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SEMIDIRECT PRODUCTS AND WREATH PRODUCTS ON LEFT AND RIGHT CLIFFORD EVENTUALLY REGULAR SEMIGROUPS

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Abstract: We introduce the concept of eventually regular semigroups S with the property $eS \subseteq Se$ or $Se \subseteq eS$ for all idempotents $e \in S$ that include all left and right Clifford eventually regular semigroups. Necessary and sufficient conditions of the semidirect products and wreath products of such semigroups are given.

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1. Introduction and Preliminaries

Nico [3] first characterized the semidirect product on regular semigroups, next Saito [4] characterized the semidirect product on orthodox semigroups. Since that, the research on the semidirect product of semigroups has aroused broadly attention. A semigroup S is called an eventually regular semigroup (also called quasi-regular semigroup) if for every $a \in S$, there exists a positive integer n such that a^n is regular. The class of eventually regular semigroups, which contains both the class of all regular semigroups and the class of all finite semigroups (and also includes all periodic semigroups) was introduced by Edwards [1]. An

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eventually regular semigroup S is called a left Clifford semigroup if $eS \subseteq Se$ for all idempotents $e \in S$. An eventually regular semigroup S called a weak Clifford semigroup if $E(S) \subseteq C(Reg(S))$. Zhang [6] and Zhang et al [5, 6] have given the necessary and sufficient conditions of the semidirect product with respect to left Clifford quasi-regular semigroups, weak Clifford quasi-regular semigroups and weak quasi-regular semigroups, respectively. Sen [5] introduced the concept of left and right Clifford semigroups. A regular semigroup is called a left and right Clifford semigroup if $eS \subseteq Se$ or $Se \subseteq eS$ for all idempotents $e \in S$. Similarly, in this paper, we introduce the concept of left and right Clifford eventually regular semigroups, and necessary and sufficient conditions on semidirect product and wreath product of such semigroups are given.

For convenience we denote all idempotents of S by $E(S)$, and denote regular elements of S by Reg S, and in place of $t^{s^{m-1}} \cdots t^s t$ by $t^{s(m)}$. Throughout this paper, the terminologies and notations that are not mentioned, see [2].

We say that S is left and right Clifford eventually semigroups, abbreviated to LRCE, if S is eventually regular and

$$(\forall e \in E(S)), \quad eS \subseteq Se \text{ or } Se \subseteq eS\,.$$

Clearly, all left and all right Clifford eventually regular semigroups are LRCE. But the converse may fail.

Example 1.1. Let $S = \{(a,b) \in \mathbb{R} \times \mathbb{R} | ab = 0\}$ with multiplication $(a,b)(c,d) = (|a|c, b|d|)$, with respect to which S becomes an eventually regular semigroup and $E(S) = \{(\pm 1, 0), (0,0), (0, \pm 1)\}$. Also $S(\pm 1, 0) \subseteq (\pm 1, 0)S, S(0,0) = (0,0)S$, $(0, \pm 1)S \subseteq S(0, \pm 1)$, $(1,0)S \nsubseteq S(1,0)$, and $S(0,1) \nsubseteq (0,1)S$. Hence S is LRCE but is neither left Clifford eventually regular nor right Clifford eventually regular.

Next we give another equivalent characterization of LRCE.

Lemma 1.2. *Let S be a semigroup. Then S is LRCE if and only if S is an eventually regular semigroup and $ese = es$ or $ese = se$ for all $e \in E(S), s \in S$.*

Proof. Necessity. Clearly, S is an eventually regular semigroup. If $eS \subseteq Se$ for all $e \in S$, then there exists $s' \in S$ such that $es = s'$ for all $s \in S$. so

$$ese = s'ee = s'e = es\,.$$

If $Se \subseteq eS$, similarly, we can show that $ese = se$.

