wavy wall, *J. Fluid Mech.*, **77**, No. 2 (1976), 243-256.

[7] K. Vajravelu, K.S. Sastri, Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, *J. Fluid Mech.*, **86**, No. 2 (1978), 365-383.