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COMMON FIXED POINTS FOR LOCALLY LIPSCHITZ MAPPINGS

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Abstract: In this paper we prove some common fixed point theorems for fuzzy locally Lipschitz mappings with respect to a mapping, that satisfy a condition of R-weak commutativity. We deduce also common fixed point results for mappings in metric space.

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1. Introduction

The notion of fuzzy metric space was introduced in different ways (see [17], [3], [10], [14]). Recently the notion of fuzzy metric space, introduced by Kramosil and Michalek [10], was modified by George and Veeramani [4], [6] that obtained a Hausdorff topology for this class of fuzzy metric spaces. In [5], [8] it was proved that the topology induced by a fuzzy metric space in the sense of George and Veeramani is metrizable. In this type of spaces, for some classes of fuzzy contractive mappings, it was proved that the *Contraction Theorem of Banach* is valid (see [1], [7], [9]). Grabiec [7] proved the contraction principle in the setting of fuzzy metric spaces introduced by Kramosil and Michalek. Gregori and Sapena [9] obtained fixed point results for complete fuzzy metric space

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in the sense of George and Veeramani, and also for Kramosil and Michalek's fuzzy metric spaces what are complete in Grabiec's sense (we say G-complete fuzzy metric space). Vasuki [15], Di Bari and Vetro [2] proved results regarding common fixed point for a family of mappings defined on a G-complete fuzzy metric space. Recently Pant [12], Pathak, Cho and Kang [13] obtained result of fixed point for discontinuous mappings in metric spaces by the notion of *R-weak commutativity*. Vasuki [16] defined R-weak commutativity of mappings in fuzzy metric spaces and prove the fuzzy version of Pant's Theorem for G-complete fuzzy metric space. In this paper we introduce a fuzzy locally Lipschitz notion for mappings and we use the fuzzy version of the notions of R-weak commutativity introduced in [13] for obtain results of fixed point for discontinuous mappings. We deduce also common fixed point results for mappings in metric space that improve results of Padaliya and Pant [11] and of Pathak, Cho and Kang [13].

2. Preliminaries on the Fuzzy Metric Spaces

In this section we recall the basic notion on fuzzy metric space in the sense of George and Veeramani. We denote with $\mathcal{N}$ the set of all positive integers and with $\mathcal{R}$ the set of all real numbers.

Definition 1. (see Schweizer and Sklar [14]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if satisfies the following conditions:

i) $*$ is associative and commutative,

ii) $*$ is continuous,

iii) $a * 1 = a$ for every $a \in [0,1]$,

iv) $a * b \leq c * d$ if $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2. (see George and Veeramani [4]) A term $(X, M, *)$ is a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X \times X \times]0, +\infty[$ satisfying, for every $x, y, z \in X$ and $s, t > 0$, the following conditions:

i) $M(x, y, t) > 0$,

ii) $M(x, y, t) = 1$ iff $x = y$,

iii) $M(x, y, t) = M(y, x, t)$,

iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

v) $M(x, y, \cdot) :]0, +\infty[\to [0, 1]$ is continuous.

George and Veeramani proved that every fuzzy metric M on X induces a Hausdorff first countable topology τ_M which has as base the family of open sets

$$\{B(x,r,t):\ x \in X,\ 0 < r < 1,\ t > 0\},$$

where

$$B(x,r,t) = \{y \in X : M(x,y,t) > 1 - r\}.$$

Let $(X, M, *)$ be a fuzzy metric space. A sequence $(x_n) \subset X$ is a Cauchy sequence if for every $0 < r < 1$ and for every $t > 0$, there is $n_0 \in \mathcal{N}$ such that $M(x_n, x_m, t) > 1 - r$ for every $n, m \geq n_0$. A sequence $(x_n) \subset X$ is a G-sequence if $M(x_n, x_{n+p}, t) \to 1$ as $n \to +\infty$ for every $p \in \mathcal{N}$ and every $t > 0$.

Remark 3. A G-sequence is a Cauchy sequence in the sense of Grabiec [7]. We observe that a Cauchy sequence is a G-sequence, in general the vice versa is not true.

Example 4. Let $X = \mathcal{R}$ and for $x, y \in X$ and $t > 0$ define

$$M(x,y,t) = \frac{t}{t + |x - y|}$$

and let $a * b = ab$ for all $a, b \in [0, 1]$. Then $(X, M, *)$ is a fuzzy metric space. For every $n \in \mathcal{N}$ let $x_n = 1 + 1/2 + \cdots + 1/n$, the sequence (x_n) is a G-sequence, but obviously is not a Cauchy sequence.