Sufficiency. If $es = ese$ for all $e \in S$ and $s \in S$, then $eS \subseteq Se$. If $se = ese$, then $Se \subseteq eS$. Thus S is LRCE. □

For arbitrary semigroup S, let

$$E_l(S) = \{e \in E(S) | (\forall f \in E(S)), efe = ef\},$$

$$E_r(S) = \{e \in E(S) | (\forall f \in E(S)), efe = fe\},$$
$$E_o(S) = \{e \in E(S) | (\forall f \in E(S)), ef = fe\}.$$

We easily know that $E_o(S) = E_l(S) \cap E_r(S)$. If S is LRCE, clearly, we have

$$E_l(S) \cup E_r(S) = E(S)\,.$$

2. Semidirect Product

Recall that the concept of semidirect product on two semigroups. Let S and T be given semigroups. Let $\mathrm{End}(T)$ be the endomorphism of T, and write endomorphism as exponents to be right of arguments. If $\alpha : S \to \mathrm{End}(T)$ is a homomorphism, and $s \in S$ and $t \in T$, write t^s for $t^{\alpha(s)}$. In what follows, α will be considered fixed and not explicitly mentioned.

The semidirect product $S \times_\alpha T$ is the semigroup with elements $\{(s,t)|s \in S$ and $t \in T\}$ and multiplication $(s,t)(s_1,t_1) = (ss_1, t^{s_1}t_1)$, where $(s,t),(s_1, t_1) \in S \times T$.

Lemma 2.1. *Let S and T be semigroups, and let $\alpha : S \to \mathrm{End}(T)$ be a given homomorphism, and the semidirect product $S \times_\alpha T$ is LRCE. then the following conditions hold:*

(i) S and T^e are LRCE for all $e \in E(S)$, where $T^e = \{t^e | t \in T\}$;

(ii) for all $s \in S$ and $t \in T$, there exists $m \in N$ such that $s^m \in \mathrm{Reg}\, S$, $t^{s(m)} \in (t^{s(m)})^{s_1 s^m} T^{s(m)}$, where $s_1 \in V(s^m)$;

(iii) for all $e \in E_l(S) \setminus E_o(S), u \in T$, if $u^e u = u$, then $u \in E_l(T), u^s u = u^s, u^s t^e = u^s t$, where $s \in S, t \in T$;

(iv) for all $e \in E_r(S) \setminus E_o(S), u \in T$, if $u^e u = u$, then $u^e \in E_r(T), u^{se} u = u$, where $s \in S, t \in T$;

(v) for all $e \in E_o(S), u \in T$, if $u^e u = u$, then $u \in E(T), u^s u = u^s, u t^e u = ut$ or $u^{se} u = u, u^e t^e u = t^e u$, where $s \in S, t \in T$.

Proof. (i) For all $e \in E(S)$, we easily show that T^e is a subsemigroup of T.

For all $t^e \in T^e$, according to the semidirect product $S \times_\alpha T$ is eventually regular, then there exists $(s_1, t_1) \in S \times_\alpha T, m \in N$ such that

$$(e, t^e)^m = (e, t^e)^m (s_1, t_1)(e, t^e)^m\,.$$

So we have $e = ese$, and

$$(t^e)^m = (t^e)^{ms_1 e} t_1^e (t^e)^m = (t^{es_1 e})^m t_1^e (t^e)^m = (t^e)^m t_1 (t^e)^m.$$

Therefore, T^e is an eventually regular semigroup.

For all $u^e \in E(T^e)$, we have $(e, u^e) \in E(S \times_\alpha T)$. For all $t^e \in T^e$, then we have

$$(e, u^e)(e, t^e)(e, u^e) = (e, u^e)(e, t^e)\,,$$

or

$$(e, u^e)(e, t^e)(e, u^e) = (e, t^e)(e, u^e)\,,$$

since $S \times_\alpha T$ is eventually regular. So $u^e t^e u^e = u^e t^e$ or $u^e t^e u^e = t^e u^e$. Consequently, T^e is LRCE.

For all $s \in S$ and $t \in T$, according to $S \times_\alpha T$ is eventually regular, then there exists $(s_2, t_2) \in S \times_\alpha T, m \in N$ such that

$$(s, t)^m = (s, t)^m (s_2, t_2)(s, t)^m,$$

we obtain

$$s^m = s^m s_2 s^m\,.$$

Therefore S is an eventually regular semigroup.

