OPTIMIZATION MODEL OF THE HIGHWAY TOLL PLAZA TRAFFIC CAPACITY

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Abstract: The traffic capacity of the highway toll plaza is studied in this paper. This model is based on M/M/c and M/G/K queuing system. The process of the vehicles pass through the toll plaza can be divided into two parts: entering system and departing system. Considering the reality that the highway traffic flow increases year by year, two models are proposed in this paper: (a) Find out the optimal number of tollbooths which maximize the traffic capacity; (b) Divide the tollbooths into quick-path and normal-path tollbooths to improve the traffic capacity. The numerical experiments proved that the two models are efficient.

AMS Subject Classification: 65Y05, 65Y10, 49J35, 90C25, 90C30, 68W10, 68W25

Key Words: optimization model, highway toll plaza, traffic capacity

1. Introduction

The serving process for vehicles of the toll plazas can be considered as two queuing systems. The first queuing system (FQS) is the process from the time that vehicles enter the decrease speed field to the time that finish paying the fee. The second queuing system (SQS) is the process from the time that finish paying the fee to the time that vehicles leave the speed up field. The output of
Figure 1: Construction of the model one. From Figure 1, one can see that the system consisted of decrease speed field, tollbooths and speed up field. The number of tollbooths is much larger than the number of lanes.

FQS is the input of SQS. The tollbooths are the connected factors of these two queuing systems. The number of tollbooths and the average serving time are the critical factors for enhance the efficiency of the two systems. The influence of the number of tollbooths and the average serving time are called service efficiency (SE). Higher the SE is, shorter the average waiting time will be. Then, the input of SQS would increase greatly, but the demand for leaving the toll plaza can not be satisfied in time and the traffic jams will happen. When the arrival condition of vehicles and the serving time of SQS are fixed, there exist upper and lower bounds of the system SE. Because the traffic capacity of the toll plaza would like to be improved because the traffic flow intensities trend to increase by time, we design the system in which some normal-paths would be converted to quick-paths. These two queuing systems provide particular serve to the drivers who use the quick-paths. The quick-paths and the normal-paths exist at the same time, which serve for different patterns of paying the fee. The former queuing system is designed for the drivers that paying in cash, while the latter is designed for the drivers who use electric toll card (ETC). In the system with quick-paths, the lay out ratio of the two types of paths and the serving time of the two queuing systems must be adjusted to improve the maximum traffic capacity. The two queuing systems can be considered as multi-server queuing system.

According to the references [5, 3, 2, 1, 4], the input of FQS follows Poisson distribution, the serving time of the tollbooths follows the negative exponent distribution. At this time, the input of SQS just follows Poisson distribution.
and the output follows negative exponent or normal distribution. The M/M/c and M/G/K queuing systems models can be used to solve these two queuing systems.

2. Symbols and Definitions

The symbols used throughout this paper are present in the following table:

<table>
<thead>
<tr>
<th>Var.</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>Traffic capacity of the system</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Average number of the coming vehicles</td>
</tr>
<tr>
<td>$\lambda_{max}$</td>
<td>Average serving time of the SQS</td>
</tr>
<tr>
<td>$\lambda_{maxq}$</td>
<td>Average serving time of SQS quick-path</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Average number of the coming vehicles of FQS</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Average served vehicles of per tollbooth</td>
</tr>
<tr>
<td>$r$</td>
<td>Serving time ratio of quick-path to the normal-path</td>
</tr>
<tr>
<td>$c$</td>
<td>Number of tollbooths</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Number of quick-path tollbooths</td>
</tr>
<tr>
<td>$L_q$</td>
<td>Average waiting line length</td>
</tr>
<tr>
<td>$W_q$</td>
<td>Average waiting time</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of lanes</td>
</tr>
<tr>
<td>$n$</td>
<td>Ratio of using quick-path to the total tollbooths</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the buffer field</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>The FQS served ratio</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>The SQS served ratio</td>
</tr>
<tr>
<td>$W_{qi}$</td>
<td>Average waiting time of FQS in the normal-path</td>
</tr>
<tr>
<td>$W_{qo}$</td>
<td>Average waiting time of SQS in the quick-path</td>
</tr>
<tr>
<td>$L_{qi}$</td>
<td>Average waiting line length of FQS</td>
</tr>
<tr>
<td>$L_{qo}$</td>
<td>Average waiting line length of SQS</td>
</tr>
</tbody>
</table>
3. Construction of the Model

Based on the M/M/c queuing system, the following optimization model is presented.

\[
\begin{align*}
\max & \quad \frac{3000}{W_{q_i} + W_{q_o} + \mu}, \\
\text{s.t.} & \quad \rho_i < 1, \\
& \quad \rho_o < 1, \\
& \quad L_{qi} < 1, \\
& \quad L_{qo} < 1.
\end{align*}
\]

The optimal model (1) is the basic model throughout this paper. This model can be used to construct four models and give the optimal solutions. All maximum traffic capacity can be given by these models.