Theorem 5. (see George and Veeramani [4]) *A sequence (x_n) in a fuzzy metric space $(X, M, *)$ converges to x if and only if $M(x_n, x, t,) \to 1$ as $n \to +\infty$.*

A fuzzy metric space $(X, M, *)$ is complete (respectively G-complete) if every Cauchy sequence (respectively G-sequence) is convergent. If $(X, M, *)$ is a complete (respectively G-complete) fuzzy metric space, then M is a complete (respectively G-complete) fuzzy metric on X. The fuzzy metric space of Example 4 is complete, but it is not G-complete.

3. Fuzzy Locally Lipschitz Mappings with Respect to a Map

In this section we introduce the notion of fuzzy locally Lipschitz mapping with respect to a map. Let $(X, M, *)$ be a fuzzy metric space and let $f, g : X \to X$ be maps. For all $x, y \in X$ and each $t > 0$, we define

$$m(f, g; x, y, t) = \min\{M(g(x), g(y), t),\ M(f(x), g(x), t),\ M(f(y), g(y), t)\}.$$

Definition 6. Let $(X, M, *)$ be a fuzzy metric space and let $f, g : X \to X$ be maps. f is a fuzzy locally Lipschitz map with respect to g if for every $\varepsilon > 0$

there exist $h(\varepsilon) \in]0, \varepsilon[$ and $\phi(\varepsilon) \in]0, 1[$, such that

$$\frac{1}{M(f(x), f(y), t)} - 1 \leq (\frac{1}{m(f, g; x, y, t)} - 1)\phi(\varepsilon) \tag{1}$$

for every $x, y \in X$ and every $t > 0$ such that

$$\frac{1}{m(f, g; x, y, t)} - 1 \in]\varepsilon - h(\varepsilon), \varepsilon + h(\varepsilon)[. \tag{2}$$

Let $f, g : X \to X$ be such that $f(X) \subset g(X)$ and $x_0 \in X$. Let x_1 be such that $f(x_0) = g(x_1)$. In general, we choose x_n so that $f(x_{n-1}) = g(x_n)$. We can do this since $f(X) \subset g(X)$. The sequence $(f(x_n))$ is said a *f-g-sequence* of initial point x_0.

Definition 7. (see Di Bari and Vetro [1]) Let $(X, M, *)$ be a fuzzy metric space. The fuzzy metric M is triangular if it satisfies the condition

$$\frac{1}{M(x, y, t)} - 1 \leq \frac{1}{M(x, z, t)} - 1 + \frac{1}{M(z, y, t)} - 1$$

for every $x, y, z \in X$ and every $t > 0$.

Lemma 8. *Let $(X, M, *)$ be a fuzzy metric space with M triangular and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If f is a fuzzy locally Lipschitz map with respect to g, then every f-g-sequence $(f(x_n))$ of initial point $x_0 \in X$ is a Cauchy sequence.*

Proof. (i). We suppose that $f(x_{n-1}) \neq f(x_n)$ for all n. We use (1) with

$$\varepsilon = \frac{1}{m(f, g; x_n, x_{n+1}, t)} - 1$$

and we deduce that

$$\begin{aligned} m(f, g; x_n, x_{n+1}, t) &= \min\{M(f(x_{n-1}), f(x_n), t),\ M(f(x_n), f(x_{n+1}), t)\} \\ &= M(f(x_{n-1}), f(x_n), t). \end{aligned}$$

It follows

$$\frac{1}{M(f(x_n), f(x_{n+1}), t)} - 1 < \frac{1}{M(f(x_{n-1}), f(x_n), t)} - 1.$$

Consequently $M(f(x_n), f(x_{n+1}), t) > M(f(x_{n-1}), f(x_n), t)$ for all n and thus $(M(f(x_{n-1}), f(x_n), t))$ is an increasing sequence of positive real numbers in [0,1]. Let

$$S(t) = \lim_{n \to +\infty} M(f(x_{n-1}), f(x_n), t)$$

and we show that $S(t) = 1$ for all $t > 0$. We suppose that there is $t > 0$ such that $S(t) < 1$ and let $\varepsilon = \frac{1}{S(t)} - 1$. For large n, we have

$$\varepsilon \leq \frac{1}{m(f,g;x_n,x_{n+1},t)} - 1 < \varepsilon + h(\varepsilon).$$

From

$$\frac{1}{M(f(x_n),f(x_{n+1}),t)} - 1 \leq (\frac{1}{m(f,g;x_n,x_{n+1},t)} - 1)\phi(\varepsilon)$$

as $n \to +\infty$, we deduce

$$\frac{1}{S(t)} - 1 \leq (\frac{1}{S(t)} - 1)\phi(\varepsilon) < \frac{1}{S(t)} - 1$$

and this is a contradiction.

