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CURVATURE FUNCTION IN THE RECOGNITION OF PEOPLE'S HANDWRITING

Biao Ou
Department of Mathematics
University of Toledo
Toledo, OH 43606, USA
e-mail: bou@math.utoledo.edu

Abstract: We discuss the curvature function in the recognition of people's handwriting by an intelligent writing pad. We address the questions of what is to be recorded, how to dissolve a parametric curve into smooth segments, and how to recognize a smooth segment using a standardized curvature function. We also examine the relevant mathematical aspects of the standardized curvature function.

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1. Introduction

In a previous paper [8] the author wrote about the curvature function of a parametric curve on the plane in the recognition of people's handwriting by an intelligent writing pad. Here we expand the discussion and supply more details.

2. The Intelligent Writing Pad

Computer scientists have been working on the making of an intelligent writing pad for a long time. Currently there are a few commercial products that can record and recognize relatively simple writing. However, there is no doubt that much more needs to and can be done in both hardware and software.

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Mathematicians would particularly like to have such a tool. When they type a paper or a book into an electronic file, it is now usually slow and inconvenient to enter mathematical symbols or characters. The intelligent writing pad would be much appreciated then. Also, when they do live presentations on mathematical computations, it is desirable to have the intelligent writing pad to turn what they write into neatly typed texts instantly.

For languages that are not alphabetically based, Chinese for example, an intelligent writing pad is much preferred for entering the text of the language into an electronic file.

3. What is to be Recorded

Each character or symbol we write is made of a few strokes. For each stroke the pen gives a parametric curve $(x(t), y(t)), a \leq t \leq b$. Here t represents the time, $x(t), y(t)$ are the coordinates of the position of the tip of the pen at each moment, and the time interval $a \leq t \leq b$ is the one in which the pen touches the pad. The time interval may alternatively be determined by the moments the pen enters or exits a designated zone in the pad.

The intelligent writing pad records strokes for each character or symbol, with each stroke represented by a parametric curve.

4. Segmentation of a Parametric Curve

Once a parametric curve is recorded, the next task is to recognize what it is. The first step is to determine if the curve is meant to be a dot by checking if all the points of the curve are in a short distance from a point, say the average position of the curve.

Next, for a parametric curve not meant to be a dot, we usually need to decompose it into regular, smooth segments. We look at the derivatives

$$(x'(t), y'(t)), \quad (x''(t), y''(t)).$$

These vector functions, in terms of physics, are the instantaneous velocity and acceleration of the tip of the pen at time t.

If at a moment t_1 the magnitude of $(x'(t_1), y'(t_1))$ is zero, we call t_1 a stagnation moment and $(x(t_1), y(t_1))$ a stagnation point. If at another moment t_2 the magnitude of $(x''(t_2), y''(t_2))$ becomes infinity, we call t_2 a shifting moment and $(x(t_2), y(t_2))$ a shifting point. Usually, at a shifting point there is a sharp change of the directions in the curve. In people's ordinary writing, there are only a few isolated stagnation or shifting points for each stroke.

We decompose the parametric curve into segments so that each segment is free of stagnation or shifting points except the beginning or ending points.

By possibly cutting off small time intervals containing the beginning or ending moments, we consider a smooth segment $(x(t), y(t)),\ c \le t \le d$ satisfying the following conditions: the functions $x(t), y(t)$ have continuous second derivatives, and there exist a positive number ν and a positive number N such that

$$|x'(t)| + |y'(t)| \ge \nu, \quad |x''(t)| + |y''(t)| \le N$$

for all time t in the interval $c \le t \le d$.

5. The Standardized Curvature Function

The essential step in the recognition of people's handwriting should be the recognition of the smooth segments. The difficulty comes from the fact that sizes, orientations, and speeds in people's writing all vary. However, we can resolve the difficulty from all these variations by using the curvature function of the curve.

It is well known that the curvature function of the curve can be calculated using the formula

$$\frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}.$$

Geometrically, the magnitude of the curvature equals the reciprocal of the radius of the tangent circle to the curve at the point $(x(t), y(t))$; the sign of the curvature indicates that the tangent direction of the curve is turning counterclockwise when positive and clockwise when negative. It is well known that if a curve has zero curvature everywhere, then it is a straight line segment. Likewise, if a curve has a constant curvature everywhere, then it is a circular arc.

To make the curvature function even more useful for our purpose we need the length parameter for the curve.

We need to first calculate the total length L of the curve by finding the definite integral

$$L = \int_c^d \sqrt{x'(t)^2 + y'(t)^2} dt.$$

Then for the rescaled curve

$$(\frac{1}{L}x(t), \frac{1}{L}y(t)), \quad c \le t \le d$$

we calculate its length function $s(t)$ and its curvature function $k(t)$:

$$\begin{aligned} s(t) &= \frac{1}{L}\int_c^t \sqrt{x'(t)^2 + y'(t)^2} dt, \\ k(t) &= L\frac{x'(t)y''(t) - x''(t)y'(t)}{(x'(t)^2 + y'(t)^2)^{3/2}}. \end{aligned}$$

On the s-k plane, $(s(t), k(t)), \ c \le t \le d$ gives the graph of a function $k = k(s), \ 0 \le s \le 1$. We call this function the *standardized curvature function* for the segment.