For all $e \in E(S)$, let $u^e \in E(T^e)$. Then $(e, u^e) \in E(S \times_\alpha T)$. For all $s \in S, t \in T$, we have

$$(e, u^e)(s, t)(e, u^e) = (e, u^e)(s, t),$$

or

$$(e, u^e)(s, t)(e, u^e) = (s, t)(e, u^e)\,.$$

So we imply that

$$ese = es \text{ or } ese = se\,.$$

Thus S is LRCE.

(ii) For all $s \in S, t \in T$, then there exists $m \in N, (s_1, t_1) \in V((s, t)^m)$ such that

$$(s, t)^m = (s, t)^m (s_1, t_1)(s, t)^m\,.$$

Thus $s^m = s^m s_1 s^m \in \text{Reg } S$, and

$$t^{s(m)} = (t^{s(m)})^{s_1 s^m} t_1^{s^m} t^{s(m)},$$

so

$$t^{s(m)} \in (t^{s(m)})^{s_1 s^m} T t^{s(m)},$$

where $s_1 \in V(s^m)$.

(iii) For all $e \in E_l(S) \setminus E_o(S), u \in T$, if $u^e u = u$, then we imply that $(e, u) \in E(S \times_\alpha T)$, and $(e, u) \in E_l(S \times_\alpha T)$. If not, suppose that $(e, u) \in E_r(S \times_\alpha T)$, for all $(s,t) \in S \times_\alpha T$, then we have $(e,u)(s,t)(e,u) = (s,t)(e,u)$, thus $ese = se$, this is contrary to $e \in E_l(S) \setminus E_o(S)$. so

$$(e,u)(e,u^e)(e,u) = (e,u)(e,u^e),$$

that is, $u^e u = u^e$, thus $u = u^e \in E(T)$.

For all $t \in T$, we have $(e,u)(e,t)(e,u) = (e,u)(e,t)$, so $u^e t^e u = u^e t$. We imply that $utu = ut$.Therefore, $u \in E_l(T)$.

For all $s \in S$, according to $(e,u) \in E_l(S \times_\alpha T)$, we have

$$(e,u)(s,u^s)(e,u) = (e,u)(s,u^s),$$

therefore

$$ese = es, u^{se} u^{se} u = u^s u^s \,.$$

On the other hand, we have $u^{se} = (u^e)^{se} = u^{es} = (u^e)^s = u^s$, since $u^e = u$, so

$$u^s u = u^{se} u = u^{se} u^{se} u = u^s u^s = u^s \,.$$

For any $s \in S, t \in T$, we have $(e,u)(s,t)(e,u) = (e,u)(s,t)$, so $ese = se, u^{se} t^e u = u^s t$, we can imply that $u^s t^e = u^s t$.

(iv) For all $e \in E_r(S) \setminus E_o(s)$ and $u \in T$, if $u^e u = u$, then $(e,u) \in E(S \times_\alpha T)$. We know that $(e,u) \in E_r(S \times_\alpha T)$ in terms of proof of above (iii). For any $s \in S$, we have

$$(e,u)(s,u^e)(e,u) = (s,u^e)(e,u),$$

so $u^{se} u^e u = u^e u$, thus

$$u^{se} u = u \,.$$

Moreover, we have $u^e \in E(T^e), (e,u^e) \in E_r(S \times_\alpha T)$, since $u^e u = u$. For all $t^e \in T^e$, we have

$$(e,u^e)(e,t^e)(e,u^e) = (e,t^e)(e,u^e),$$

so $u^e t^e u^e = t^e u^e$. Therefore $u^e \in E_r(T^e)$.

(v) For all $e \in E_o(S)$ and $u \in T$, if $u^e u = u$, then $(e,u) \in E(S \times_\alpha T)$. If $(e,u) \in E_l(S \times_\alpha T)$, then we have

$$(e,u)(e,u^e)(e,u) = (e,u)(e,u^e) \,.$$

So $u^e u = u^e$, that is, $u = u^e \in E(T)$.