To prove that $(f(x_n))$ ia a Cauchy sequence, it is sufficient to show that the subsequence $(f(x_{2n}))$ of $(f(x_n))$ is a Cauchy sequence. For convenience, let $y_n = f(x_n)$ for $n = 0, 1, \cdots$. We suppose that (y_n) is not a Cauchy sequence. Then there is an $\varepsilon > 0$ and one $t > 0$ such that for each positive integer k, there exist $m(k)$ and $n(k)$, with $m(k) > n(k) \geq k$, such that

$$M(y_{2n(k)}, y_{2m(k)}, t) \leq 1 - \varepsilon.$$

For each positive integer k, let $m(k)$ be the least integer greater than $n(k)$ satisfying

$$M(y_{2n(k)}, y_{2m(k)}, t) \leq 1 - \varepsilon, \quad M(y_{2n(k)}, y_{2m(k)-2}, t) > 1 - \varepsilon.$$

Being M triangular, for each positive integer k, we have

$$\frac{\varepsilon}{1-\varepsilon} \leq \frac{1}{M(y_{2n(k)}, y_{2m(k)}, t)} - 1 \leq \frac{1}{M(y_{2n(k)}, y_{2m(k)-2}, t)} - 1$$

$$+ \frac{1}{M(y_{2m(k)-2}, y_{2m(k)-1}, t)} - 1 + \frac{1}{M(y_{2m(k)-1}, y_{2m(k)}, t)} - 1.$$

Since $M(y_n, y_{n+1}, t) \to 1$ as $n \to +\infty$, it follows that

$$\lim_{k \to +\infty} \frac{1}{M(y_{2n(k)}, y_{2m(k)}, t)} - 1 = \frac{\varepsilon}{1-\varepsilon}.$$

Now the hypothesis that M is triangular assures also that

$$\lim_{k \to +\infty} \frac{1}{M(y_{2n(k)}, y_{2m(k)-1}, t)} - 1 = \frac{\varepsilon}{1-\varepsilon}.$$

We remark that

$$\lim_{k \to +\infty} m(f,g;x_{2n(k)+1}, x_{2m(k)}, t) = 1 - \varepsilon$$

and for n large

$$\frac{\varepsilon}{1-\varepsilon} - h(\frac{\varepsilon}{1-\varepsilon}) < \frac{1}{m(f,g;x_{2n(k)+1}, x_{2m(k)}, t)} - 1 < \frac{\varepsilon}{1-\varepsilon} + h(\frac{\varepsilon}{1-\varepsilon}).$$

Then

$$\frac{1}{M(y_{2n(k)}, y_{2m(k)}, t)} - 1$$
$$\leq \frac{1}{M(y_{2n(k)}, y_{2n(k)+1}, t)} - 1 + \frac{1}{M(y_{2n(k)+1}, y_{2m(k)}, t)} - 1$$
$$\leq \frac{1}{M(y_{2n(k)}, y_{2n(k)+1}, t)} - 1 + (\frac{1}{m(f, g; x_{2n(k)+1}, x_{2m(k)}, t)} - 1)\phi(\frac{\varepsilon}{1-\varepsilon})$$

and so, as $k \to +\infty$,

$$\frac{\varepsilon}{1-\varepsilon} \leq \frac{\varepsilon}{1-\varepsilon}\, \phi(\frac{\varepsilon}{1-\varepsilon}) < \frac{\varepsilon}{1-\varepsilon}.$$

This is a contradiction and thus $(f(x_{2n}))$ is a Cauchy sequence.

(ii). If there exists $m \in \mathcal{N}$ such that $f(x_m) = f(x_{m+1})$, then $f(x_n) = f(x_{n+1})$ for all $n \geq m$. Consequently $(f(x_n))$ is a Cauchy sequence. In fact $m(f, g; x_{m+1}, x_{m+2}, t) = M(f(x_{m+1}), f(x_{m+2}), t)$ and if $f(x_{m+1}) \neq f(x_{m+2})$, then for

$$\varepsilon = \frac{1}{m(f, g; x_{m+1}, x_{m+2}, t)} - 1 > 0$$

we deduce

$$\frac{1}{M(f(x_{m+1}), f(x_{m+2}), t)} - 1 < \frac{1}{M(f(x_{m+1}), f(x_{m+2}), t)} - 1.$$

It follows that $f(x_{m+1}) = f(x_{m+2})$ and this assures that $f(x_n) = f(x_{n+1})$ for all $n \geq m$. □

