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PERIODICITY OF A HOLLING-TANNER PREDATOR-PREY SYSTEM WITH IMPULSIVE EFFECT

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Abstract: In this paper, a Holling-Tanner predator-prey system with impulsive ratio-harvested prey is investigated. Using Floquet theory and small amplitude perturbation skills, we obtain the condition under which the boundary periodic solution is stable or unstable. Further, using the Coincidence Degree Theorem and its related continuous theorem, we prove existence of the positive periodic solution of the system.

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1. Introduction

The simple Holling-Tanner predator-prey system is as follows:

$$\begin{cases} \dfrac{dx_1}{dt} = rx_1(1 - \dfrac{x_1}{K}) - \dfrac{mx_1}{A + x_1}x_2, \\ \dfrac{dx_2}{dt} = sx_2(1 - \dfrac{hx_2}{x_1}), \end{cases} \tag{1.1}$$

where r, m, s, h, A and K are positive constants, x_1 and x_2 are densities of prey and predator, respectively. System (1.1) assumes that the prey grows logistically with intrinsic growth rate r and carrying capacity K in the absence of predation. The predator consumes the prey according to Holling type-II functional response and grows logistically with intrinsic rate s and carrying capacity proportional to the population size of the prey. System (1.1) is an important and interesting model of predator-prey system in both biological and mathematical sense and is studied in many papers. For example, Hsu and Hwang in [5] carried out the stability analysis of position equilibrium point $E^* = (x_1^*, x_2^*)$ and obtained some conditions under which E^* is globally asymptotically stable in the interior of the first quadrant; in [6] showed that for some parameter range there exists a small-amplitude repelling periodic orbit enclosing a stable equilibrium and hence there are multiple limit cycles. Sunita Gakkhar et al [4] detected the abundance of steady state chaotic solutions when seasonality is super imposed on the system.

However, as Cushing [1] pointed out, that it is necessary and important to consider models with periodic ecological parameters or perturbations which might be quite naturally exposed (for example, those due to seasonal effects of weather, food supply, mating habits, hunting or harvesting seasons, etc.). There is evidence, for example, the seasonal variation in contact rates derives the dynamics of childhood disease epidemics [11]. Recently impulsive differential equations (IDE) have been introduced into almost every domain of applied sciences: chemotherapeutic treatment of disease [9], pulse vaccination [8], birth pulse [12], [10], population ecology [2], [14], [13], neural network [7], etc.

Unexceptionally, the models of predator-prey system in periodic forcing and impulsive effect environments have attracted new attention because of their propensity for chaos [14], [13]. But to our knowledge there have been no results on the Holling-Tanner predator-prey system with impulsive ratio-harvested prey. The main feature of the present paper is to study the oscillatory behavior of the following Holling-Tanner predator-prey system with impulse effect:

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= rx_1(1-\frac{x_1}{K}) - \frac{mx_1}{A+x_1}x_2, \\ \frac{dx_2}{dt} &= sx_2(1-\frac{hx_2}{x_1}), \end{aligned}\right\} & t \neq t_k = kT, \\ \triangle x_1 = -px_1, & t = t_k = kT, \end{cases} \tag{1.2}$$

where $0 < p < 1$, T is the period of impulsive effect.

The organizations of the paper are as follows. In the next section, we discuss the stability of boundary periodic solution under periodic pulsed conditions. In Section 3, using the Coincidence Degree Theorem and its related continuous theorem we prove the existence of positive periodic solution of system (2.1). In the last section, we give out the conclusion of this paper.