This standardized curvature function $k = k(s), \ 0 \le s \le 1$ is characteristic of the segment in the sense that two segments are geometrically similar if and only if their standardized curvature functions are the same. In other words, if two segments have the same standardized curvature function, then after a transformation consisting of a scaling, a translation, and a rotation, the points of one segment coincide with the points of the other.

Furthermore, two segments with close standardized curvature functions "resemble" each other in the sense we will specify. This closeness ensures the stability of the algorithm of using the curvature function to identify a smooth segment.

Here we note that in our calculations we need to discretize all the functions and their derivatives and integrals. We can make use of many advanced tools developed in numerical analysis such as splines and wavelets.

We also note that the software part of the intelligent writing pad would surely need to contain a database of all the standardized curvature functions for all regular, smooth segments of all strokes of the symbols and characters that are of interest. Because writing differs greatly from person to person, there should be a need for individualized database for all the curvature functions. By comparing the standardized curvature functions from what we write with those standardized curvature functions in the database, the intelligent writing pad would then recognize the symbol or character we write and have the task performed accordingly.

It goes without saying that other features in a character or symbol should be used in addition. Some, such as the number of strokes and the number of dots, can be determined without involving the curvature function. Others – such as the numbers of straight line segments, circular segments, stagnation points, and shifting points – can be determined at the same time when the curvature functions are calculated. The discrete set made of special points, subject to variations in size and orientation, should be very useful. The shifting angle at a shifting point, if present, should also be of value.

6. Mathematics of the Standardized Curvature Function

The mathematics for all our needs here is advanced for most engineering undergraduate students. We refer the interested reader to the books of DoCarmo [3], Birkhoff-Rota [1], or other textbooks on classical differential geometry and ordinary differential equations. For the numerical aspect of our work we refer to the book of Burden-Faires [2] or other textbooks on numerical analysis.

As discussed in the provious section, for any smooth segment of a parametric curve, there is a standardized curvature function $k(s),\ 0 \leq s \leq 1$. Now let us consider in the opposite direction. Suppose the standardized curvature function is known, and we see how we can recover the segment.

The system of linear ordinary differential equations

$$\begin{aligned} \frac{d^2}{ds^2}x(s) &= -k(s)\frac{d}{ds}y(s), \\ \frac{d^2}{ds^2}y(s) &= k(s)\frac{d}{ds}x(s) \end{aligned}$$

on $0 \leq s \leq 1$ with the initial condition

$$(x(0), y(0)) = (0,0), \quad (x'(0), y'(0)) = (1,0)$$

has a unique solution $(x(s), y(s))$ on the interval $0 \leq s \leq 1$. The points of the solution – after going through a transformation consisting of a scaling, a translation, and a rotation – coincide with the points of any segment whose standardized curvature function is $k(s),\ 0 \leq s \leq 1$.

The curve $(x(s), y(s)),\ 0 \leq s \leq 1$ has a total length of one, and the vector $(x'(s), y'(s))$ is of unit length for every s in the interval.

This basic existence and uniqueness theorem for a curve with a known curvature function is proved in differential geometry. Usually it is part of the so-called fundamental theorem for the differential geometry of space curves. It is the result of an application of the basic existence and uniqueness theorem for a system of linear ordinary differential equations with an initial condition.

Now suppose two continuous standardized curvature functions $k_1(s), k_2(s)$ satisfy

$$|k_1(s) - k_2(s)| \leq \epsilon \quad \text{for all} \;\; 0 \leq s \leq 1$$

for some small positive number ϵ. Also suppose that the two functions are uniformly bounded satisfying $|k_1(s)|, |k_2(s)| \leq K$ for a constant K and for all $0 \leq s \leq 1$. Let $(x_1(s), y_1(s)), (x_2(s), y_2(s))$ be the two curves from the two standardized curvature functions. Then

$$\begin{aligned} & |x_1(s) - x_2(s)| + |y_1(s) - y_2(s)| \\ & |x_1'(s) - x_2'(s)| + |y_1'(s) - y_2'(s)| \\ & |x_1''(s) - x_2''(s)| + |y_1''(s) - y_2''(s)| \\ \leq \;\; & \delta(\epsilon) \;\; \text{for all} \;\; 0 \leq s \leq 1 \end{aligned}$$

for some number $\delta(\epsilon)$ satisfying that $\delta(\epsilon)$ approaches zero as ϵ does.

Again, this resemblance between the two curves with close curvature functions follows from the stability of solutions of a system of ordinary differential equations.

7. Remarks

We point out that the computer recognition of people's handwriting recorded on a piece of paper, or in an electronic file such as a ps file serving the same purpose as the paper does, is a much more difficult problem. Note that valuable information involving the time is lost when people's handwritings are recorded on a piece of paper.

The computer recognition of special objects in a picture, or a digital file such as a jpeg file, is similarly difficult. For example, even when human eyes can tell easily that there is a horse in a picture, it is hard to write a computer program to perform the same act.

I learned about these problems years back from books and journals on popular computer sciences. For examples, in the book *The Road Ahead* by the well-known software architect Gates [5], there is a paragraph on the intelligent writing pad. In [4], computer scientists Forsyth, Malik, and Wilensky wrote about the computer recognition of objects in digital pictures. I was led to the idea of using the curvature function by these writers as well as by my previous mathematical works. In addition to [8] on an equality for the curvature function of a simple and closed curve on the plane, I used the curvature function to theoretically recognize certain curves as straight line segments in [6] and circular arcs in [7].

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