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COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACES

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Abstract: This paper presents some common fixed point theorems using weakly compatible mappings in fuzzy metric spaces.

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1. Introduction

Zadeh [15] introduced the notion of fuzzy set. Many researchers have been worked on fuzzy set and investigated new concepts in fuzzy set theory. Kramosil and Michalek [10] introduced concept of fuzzy metric space and George and Veeramani [4] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Many researchers have been proved common fixed point theorems for mappings satisfying different types of commutativity conditions. Vasuki [14] proved fixed point theorems for R-weakly commutating mappings. Recently Seong Hoon Cho [1] established fixed point theorems for compatible mappings in complete fuzzy metric spaces.

In this paper we generalized some common fixed point theorems in [1] using more general commutative condition, i.e. weakly compatible mappings in fuzzy metric space.

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2. Preliminary Notes

Definition 2.1. (see [15]) A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.2. (see [11]) A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norms if * is satisfying conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Definition 2.3. (see [4]) A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$,

$(f1)$ $M(x, y, t) > 0$;

$(f2)$ $M(x, y, t) = 1$ if and only if $x = y$;

$(f3)$ $M(x, y, t) = M(y, x, t)$;

$(f4)$ $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;

$(f5)$ $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.4. (Induced Fuzzy Metric, see [4]) Let (X, d) be a metric space. Denote $a * b = ab$ for all $a, b \in [0, 1]$ and let M_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

Example 2.5. (see [4]) Let $X = N$. Denote $a * b = \max\{0, a + b - 1\}$ for all $a, b \in [0, 1]$ and let M be fuzzy set on $X^2 \times (0, \infty)$ as follows;

$$M(x, y, t) = \begin{cases} \frac{x}{y} & \text{if } x \leq y, \\ \frac{y}{x} & \text{if } y \leq x, \end{cases}$$

for all $x, y \in X$ and $t > 0$. Then $(X, M, *)$ is a fuzzy metric space. Note that, in the above example, there exists no metric d on X satisfying

$$M(x,y,t) = \frac{t}{t + d(x,y)},$$

where $M(x,y,t)$ is as defined in the above example. Also note the above function M is not a fuzzy metric with the t-norm defined as $a * b = \min\{a,b\}$.

Definition 2.6. (see [4]) Let $(X, M, *)$ be a fuzzy metric space. Then:

(a) a sequence $\{x_n\}$ in X is said to be convergent to x in X if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x, t) > 1-\epsilon$ for all $n \geq n_0$.

(b) a sequence $\{x_n\}$ in X is said to be Cauchy if for each $\epsilon > 0$ and each $t > 0$, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$.

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.7. A pair of self-mappings (f, g) of a fuzzy metric space $(X, M, *)$ is said to be:

(i) weakly commuting [14] if $M(fgx, gfx, t) \geq M(fx, gx, t)$ for all $x \in X$ and $t > 0$.

(ii) R-weakly commuting [14] if there exists some $R > 0$ such that

$$M(fgx, gfx, t) \geq M(fx, gx, t/R)$$

for all $x \in X$ and $t > 0$.

Clearly here weak commutativity implies R-weakly commutativity but converse is not true.

Definition 2.8. (see [7]) Two self mappings f and g of a fuzzy metric space $(X, M, *)$ are called compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gxn = x$ for some x in X.

Lemma 2.9. *Let $(X, M, *)$ be a fuzzy metric space. If there exists $q \in (0,1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

Definition 2.10. (see [8]) A pair of maps S and T is called weakly compatible pair if they commute at coincidence points.

Once note that here compatible implies weakly compatible but reverse is not true, see example.