Figure 2: Construction of the model two. One can see that except for the Quick-path, the model two is similar to the model one. The Quick-path can make the vehicles pass the tollbooths in a very short time.

3.1. Model One

The number of the tollbooths are much larger than the lanes. All vehicles choose tollbooths freely. The model is used to obtain the optimal solution and calculate the average length of waiting queue (see Figure 1).

The vehicles coming into the decrease speed field are the customers of FQS. The variables of the system are \( \lambda = N_\lambda, \ c = M. \ \mu \) is determined by the real data.
The vehicles coming out of the tollbooths are the customers of SQS. The variables of the system are \( \lambda = M\mu, \mu = \lambda_{\text{max}}, c = N \). The objective function of the model one is

\[
T = \frac{3600}{W_{qi} + W_{qo} + \mu^{-1}}. \tag{2}
\]

\( W_{qi} \) and \( W_{qo} \) can be expressed by (1).

The toll plaza obtains its maximum traffic capacity when the average waiting time reaches its minimum. The optimal solutions are calculated and presented in Figure 3 and Figure 4.

### 3.2. Model Two

In the model two, some of the tollbooths are set to be quick-paths which use ETC. Suppose the average serving time of the quick-path is much shorter than that of the normal-path. The character of the model two is the fee may be paid automatically by ETC. In order to make the vehicles pass the toll plaza as quickly as possible, the average time that the vehicles consume in the toll plaza system must be very short. We change some tollbooths to be quick-paths and their serving time is very short. The vehicles passing through the quick-paths must use ETC to pay the fee, the others passing through the normal-paths pay the fee in cash. The traffic capacity of the model two consists of the vehicles passing through the quick-paths and the normal-paths. Our task become to confirm the optimal ratio of the quick-paths to the normal-paths to maximize the system’s traffic capacity (see Figure 2).

The variables in the model two queuing system are \( \lambda = aN, \mu = r\mu, c = bM \). Because the traffic capacity of the quick-path tollbooths has been improved greatly, the average serving time in SQS also must be improved consequently. Let average serving time of SQS in the quick-path be \( \mu = \lambda_{\text{max}} \). The objective function of the model two is

\[
T = T_q + T_n = \frac{3600}{W_{qin} + W_{qon} + \mu^{-1}} + \frac{3600}{W_{qiq} + W_{qoq} + \mu_1^{-1}}. \tag{3}
\]

Because the total number of the tollbooths is equal to that of the model one, the variables in the normal-paths are \( \lambda = (1 - a)N, \mu = \mu, c = (1 - b)M \). The model two is described in Figure 2.
Figure 3: Average line length of FQS and SQS to different number of tollbooths. From Figure 3, one can get that when the number of tollbooths is 12 or 13, the average waiting time of FQS and SQS in the model one are both very short, which are less than 1 second.

4. Numerical Experiments

In the model one, when $\lambda_{\text{max}} = 1$, $\lambda = 0.4$, $\mu = 0.33$ and $N = 6$, one can obtain that when the number of tollbooths is equal to 13, the system reaches its maximum traffic capacity. The average line length of FQS and SQS are presented in Figure 3 and the maximum traffic capacity is presented in Figure 4.

5. Conclusion and Discussion

In summary, the optimization models of the highway toll plaza are presented in this paper. The basic idea of the models is that the queuing system of the toll plaza can be decomposed into two sub-systems. One is FQS, the other is SQS. Based on the two systems, four models are presented corresponding to different conditions. The parameters of the model one are adjusted, then the
Figure 4: The maximum traffic capacity to different number of tollbooths of model one. From Figure 4, one can get the conclusion that when the number of tollbooths is 12 or 13, the traffic capacity of the highway toll plaza reaches its maximum value.

The model can give the optimal solution to different parameter values. To different vehicles flow, the model one can adjust parameter values to improve its traffic capacity. The model two improves the traffic capacity efficiently. The traffic capacity of the model two is larger than the one obtained by model one.

References