For all $t \in T$, then $(e,u)(e,t)(e,u) = (e,u)(e,t)$, so $u^e t^e u = u^e t$, we imply that $u t^e u = ut$.

For all $s \in S$, then we have $(e,u)(s,u^s)(e,u) = (e,u)(s,u^s)$, so $ese = es, u^{se}u^{se}u = u^s u^s$. We know that $u^e = u \in E(T)$ in terms of proof of above. Thus $u^s u = (u^e)^s u = u^{ese}u = u^{se}u = u^{se}u^{se}u = u^s u^s = u^s$.

If $(e,u) \in E_r(S \times_\alpha T)$, then $(e,u)(s,u^e)(e,u) = (s,u^e)(e,u)$ for any $s \in S$. Therefore $u^{se}u^e u = u^e u$, that is, $u^{se}u = u$. For all $t \in T$, we have $(e,u)(e,t)(e,u) = (e,t)(e,u)$, so $u^e t^e u = t^e u$. □

Thus we have the following:

Theorem 2.2. *Let S and T be semigroups, and let $\alpha : S \to \mathrm{End}(T)$ be a given homomorphism. the semidirect product $S \times_\alpha T$ is LRCE if and only if:*

(i) S and T^e are LRCE for all $e \in E(S)$, where $T^e = \{t^e | t \in T\}$;

(ii) for all $s \in S$ and $t \in T$, there exists $m \in N$ such that $s^m \in \mathrm{Reg}\ S$, $t^{s(m)} \in (t^{s(m)})^{s_1 s^m} T^{s(m)}$, where $s_1 \in V(s^m)$;

(iii) for all $e \in E_l(S) \setminus E_o(S), u \in T$, if $u^e u = u$, then $u = u^e \in E_l(T), u^s u = u^s, u^s t^e = u^s t$, where $s \in S, t \in T$;

(iv) for all $e \in E_r(S) \setminus E_o(S), u \in T$, if $u^e u = u$, then $u^e \in E_r(T), u^{se}u = u$, where $s \in S, t \in T$;

(v) for all $e \in E_o(S), u \in T$, if $u^e u = u$, then $u \in E(T), u^s u = u^s, ut^e u = ut$ or $u^{se}u = u, u^e t^e u = t^e u$, where $s \in S, t \in T$.

Proof. By Lemma 2.1, necessity is obvious. We only show that sufficiency.

For all $(s,t) \in S \times_\alpha T$, by condition (ii), there exits $m \in N$ such that $s^m \in \mathrm{Reg}\ S$, $t^{s(m)} \in (t^{s(m)})^{s_1 s^m} T t^{s(m)}$, where $s_1 \in V(s^m)$.

Let $t_1 \in T$, and $t^{s(m)} = (t^{s(m)})^{s_1 s^m} t_1 t^{s(m)}$. Then we have

$$(t^{s(m)})^{s_1 s^m} = (t^{s(m)})^{s_1 s^m} t_1^{s_1 s^m} (t^{s(m)})^{s_1 s^m},$$

so

$$\begin{aligned} t^{s(m)} = (t^{s(m)})^{s_1 s^m} t_1 t^{s(m)} &= (t^{s(m)})^{s_1 s^m} t_1^{s_1 s^m} (t^{s(m)})^{s_1 s^m} t_1 t^{s(m)} \\ &= (t^{s(m)})^{s_1 s^m} t_1^{s_1 s^m} t^{s(m)}. \end{aligned}$$

Hence

$$\begin{aligned} (s,t)^m (s_1, t_1^{s_1})(s,t) &= (s^m s_1 s^m, (t^{s(m)})^{s_1 s^m} t_1^{s_1 s^m} t^{s(m)}) \\ &= (s^m, t^{s(m)}) = (s,t)^m, \end{aligned}$$

that is, $S \times_\alpha T$ is an eventually regular semigroup.