2. Stability of Boundary Periodic Solution

We write system (1.2) in a nondimensional form. Let

$$\begin{aligned} &\tilde{t} = rt, \quad \tilde{x_1}(\tilde{t}) = \frac{x_1(t)}{K}, \quad \tilde{x_2}(\tilde{t}) = \frac{mx_2(t)}{rK}, \\ &\delta = s/r, \qquad \beta = \frac{sh}{m}, \qquad a = A/K, \qquad \omega = rT. \end{aligned}$$

Then (1.2) takes the form

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= x_1 - x_1^2 - \frac{x_1x_2}{a+x_1}, \\ \frac{dx_2}{dt} &= x_2(\delta - \beta\frac{x_2}{x_1}), \end{aligned}\right\} & t \neq t_k = k\omega, \\ \triangle x_1 = -px_1, & t = t_k = k\omega, \end{cases} \tag{2.1}$$

where $0 < p < 1$, ω is the period of impulsive effect.

Lemma 2.1. *Both the nonnegative cone $R_+^2 := \{(x_1, x_2) \in R^2 | x_i \geq 0, i = 1, 2\}$ and the positive cone R_+^2 are positively invariant with respect to system (2.1).*

We know that system (2.1) has the nonnegative boundary ω-period solution

$$(x_{1e}(t), x_{2e}(t)) = (0, 0).$$

For system (2.1), we have the following results.

Theorem 2.1. *Let $x(t) = (x_1(t), x_2(t))$ be any solution of system (2.1) with $x(0) > 0$. Then we have that:*

(1) if $\mu_1 := (1-p)e^{\omega} < 1$, then system (2.1) has a unique globally asymptotically stable positive ω-periodic solution $(x_{1e}(t), x_{2e}(t))$, that is say, for any solution $x(t)$, we have $\lim_{t\to\infty} x(t) = (0,0)$;

(2) if $\mu_1 := (1-p)e^{\omega} > 1$, then system (2.1) has a boundary ω-periodic solution $(x_{1s}, 0)$, which is always unstable. At same time, we have that

$$\int_0^{\omega} x_{1s}(t)dt = \omega + \ln(1-p).$$

Proof. (1) From system (2.1), we can deduce that

$$\begin{aligned} x_1(t) &= x_1(0)(1-p)^n \exp\left(\int_0^t (1 - x_1(l) - \frac{x_2(l)}{a + x_1(l)})dl\right) \\ &\leq x_1(0)(1-p)^n \exp[(n+1)\omega], \quad t \in (n\omega, (n+1)\omega]. \end{aligned}$$

If $\mu_1 := (1-p)e^{\omega} < 1$, then $\lim_{t\to\infty} x_1(t) = 0$. For any $\varepsilon_1 > 0$, there exists $T_1 > 0$ such that for any $t > T_1$, $x_1(t) < \varepsilon_1$. We consider the following comparing system

$$\frac{dx_2(t)}{dt} \leq x_2(t)(\delta - \beta \frac{x_2(t)}{\varepsilon_1}).$$

Hence we have that there exists $T_2 > T_1$ such that for any $t > T_2$, $x_2(t) < 2\frac{\delta}{\beta}\varepsilon_1$ holds. So we have that $\lim_{t\to\infty} x_2(t) = 0$.

(2) Let us consider the impulsive equations

$$\begin{cases} \dfrac{dx_1}{dt} = x_1 - x_1^2(t), & t \neq t_k = k\omega, \\ \triangle x_1 = -px_1, & t = t_k = k\omega. \end{cases} \tag{2.2}$$

By simply calculating, we know system (2.2) has a unique positive ω-periodic solution

$$\begin{aligned} x_{1s}(t) &= \frac{x_{1s}(0)\exp[t-(k-1)\omega]}{1 - x_{1s}(0) + x_{1s}(0)\exp[t-(k-1)\omega]}, \quad t \in ((k-1)\omega, k\omega], \\ x_{1s}(0) &= 1 - \frac{pe^{\omega}}{e^{\omega}-1}. \end{aligned} \tag{2.3}$$