Lemma 9. *Let $(X, M, *)$ be a fuzzy metric space and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If f is a fuzzy locally Lipschitz map with respect to g, then every f-g-sequence $(f(x_n))$ of initial point $x_0 \in X$ is a G-sequence.*

Proof. We say that $M(f(x_n), f(x_{n+1}), t) \to 1$ for every $t > 0$. Thus, for each positive integer p, we have

$$M(f(x_n), f(x_{n+p}), t)$$
$$\geq M(f(x_n), f(x_{n+1}), \frac{t}{p}) * \cdots * M(f(x_{n+p-1}), f(x_{n+p}), \frac{t}{p}).$$

It follows that

$$\lim_{n\to+\infty} M(f(x_n), f(x_{n+p}), t) \geq \underbrace{1 * \cdots * 1}_{p} = 1$$

and $(f(x_n))$ is a G-sequence. □

4. Common Fixed Points in Fuzzy Metric Spaces

In this section we prove that if f, g verify a fuzzy version of the R-weak commutativity (condition (3) of Theorem 10) and f is a fuzzy locally Lipschitz map with respect to g, then f and g have a unique common fixed point.

Theorem 10. *Let $(X, M, *)$ be a fuzzy metric space with M triangular and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If f is a fuzzy locally Lipschitz map with respect to g, $f(X)$ or $g(X)$ is a complete subspace of X and there exists a constant $R > 0$ such that*

$$\frac{1}{M(f(g(x)), g(g(x)), t)} - 1 \leq \frac{1}{M(f(x), g(x), \frac{t}{R})} - 1 \tag{3}$$

in each x, where $f(x) = g(x)$, then f and g have a unique common fixed point.

Proof. Let x_0 be any point in X. By Lemma 8, the f-g-sequence $(f(x_n))$ of initial point x_0 is a Cauchy sequence and if $f(X)$ is a complete subspace of X, then there is $z \in f(X)$ such that $f(x_n) = g(x_{n+1}) \to z$ (this holds also if $g(X)$ is complete with $z \in g(X)$). Let $u \in X$ be such that $g(u) = z$. If $f(u) \neq g(u)$, for large value of n

$$m(f, g; u, x_n, t) = M(f(u), g(u), t)$$

and so

$$\frac{1}{M(f(u), f(x_n), t)} - 1 \leq \left(\frac{1}{M(f(u), g(u), t)} - 1\right)\phi\left(\frac{1}{M(f(u), g(u), t)} - 1\right).$$

Letting $n \to +\infty$, we obtain

$$\frac{1}{M(f(u), g(u), t)} - 1 < \frac{1}{M(f(u), g(u), t)} - 1$$

for every $t > 0$. It follows that $f(u) = z$.

Being $f(u) = g(u)$, we use (3) and from

$$\frac{1}{M(f(g(u)), g(g(u)), t)} - 1 \leq \frac{1}{M(f(u), g(u), \frac{t}{R})} - 1 = 0,$$

we deduce that $f(z) = g(z)$. If $z \neq f(z)$, we obtain

$$\frac{1}{M(f(u), f(z), t)} - 1 \leq \left(\frac{1}{M(f(u), f(z), t)} - 1\right)\phi\left(\frac{1}{M(f(u), f(z), t)} - 1\right)$$

$$< \frac{1}{M(f(u), f(z), t)} - 1$$

and this is a contradiction.

It follows that $z = f(z) = g(z)$ and hence z is a common fixed point of f

and g. Finally if $w \in X$ is another common fixed point, using (1) with

$$\varepsilon = \frac{1}{M(f(z), f(w), t)} - 1 > 0,$$

we deduce that $z = w$. □

If we suppose that in the fuzzy metric space $(X, M, *)$ the f-g-sequence are of Cauchy, then we deduce the following result.

Theorem 11. *Let $(X, M, *)$ be a fuzzy metric space and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If f is a fuzzy locally Lipschitz map with respect to g, $f(X)$ or $g(X)$ is a complete subspace of X, the f-g-sequence $(f(x_n))$ are of Cauchy and there is a constant $R > 0$ such that*

$$\frac{1}{M(f(g(x)), g(g(x)), t)} - 1 \leq \frac{1}{M(f(x), g(x), \frac{t}{R})} - 1,$$

in each x, where $f(x) = g(x)$, then f and g have a unique common fixed point.