Let $(e,u) \in E(S \times_\alpha T)$. Then $u^e u = u$. For all $(s,t) \in S \times_\alpha T$, we have $(e,u)(s,t)(e,u) = (ese, u^{se} t^e u)$.

If $e \in E_l(S) \setminus E_o(S)$, then $ese = es$, by condition (iii), we know that $u \in E_l(T), u^s u = u^s, u^s t^e = u^s t$. Therefore, $u^e = u$, and $u^{se} = (u^e)^{se} = u^{ese} = u^{es} = u^s$. So we have

$$u^{se} t^e u = u^s t^e u = u^s t u = u^s u t = u^s t\,.$$

Therefore

$$(e,u)(s,t)(e,u) = (es, u^s t) = (e,u)(s,t)\,.$$

If $e \in E_r(S) \setminus E_o(S)$, then $ese = se$, by condition (iv), we have $u^{se}u = u, u^e \in E_r(T^e)$. So

$$u^{se} t^e u = u^{se} t^e u^e u = u^{se} u^e t^e u^e u = (u^{se} u)^e t^e u = u^e t^e u = u^e t^e u^e u = t^e u\,.$$

Thus

$$(e,u)(s,t)(e,u) = (se, t^e u) = (s,t)(e,u)\,.$$

If $e \in E_o(S)$, then $es = se$, by condition (v), we know that $u^e u = u^e = u \in E(T)$, and $u^{se} = u^{es} = (u^e)^s = u^s$ for $u \in E(T), u^s u = u^s, u t^e u = ut$. Thus

$$u^{se} t^e u = u^s t^e u = (u^s u) t^e u = u^s u t = u^s t,$$

so

$$(e,u)(s,t)(e,u) = (es, u^s t) = (e,u)(s,t)\,.$$

Similarly, we show that $(e,u)(s,t)(e,u) = (se, t^e u) = (s,t)(e,u)$ for $u^{se}u = u, u^e t^e u = t^e u$.

In conclusion, $S \times_\alpha T$ is LRCE. □

3. Wreath Product

Let S and T be semigroups, and S acts on a set X from the left, and $T^X = \{f | f : X \to T\}$. Let $(fg)(x) = f(x)g(x)$ for all $f, g \in T^X$ and $x \in X$. Then T^x is a semigroup. Let $\alpha : S \to \mathrm{End}(T), s \mapsto \alpha(S)$ be a homomorphism, where $f^s(x) = f(sx)$ for $s \in S, f \in T^X$. Then the semidirect product $S \times_\alpha T$ is called wreath product of S and T, and we denote it by $S_{W_X} T$.

By Theorem 2.1, we have the following:

Theorem 3.1. *Let S, T, α, T^X be defined as above. Then the wreath product $S_{W_X} T$ is LRCE if and only if:*

(i) S and $(T^X)^e$ are LRCE for all $e \in E(S)$, where $(T^X)^e = \{f^e | f \in T^X\}$;

(ii) for all $s \in S$ and $f \in T^X$, there exists $m \in N$ such that $s^m \in \mathrm{Reg}\ S$, $f^{s(m)} \in (f^{s(m)})^{s_1 s^m} T^X f^{s(m)}$, where $s_1 \in V(s^m)$;

(iii) for all $e \in E_l(S) \setminus E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f \in E_l(T^X), f^s f = f^s, f^s g^e = f^s g$, *where* $s \in S, g \in T^X$*;*

(iv) for all $e \in E_r(S) \setminus E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f^e \in E_r(T^X)$, $f^{se} f = f$, *where* $s \in S, f \in T^X$*;*

(v) for all $e \in E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f \in E(T^X), f^s f = f^s, f g^e f = f g$ *or* $f^{se} f = f, f^e g^e f = g^e f$, *where* $s \in S, g \in T^X$.