If $\mu_1 := (1-p)e^{\omega} > 1$, it follows from (2.2) that $\int_0^{\omega} x_{1s}(t)dt = \omega + \ln(1-p) > 0$. We calculate the multiplier λ_1 of the ω-period solution $x_{1s}(t)$ and obtain that

$$\begin{aligned}\lambda_1 := (1-p)\exp(\int_0^\omega (1-2x_{1s}(t))dt) \\ &= (1-p)\exp(\omega - 2\ln(1-p) - 2\omega) \\ &= \exp(-\omega - \ln(1-p)) = \exp(-\int_0^\omega x_{1s}(t)dt) < 1. \quad (2.4)\end{aligned}$$

Hence the ω-period solution $x_{1s}(t)$ of system (2.2) is globally asymptotically stable.

We know that the ω-period solution $(x_{1s}(t), 0)$ is a boundary periodic solution of system (2.1). The local stability of periodic solution $(x_{1s}(t), 0)$ may be determined by considering the behavior of small amplitude perturbations of the solution. Define

$$x_1(t) = u(t) + x_{1s}(t), x_2(t) = v(t),$$

there may be written

$$\begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \end{pmatrix},$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} 1-2x_{1s} & \frac{x_{1s}}{a+x_{1s}} \\ 0 & \delta \end{pmatrix} \Phi(t).$$

Hence the fundamental solution matrix is

$$\Phi(\omega) = \begin{pmatrix} \exp(\int_0^\omega (1-2x_{1s}(t))dt) & * \\ 0 & e^{\delta\omega} \end{pmatrix}.$$

It is no need to give the exact form of $(*)$ as it is not required in the analysis that follows. The linearization of impulsive subsystem (2.1) gives

$$\begin{pmatrix} u(n\omega^+) \\ v(n\omega^+) \end{pmatrix} = \begin{pmatrix} 1-p & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(n\omega) \\ v(n\omega) \end{pmatrix}.$$

We denote that

$$M = \begin{pmatrix} 1-p & 0 \\ 0 & 1 \end{pmatrix} \Phi(\omega),$$

hence we have

$$M = \begin{pmatrix} (1-p)\exp(\int_0^\omega (1-2x_{1s}(t))dt) & ** \\ 0 & e^{\delta\omega} \end{pmatrix}.$$

It is no need to give the exact form of $(**)$ as it is not required in the analysis that follows. The eigenvalues of the matrix M are the eigenvalues $\lambda_1 = (1-p)\exp(\int_0^\omega(1-2x_{1s}(t))dt)$, $\lambda_2 = e^{\delta\omega}$. From (2.4), $\lambda_1 < 1$. But $\lambda_2 > 1$ is obvious. Hence, the periodic solution $(x_{1s}(t), 0)$ is unstable. We complete the proof. $\square$

3. Existence of Positive Periodic Solution

In this section, we shall confine ourselves to prove that system (2.1) has an ω-periodic solution under some conditions. The proof of the theorem is based on the Gains and Mawhin's Continuation Theorem, see [3]. For the reader's convenience, we introduce this theorem as follows.

Set X and Y are two real Banach spaces. Consider the operator equation

$$Lx = Nx\,,$$

where $L : \operatorname{Dom} L \subset X \to Y$ is a linear bounded operator, $N : Y \to Y$ is a continuous operator. The mapping L will be called a Fredholm mapping of index zero if $\dim \operatorname{Ker} L = \operatorname{codim} \operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Y. If L is a Fredholm mapping of index zero, there exist continuous projectors $P : X \to X$ and $Q : Y \to Y$ such that

$$\operatorname{Im} P = \operatorname{Ker} L, \quad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I-Q).$$

It follows that $L|_{\operatorname{dom} L \subset \operatorname{Ker} P} : (I-P)X \to \operatorname{Im} L$ is invertible. We denote the inverse of that map by K_p. If Ω is an open subset of X, the mapping N will be called L-compact on Ω if $QN(\overline{\Omega})$ is bounded and $K_p(I-Q)N : \overline{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$ there exist isomorphisms $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 3.1. *Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$. Assume:*

(a) for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L, Lx \neq \lambda Nx$;

(b) for each $x \in \partial\Omega \cap \operatorname{Ker} L, QNx \neq 0$;

(c) $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$.