Example 2.11. Let $(X, d) = ([0, 10], |.|)$ and $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all

$t > 0$ and $M(x, y, 0) = 0$ for all $x, y \in X$. Define S and T by

$$Sx = \begin{cases} 3 & \text{if } 0 \in (0, 2], \\ 0 & \text{if } x \in \{0\} \cup (2, 10], \end{cases} \qquad Tx = \begin{cases} 0 & \text{if } x = 0, \\ x + 8 & \text{if } x \in (0, 2], \\ x - 2 & \text{if } x \in (2, 10]. \end{cases}$$

We have $Sx = Tx$ iff $x = 0$. $ST(0) = TS(0) = 0$. Then, (S, T) is weakly compatible. Let x_n be a sequence in X defined by $x_n = 2 + \frac{1}{n}, n \geq 1$. $Sx_n = S(2 + \frac{1}{n}) = 0$, $Tx_n = T(2 + \frac{1}{n}) = \frac{1}{n}$. $Sx_n, Tx_n \to 0$ as $n \to \infty$. $STx_n = S(\frac{1}{n}) = 3, TSx_n = T(0) = 0$. Since $\lim_{n\to\infty} |STx_n - TSx_n| = 3 \neq 0$, $M(STx_n, TSx_n, t) = \frac{t}{t+d(STx_n,TSx_n)} = \frac{t}{t+3} \neq 0$. (S, T) is not compatible.

Lemma 2.12. *Let $(X, M, *)$ be a fuzzy metric space. Then for all $x, y \in X$, $M(x, y, .)$ is nondecreasing.*

Lemma 2.13. *Let $(X, M, *)$ be a fuzzy metric space. If there exists $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$, then $x = y$.*

Seong Hoon Cho [1] proved the following theorem.

Theorem 2.14. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X such that the following conditions are satisfied:*

(i) *$AX \subset TX, BX \subset SX$,*

(ii) *S and T are continuous,*

(iii) *the pairs $[A, S]$ and $[B, T]$ are compatible,*

(iv) *there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X.

3. Main Results

Theorem 3.1. *Let $(X, M, *)$ be a fuzzy metric space and let A, B, S and T be self mappings of X such that the following conditions are satisfied:*

(i) *$A(X) \subset T(X)$, $B(X) \subset S(X)$.*

(ii) *There exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t).$$

(iii) If one of $A(X), B(X), S(X)$, or $T(X)$ is complete subspace of X, then:

(a) A and S have a coincidence.

(b) B and T have a coincidence.

Further, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof. Given that $A(X) \subset T(X)$ and $B(X) \subset S(X)$. Let $x_0 \in X$ be any arbitrary. There exists some $x_1 \in X$ such that $Ax_0 = Tx_1$ and for $x_1 \in X$. There exists $x_2 \in X$ such that $Bx_1 = Sx_2$. Continuing in this way, we get a sequence $\{y_n\}$ in X as follows: $y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$ for $n = 1, 2, \cdots$. Therefore from (ii),

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, qt) &= M(Ax_{2n}, Bx_{2n+1}, qt) \\ &\geq *M(Sx_{2n}, Tx_{2n+1}, t)M(Ax_{2n}, Sx_{2n}, t) \\ &\quad * M(Bx_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t) \\ &= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+2}, y_{2n+1}, t) \\ &\quad * M(y_{2n+1}, y_{2n+1}, t) \\ &\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t). \end{aligned}$$

Therefore we have that

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t). \tag{1}$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2,t}). \tag{2}$$

From equations (1) and (2), we have

$$M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t). \tag{3}$$

From (3)

$$\begin{aligned} M(y_n, y_{n+1}, t) &\geq M(y_n, y_{n-1}, t/q) \geq M(y_{n-2}, y_{n-1}, t/q^2) \\ &\geq \cdots \geq M(y_1, y_2, t/q^n) \to 1 \end{aligned} \tag{4}$$

as $n \to \infty$. So $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$ for any $t > 0$. For each $\epsilon > 0$ and each $t > 0$, we can choose $n_0 \in N$ such that $M(y_n, y_{n+1}, t) > 1 - \epsilon$ for all $n > n_0$. For $m, n \geq N$, we suppose $m \geq n$. Then we have that

$$\begin{aligned} M(y_n, y_m, t) &\geq M(y_n, y_{n+1}, t/m-n) * M(y_{n+1}, y_{n+2}, t/m-n) \\ &\quad * \cdots * M(y_{m-1}, y_m, t/m-n) \\ &> \overbrace{(1-\epsilon) * (1-\epsilon) * \cdots * (1-\epsilon)}^{m-n} \geq (1-\epsilon) \end{aligned}$$

and hence $\{y_n\}$ is a Cauchy sequence in X.