Lemma 3.2. *Let* S *and* T *be semigroups, and* S *acts on a set* X *from the left, and* T^X *is defined as above. Let* $(T^X)^e = \{f^e | f \in T^X\}$. *Then* $(T^X)^e$ *is LRCE if and only if* T *is LRCE.*

Proof. Necessity. Let $(T^X)^e$ is LRCE. We define $f : X \longrightarrow T, f(x) = t$ for all $t \in T$. Then $f^e(x) = f(ex) = t$. So $f = f^e \in (T^X)^e$. By $(T^X)^e$ is eventually regular, there exists $m \in N$ such that $(f^e(x))^m \in \mathrm{Reg}\ T$. So there exists f'^e such that $(f^e)^m = (f^e)^m f'^e (f^e)^m$. Thus we have $(f^e(x))^m = (f^e(x))^m f'^e(x)(f^e(x))^m$ for all $x \in X$, this imply that $t^m = t^m f'^e(x) t^m$. Consequently, T is an eventually regular semigroup.

We define $g(X) = i$ for all $i \in E(T)$, $h(X) = t$ for any $t \in T$, according to proof of above, we know that $g = g^e \in E((T^X))^e$ and $h^e \in (T^X)^e$. By $(T^X)^e$ is LRCE, we have $g^e h^e g^e = g^e h^e$ or $g^e h^e g^e = h^e g^e$. So for any $x \in X$, we obtain

$$g^e(x)h^e(x)g^e(x) = g^e(x)h^e(x) \text{ or } g^e(x)h^e(x)g^e(x) = h^e(x)g^e(x),$$

that is $iti = it$ or $iti = ti$. Therefore, T is LRCE.

Sufficiency. Let $g^e \in (T^X)^e$. Then we have $g^e(x) \in T$ for any $x \in X$. By T is LRCE, then there exists $m \in N$ such that $(g^e(x))^m \in \mathrm{Reg}\ T$, so there exists $t_1 \in T$ such that $(g^e(x))^m = (g^e(x))^m t_1 (g^e(x))^m$. We define $h(x) = t_1$, then $h^e = h$. so $(g^e(x))^m = (g^e(x))^m h(x)(g^e(x))^m$ for any $x \in X$, thus $(g^e)^m = (g^e)^m h^e (g^e)^m$, that is, $(g^e)^m \in \mathrm{Reg}\ (T^X)^e$. Therefore $(T^X)^e$ is an eventually regular semigroup.

Let every $f^e \in E(T^X)^e$ and $g^e \in T^X$. Then we have $f^e(x) \in E(T)$, $g^e(x) \in T$ for any $x \in X$. By T is LRCE, we obtain

$$f^e(x)g^e(x)f^e(x) = f^e(x)g^e(x) \text{or} f^e(x)g^e(x)f^e(x) = g^e(x)f^e(x),$$

so $f^e g^e f^e(x) = f^e g^e(x)$ or $f^e g^e f^e(x) = g^e f^e(x)$, that is, $f^e g^e f^e = f^e g^e$ or $f^e g^e f^e = g^e f^e$. Thus, $(T^X)^e$ is LRCE. □

By this lemma, we know that Theorem 3.1 is equivalent to the following:

Theorem 3.3. *Let* S, T, α, T^X *be defined as above. Then* $S_{W_X} T$ *is LRCE if and only if:*

(i) S *and* T *are LRCE;*

(ii) for all $s \in S$ *and* $f \in T^X$, *there exists* $m \in N$ *such that* $s^m \in \mathrm{Reg}\ S$, $f^{s(m)} \in (f^{s(m)})^{s_1 s^m} T^X f^{s(m)}$, *where* $s_1 \in V(s^m)$*;*

(iii) for all $e \in E_l(S) \setminus E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f = f^e \in E_l(T^X)$, $f^s f = f^s$, $f^s g^e = f^s g$, *where* $s \in S, g \in T^X$*;*

(iv) for all $e \in E_r(S) \setminus E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f^e \in E_r(T^X)$, $f^{se} f = f$, *where* $s \in S, f \in T^X$*;*

(v) for all $e \in E_o(S), f \in T^X$, *if* $f^e f = f$, *then* $f \in E(T^X), f^s f = f^s, f g^e f = fg$ or $f^{se} f = f, f^e g^e f = g^e f$, *where* $s \in S, g \in T^X$.

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