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

In this section, for convenance, we denote the impulsive time points as $t_k = (k - \frac{1}{2})\omega$, $k = 1, 2, \cdots$.

Lemma 3.2. *$x^*(t)$ is an ω-periodic solution of (2.1) with strictly positive* *components if and only if* $\ln x^*(t)$ *is an* ω*-periodic solution of*

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= 1 - \exp(x_1(t)) - \frac{\exp(x_2(t))}{a + \exp(x_1(t))}, \\ \frac{dx_2}{dt} &= \delta - \beta\frac{\exp(x_2(t))}{\exp(x_1(t))}, \end{aligned}\right\} & t \neq t_k = (k - \frac{1}{2})\omega, \\ \left.\begin{aligned} \triangle x_1 &= x_1(t_k^+) - x_1(t_k) = \ln(1-p), \\ \triangle x_2 &= x_2(t_k^+) - x_2(t_k) = 0, \end{aligned}\right\} & t = t_k = (k - \frac{1}{2})\omega, \end{cases} \tag{3.1}$$

where $\ln\{x^*(t)\} = (\ln\{x_1^*(t)\}, \ln\{x_2^*(t)\})$.

With the help of Lemma 3.1, we can explore the existence of positive periodic solution of (2.1) in a more direct way. In order to apply Lemma 3.1, we shall first embed our existence problem into the frame of the continuation theorem. Define

$$C[0,\omega;t_1] = \Big\{ x : [0,\omega] \to R^2 \Big| x(t) \text{ is continuous with respect to } t \neq t_1; \\ x(t+0) \text{ and } x(t+0) \text{ exist at } t_1;\ x(t_1) = x(t_1 - 0). \Big\}$$

We introduce

$$X = \{C[0,\omega;t_1] | x(0) = x(\omega)\}, \quad \| x \|_c = \left\{ \sup_{t\in[0,\omega]} \| x \|, x \in X \right\},$$

where $\| \cdot \|$ is any norm in R^2, and

$$Z = X \times R^2,\ \| z \|_Z = \| z \|_c + \| y \|,\ z = (x,y) \in Z,\ \text{with } x \in X,\ y \in R^2,$$

where $\| \cdot \|$ is any given norm of R^2. Then it is standard to show that both X and Z are Banach spaces when they are endowed with the norms $\| \cdot \|_c$ and $\| \cdot \|_Z$, respectively.

Let $\mathrm{Dom}\, L = X$, $L : \mathrm{Dom}\, L \to Z$, $Lx = (x', \triangle x(t_1))$. We define $N : X \to Z$ as follows

$$Nx = \left\{ \begin{pmatrix} f_1 := 1 - \exp(x_1(t)) - \dfrac{\exp(x_2(t))}{a + \exp(x_1(t))} \\ f_2 := \delta - \beta\dfrac{\exp(x_2(t))}{\exp(x_1(t))} \end{pmatrix}, \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right\}, \quad x \in X.$$

We denote $b := (b_1, 0)^T, b_1 = \ln(1-p)$. It is trivial to see that L is a bounded linear operator and $\mathrm{Ker}\,(L) = R^2$,

$$\mathrm{Ker}\,(L) = \{x \in X | x = h \in R^2, t \in [0, \frac{\omega}{2}]; x = g \in R^2, t \in (\frac{\omega}{2}, \omega]\},$$

$$\mathrm{Im}\, L = \left\{ z = (f,b) \in Z | \int_0^\omega f(s)ds + b = 0 \right\},$$

and $\operatorname{Im} L$ is closed in Z, therefore, L is a Fredholm mapping of index zero. Define two projects P, Q as

$$Px = \frac{1}{\omega}\int_0^\omega x(s)ds, \qquad x \in X;$$
$$Qz = Q(f,b) = \left(\frac{1}{\omega}[\int_0^\omega x(s)ds + b], 0\right).$$