Proof. Fix $x_0 \in X$, by hypothesis the f-g-sequence $(f(x_n))$ of initial point x_0 is of Cauchy and being $f(X)$ or $g(X)$ a complete subspace of X there is $z \in g(X)$ such that $f(x_n) \to z$. Proceeding as in Theorem 10, we prove Theorem 11. □

Proceeding as in Theorem 10, we prove the following theorem.

Theorem 12. *Let $(X, M, *)$ be a fuzzy metric space and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If f is a fuzzy locally Lipschitz map with respect to g, $f(X)$ or $g(X)$ is a G-complete subspace of X and there is a constant $R > 0$ such that*

$$\frac{1}{M(f(g(x)), g(g(x)), t)} - 1 \leq \frac{1}{M(f(x), g(x), \frac{t}{R})} - 1,$$

in each x where $f(x) = g(x)$, then f and g have a unique common fixed point.

Example 13. Let $X = [0, 1]$ and $a * b = ab$ for all $a, b \in [0, 1]$. For every $x, y \in X$ and $t > 0$, define

$$M(x, y, t) = \frac{t}{t + |x - y|},$$

then $(X, M, *)$ is a complete fuzzy metric space. Let $f, g : X \to X$ defined by $f(x) = 1$ for each $x \in X$ and $g(x) = 1$ if x is rational and $g(x) = x$ if x is irrational. For every x, y and each $t > 0$, we have

$$M(f(x), f(y), t) = 1 \quad \text{and} \quad m(f, g; x, y, t) > 0.$$

If we choose $\phi(\varepsilon) = 1/2$ and $h(\varepsilon) \in]0, \varepsilon[$, for every $\varepsilon > 0$, we deduce that f is a fuzzy locally Lipschitz map with respect to g.

In this example every f-g-sequence is constant and so is a Cauchy sequence. The hypotheses of Theorem 10 or Theorem 11 are verified for $R \geq 1$ and so f and g have a unique common fixed point $x = 1$.

5. Fixed Points in Metric Spaces

In this section we use Theorem 10 to deduce a result of common fixed point in metric spaces, that is an improvement of Theorem 2 of Padaliya and Pant [11] and of Theorem 2.1 of Pathak, Cho and Kang [13]. Let (X, d) be a metric space and let $f, g : X \to X$. For each $x, y \in X$ define

$$D(f, g; x, y) := \max\{d(g(x), g(y)), d(f(x), g(x)), d(f(y), g(y))\}.$$

Theorem 14. *Let (X, d) be a metric space and let $f, g : X \to X$ be such that $f(X) \subset g(X)$. If $f(X)$ or $g(X)$ is a complete subspace of X and the following conditions hold:*

(i) for every $\varepsilon > 0$ there exist $h(\varepsilon) \in]0, \varepsilon[$ and $\phi(\varepsilon) \in]0, 1[$, such that

$$d(f(x), f(y)) \leq D(f, g; x, y)\phi(\varepsilon)$$

for every $x, y \in X$ such that $D(f, g; x, y) \in]\varepsilon - h(\varepsilon), \varepsilon + h(\varepsilon)[$,

(ii) for each x, where $f(x) = g(x)$, there exists a constant $R > 0$ such that

$$d(f(g(x)), g(g(x))) \leq Rd(f(x), g(x)),$$

then f and g have a unique common fixed point.

Proof. We consider the fuzzy metric space $(X, M_d, *)$, where for all $x, y \in X$ and each $t > 0$

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

and $a * b = ab$ for every $a, b \in]0, 1[$. The hypotheses of Theorem 14 assure that the mappings f and g satisfy the hypotheses of the Theorem 10. Consequently the mappings f and g have a unique common fixed point. □

Remark 15. Condition (i) of Theorem 14 is satisfied if there exists an upper semicontinuous function $r : [0, +\infty[\to [0, +\infty[$ such that $r(t) < t$ for all $t > 0$ and such that

$$d(f(x), f(y)) \leq r(D(f, g; x, y))$$

for every $x, y \in X$.

From Theorem 14 if we choose $g = I_X$ the identity mapping on X, we obtain the following corollary.

Corollary 16. *Let (X, d) be a metric space and let $f : X \to X$. If $f(X)$ is a complete subspace of X or X is complete and the following conditions hold:*

(i) for every $\varepsilon > 0$ there exist $h(\varepsilon) \in]0, \varepsilon[$ and $\phi(\varepsilon) \in]0, 1[$, such that

$$d(f(x), f(y)) \leq D(f, I_X; x, y)\phi(\varepsilon)$$

for every $x, y \in X$ such that $D(f, I_X; x, y) \in]\varepsilon - h(\varepsilon), \varepsilon + h(\varepsilon)[$, then f and g have a unique common fixed point.

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