It is not difficult to show that P and Q are continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L \quad and \quad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I-Q),$$

and hence, the generalized inverse (to L) K_p exists. In the following, we first devote ourselves to deriving the explicit expression of $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$. Take $z = (f,b) \in \operatorname{Im} L$, then there exists an $x \in \operatorname{Dom} L \subset X$ such that

$$\dot{x}(t) = f(t), \quad t \neq t_k = (k - \frac{1}{2})\omega,$$
$$\triangle x(t) = b, \quad t = t_k = (k - \frac{1}{2})\omega.$$

Then direct integration produces

$$x(t) = \int_0^t f(s)ds + \sum_{t>t_k} b + x(0). \tag{3.2}$$

Note that $x(t) \in \operatorname{Ker} P$, i.e., $\int_0^\omega x(s)ds = 0$, which, together with (3.2), implies

$$\int_0^\omega \int_0^t f(s)dsdt + \omega x(0) = 0.$$

Then

$$x(t) = \int_0^t f(s)ds + \sum_{t>t_k} b - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)dsdt,$$

that is,

$$K_p z = \int_0^t f(s)ds + \sum_{t>t_k} b - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)dsdt\,.$$

Then

$$QNx = \left\{\frac{1}{\omega}[\int_0^\omega f(s)ds + b], 0\right\}, \quad x \in X.$$

$$K_p(I-Q)Nx \quad = \int_0^t f(s)ds - \frac{t}{\omega} + \sum_{t>t_k} b + (\frac{1}{2} - \frac{t}{\omega}) \int_0^\omega f(s)ds + \frac{b}{2} - \frac{1}{\omega}\int_0^\omega \int_0^l f(s)dsdl, \qquad x \in X.$$

It is easy to check that QN and $K_p(I-Q)N$ are continuous by the Lebesgue Convergence Theorem and moreover, by the Arzera-Ascoli Theorem, $QN(\bar{\Omega})$ and $\overline{K_p(I-Q)N(\bar{\Omega})}$ compact for any open bounded set $\Omega \subset X$. Hence, N is L-compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

Now we are at the point to find an open, bounded subset Ω which has been given in the above paragraph to satisfy the three conditions of Gaines and Mawhin's Continuation Theorem.

Theorem 3.1. *For system (2.1), if $\omega + \ln(1-p) > 0$, then (2.1) has at least one ω-periodic solution with strictly positive components.*

Proof. To complete the proof, we only need to search for an appropriate open bounded subset $\Omega \subset X$ verifying all the requirements in Lemma 3.1.

Consider the operator equation $Lx = \lambda Nx$, $\lambda \in (0,1)$, i.e.,

$$\begin{cases} \left.\begin{aligned} \frac{dx_1}{dt} &= \lambda(1-\exp(x_1(t)) - \frac{\exp(x_2(t))}{a+\exp(x_1(t))}), \\ \frac{dx_2}{dt} &= \lambda(\delta - \beta\frac{\exp(x_2(t))}{\exp(x_1(t))}), \end{aligned}\right\} & t \neq (k-\frac{1}{2})\omega, \\ \triangle x_1 = \lambda \ln(1-p), & t = (k-\frac{1}{2})\omega. \end{cases}$$

Assume that $x = (x_1(t), x_2(t)) \in X$ is a solution of (3.1) for a certain $\lambda \in (0,1)$. Integrating (3.1) over the interval $[0,\omega]$, we obtain

$$\begin{cases} \lambda \int_0^\omega (1-\exp(x_1(t)) - \frac{\exp(x_2(t))}{a+\exp(x_1(t))})dt + \lambda\ln(1-p) = 0, \\ \lambda \int_0^\omega (\delta - \beta\frac{\exp(x_2(t))}{\exp(x_1(t))})dt = 0. \end{cases} \tag{3.3}$$

That is

$$\begin{cases} \int_0^\omega (1-\exp(x_1(t)) - \frac{\exp(x_2(t))}{a+\exp(x_1(t))})dt + \ln(1-p) = 0, \\ \int_0^\omega (\delta - \beta\frac{\exp(x_2(t))}{\exp(x_1(t))})dt = 0. \end{cases} \tag{3.4}$$

Notice the assumption $\omega+\ln(1-p)>0$. From (3.3) and (3.4), one can derive

$$\int_0^\omega |\dot{x}_1(t)|dt \le \omega + \int_0^\omega \exp(x_1(t))dt + \int_0^\omega \frac{\exp(x_2(t))}{a+\exp(x_1(t))}dt$$
$$= 2(\omega+\ln(1-p)),$$

$$\int_0^\omega |\dot{x}_2(t)|dt \le \int_0^\omega (\delta+\beta\frac{\exp(x_2(t))}{\exp(x_1(t))})dt = 2\delta\omega.$$

Since $x(t)\in X$, there exist $\xi_i\in[0,\omega]$ and $\zeta_i\in[0,\omega]$ such that

$$x_i(\xi_i)=\min_{t\in[0,\omega]} x_i(t), \quad x_i(\zeta_i)=\max_{t\in[0,\omega]} x_i(t).$$

We notice that

$$x_i(\zeta_i) - \int_0^\omega |\dot{x}_i(t)|dt \le x_i(t) \le x_i(\xi_i) + \int_0^\omega |\dot{x}_i(t)|dt.$$

By comparing theorem, notice that in system (2.1), $x_1(t)\le 1$, so in system (3.1), we can choose $\exp(x_1(\zeta_1))\le 1$. From (3.4), we have that

$$\begin{cases} \int_0^\omega (1-\exp(x_1(\xi_1)) - \frac{\exp(x_2(t))}{1+a})dt + \ln(1-p) \ge 0, \\ \int_0^\omega (\delta-\beta\frac{\exp(x_2(t))}{\exp(x_1(\xi_1))})dt \le 0. \end{cases} \tag{3.5}$$

So we have that

$$\frac{\delta}{\beta}\exp(x_1(\xi_1)) \le \int_0^\omega \exp(x_2(t))dt \le (1+a)[\omega-\omega\exp(x_1(\xi_1))+\ln(1-p)].$$

Hence we have

$$x_1(\xi_1) \le \ln(\frac{\beta(a+1)[\omega+\ln(1-p)]}{\beta\omega(a+1)+\delta}).$$

We consider that

$$\begin{cases} \int_0^\omega (1-\exp(x_1(\zeta_1)) - \frac{\exp(x_2(t))}{a})dt + \ln(1-p) \le 0, \\ \int_0^\omega (\delta-\beta\frac{\exp(x_2(t))}{\exp(x_1(\zeta_1))})dt \ge 0. \end{cases}$$

So we have that

$$a[\omega-\omega\exp(x_1(\zeta_1))+\ln(1-p)] \le \int_0^\omega (\exp(x_2(t)))dt \le \frac{\delta}{\beta}\exp(x_1(\zeta_1)).$$

Hence we have

$$x_1(\zeta_1) \geq \ln(\frac{\beta a[\omega + \ln(1-p)]}{\beta a\omega + \delta}).$$

We denote that

$$P_1 := \ln(\frac{\beta a[\omega + \ln(1-p)]}{\beta a\omega + \delta}) - 2\omega - 2\ln(1-p);$$
$$H_1 := \ln(\frac{\beta(a+1)[\omega + \ln(1-p)]}{\beta\omega(a+1) + \delta}) + 2\omega + 2\ln(1-p).$$

We have that

$$P_1 \leq x_1(\zeta_1) - \int_0^\omega |\dot{x}_1(t)|dt \leq x_1(t) \leq x_1(\xi_1) + \int_0^\omega |\dot{x}_1(t)|dt \leq H_1.$$

From (3.4), we have that

$$\int_0^\omega (\frac{\delta}{\beta} - \frac{\exp(x_2(\xi_2))}{\exp(x_1((t))})dt \geq 0.$$

So we have that

$$\exp(x_2(\xi_2)) \leq \frac{\delta}{\beta}\exp(x_1(\zeta_1)) \leq \frac{\delta}{\beta}\exp(H_1).$$

From (3.4), we have that

$$\int_0^\omega (\frac{\delta}{\beta} - \frac{\exp(x_2(\zeta_2)}{\exp(x_1(t))})dt \leq 0.$$

So we have that

$$\exp(x_2(\zeta_2)) \geq \frac{\delta}{\beta}\exp(x_1(\xi_1)) \geq \frac{\delta}{\beta}\exp(P_1).$$

We denote that

$$P_2 := \ln(\frac{\delta}{\beta}\exp(P_1)) - 2\frac{\delta\omega}{\beta},$$
$$H_2 := \ln(\frac{\delta}{\beta}\exp(H_1)) + 2\frac{\delta}{\beta}\omega.$$

We have that

$$P_2 \leq x_2(\zeta_2) - \int_0^\omega |\dot{x}_2(t)|dt \leq x_2(t) \leq x_2(\xi_2) + \int_0^\omega |\dot{x}_2(t)|dt \leq H_2.$$

From (3.5), it follows that

$$\sup_{t\in[0,\omega]} |x_i(t)| \le \sup\{|H_i|, |P_i|\} := M_i.$$

Clearly, M_i are independent of λ. Set $M = M_1 + M_2 + M_0$, where M_0 is taken sufficiently large such that $\| \ln\{x^*\} \| = \| (\ln\{x_1^*\}, \ln\{x_2^*\})^T \| = |\ln\{x_1^*\}| + |\ln\{x_2^*\}| < M_0$, then $\| x \|_c < M$.

Let $\Omega = \{x \in X : \| x \|_c < M$, it is clear that Ω verifies the requirement (1) in Lemma 3.1. When $x \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap R^2$, x is a constant vector in R^2 with $\| x \|_c = M$, then

$$QNx = \left\{ \begin{pmatrix} 1 - e^{x_1} - \dfrac{e^{x_2}}{a + e^{x_1}} + \dfrac{1}{\omega}\ln(1-p) \\ \delta - \beta\dfrac{e^{x_2}}{e^{x_1}} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\} \neq 0.$$

Take

$$J : \operatorname{Im} Q \to x, (f, 0) \to f,$$

then when $x \in \partial\Omega \cap \operatorname{Ker} L$, one obtains

$$JQNx = \begin{pmatrix} 1 - e^{x_1} - \dfrac{e^{x_2}}{a + e^{x_1}} + \dfrac{1}{\omega}\ln(1-p) \\ \delta - \beta\dfrac{e^{x_2}}{e^{x_1}} \end{pmatrix}.$$

Notice the condition $w + \ln(1-p) > 0$, the equation $JQNx = 0$ has unique solution (x_1^*, x_2^*), where

$$x_1^* := \ln(a - B - \frac{\delta}{\beta} + \sqrt{(a - B - \frac{\delta}{\beta})^2 + 4aB}),$$
$$x_2^* := \ln(\frac{\delta}{\beta}[a - B - \frac{\delta}{\beta} + \sqrt{(a - B - \frac{\delta}{\beta})^2 + 4aB}])),$$
$$B := 1 + \frac{1}{w}\ln(1-p) > 0.$$

We know that

$$\deg\{JQNx, \Omega \cap \operatorname{Ker} L, 0\} = \operatorname{sgn}\{-e^{x_1^*} + \frac{e^{x_1^*+x_2^*}}{(a + e^{x_1^*})^2} + \frac{e^{x_2^*}}{a + e^{x_1^*}}\} \neq 0.$$

By now we have proved that Ω verifies all the requirements in Lemma 2.1. Hence, (3.1) has at least one periodic solution $x^*(t) = (x_1^*(t), x_2^*(t))^T$ in $\operatorname{Dom} L \cap \bar{\Omega}$. Set $y^*(t) = \exp\{x^*(t)\}$, then $y^*(t) = (y_1^*(t), y_2^*(t))^T$ is an ω-periodic solution of (2.1) with strictly positive components. The proof is complete. □

4. Conclusion

In this paper, we introduced and studied the Holling-Tanner predator-prey system with impulsive ratio-harvested prey. By using Floquet Theorem and small amplitude perturbation skills, we have proved that if $\mu_1 := (1-p)e^{\omega} < 1$, the periodic solution $(0,0)$ is globally asymptotically stable; if $\mu_1 := (1-p)e^{\omega} > 1$, the boundary ω-periodic solution $(x_{1s}(t), 0)$ is always unstable and if $\omega + \ln(1-p) > 0$, system (2.1) has a positive ω-periodic solution.

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References

[1] J.M. Cushing, Two species competition in a periodic environment, *J. Math. Biol.*, **10** (1980), 348-400.

[2] L. Dong, L. Chen, L. Sun, Extinction and permanence of the predator-prey system with stocking of prey and harvesting of predator impulsively, *Mathematical Methods in the Applied Sciences*, **29** (2006), 415-425.

[3] R.E. Gaines, J.L. Mawhin, *Coincidence Degree, and Nonlinear Differential Equations*, Springer-Verlag, New York (1977).

[4] Sunita Gakkhar, Ra'id Kamel Naji, Chaos in seasonally perturbed ratio-dependent prey-predator system, *Chaos, Solitons and Fractals*, **15** (2003), 107-118.

[5] S.B. Hsu, T.W. Hwang, Global stability for a class of predator-prey systems, *SIAM J. Appl. Math.*, **55** (1995), 763-783.

[6] S.B. Hsu, T.W. Hwang, Hopf bifurcation analysis for a predator-prey system of Holling and Leslie type, *Taiwanese J. Math.*, **3** (1999), 35-53.

[7] Z. Huang, Q. Yang, X. Luo, Exponential stability of impulsive neural networks with time-varying delays, *Chaos, Solitons and Fractals*, In Press, Available online 27 July 2006.

[8] A. d'Onofrio, Stabil1ty properties of pulse vaccination strategy in the SIR epidemic model, *Bull. Math. Biol.*, **60** (2002), 1-26.

[9] J.C. Panetta, A mathematical model of periodically pulsed chemotherapy: tumor recurrence and metastasis in a competition environment, *Bull. of Math. Biol.*, **58** (1996), 425-447.

[10] M.G. Roberts, R.R. Kao, The dynamics of an infectious disease in a population with birth pulses, *Math. Biosci*, **149** (1998), 23-36.

[11] W.M. Schaffer, Can nonlinear dynamics clucidate mechanisms in ecology and epidemiology? *IMA J. Math. Appl. Med. Biol.*, **2** (1985), 221-252.

[12] S. Tang, L. Chen, Density-dependent birth rate, birth pulse and their population dynamic consequences, *J. Math. Biol.*, **44** (2002), 185-199.

[13] F. Wang, G. Zeng, Chaos in a Lotka-Volterra predator-prey system with periodically impulsive ratio-harvesting the prey and time delays, *Chaos, Solitons and Fractals*, In Press, Available online 7 February 2006.

[14] Wang F, Zhang S, Chen L, Sun L, Permanence and complexity of a three species food chain with impulsive effect on the top predator, *Int. J. Nonlinear Sci. Numer. Simul.*, **6** (2005), 169-180.