Since $(X, M, *)$ is complete, $\{y_n\}$ converges to some point $z \in X$, and so $\{Ax_{2n-2}\}, \{Sx_{2n}\}, \{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converges to z.

Suppose $S(X)$ is complete, then the subsequence $\{Sx_{2n}\}$ has limit in $S(X)$, i.e. $Sx_{2n} \to u$ for some $u \in S(X)$ and there exist $v \in X$ such that $Sv = u$. We claim that $Av = u$. We put $x = v$ and x_{2n-1} in (ii)

$$M(Av, Bx_{2n-1}, qt) \geq M(Sv, Tx_{2n-1}, t) * M(Av, Sv, t) * M(Bx_{2n-1}, Tx_{2n-1}, t) * M(Av, Tx_{2n-1}, t).$$

As $n \to \infty$

$$\begin{aligned} M(Av, u, qt) &\geq M(u, u, t) * M(Av, u, t) * M(u, u, t) * M(Av, u, t) \\ &\geq M(Av, u, t) \end{aligned}$$

which implies that $Av = u$. Since $A(u) \subset T(X)$, so $u \in T(X)$ and therefore there exists $w \in X$ such that $Tw = u$. We claim that $Bw = u$. In (ii) put $x = x_{2n}$ and $y = w$

$$M(Ax_{2n}, Bw, qt) \geq M(Sx_{2n}, Tw, t) * M(Ax_{2n}, Sx_{2n}, t) * M(Bw, Tw, t) * M(Ax_{2n}, Tw, t)$$

as $n \to \infty$

$$M(u, Bw, qt) \geq M(u, u, t) * M(u, u, t) * M(Bw, u, t) * M(u, u, t)$$

which implies $Bw = u$.

If we assume $T(X)$ is complete, then arguments analogous to the previous completeness argument establish (a) and (b).

If $A(X)$ is complete, then by (i) $u \in A(X) \subset T(X)$. Similarly if $B(X)$ is complete, then by (i) $u \in B(X) \subset S(X)$. Thus in any case of completeness of $A(X), B(X), S(X)$ and $T(X)$ we have $u = Av = Sv = Bw = Tw$. Note that (A, S) and (B, T) are weakly compatible at v and w respectively. So $Au = ASv = SAv = Su$ and $Bu = BTw = TBw = Tu$. If $Bu \neq u$, then by (ii)

$$\begin{aligned} M(u, Bu, qt) &= M(Av, Bu, qt) \\ &\geq M(Sv, Tu, t) * M(Av, Sv, t) * M(Bu, Tu, t) * M(Av, Tu, t) \\ &= M(u, Bu, t) * M(u, u, t) * M(Bu, Bu, t) * M(u, Bu, t) \\ &\geq M(u, Bu, t) \end{aligned}$$

which implies that $Bu = u$. Similarly $Au = u$. So $Au = Su = Bu = Tu = u$. Thus u is a common fixed point of A, B, S and T. Uniqueness follows from (ii) easily.

Corollary 3.2. *Let $(X, M, *)$ be a complete fuzzy metric space and let*

A, B, S and T be self mappings of X satisfying:

(i) $A(X) \subset T(X), B(X) \subset S(X)$.

(ii) *There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t).$$

(iii) *If one of $A(X), B(X), S(X)$, or $T(X)$ is complete subspace of X, then:*

(a) *A and S have a coincidence.*

(b) *B and T have a coincidence.*

Further, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof. We have

$$\begin{aligned} M(Ax, By, qt) &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) \\ &\quad * M(By, Sx, 2t) * M(Ax, Ty, t) \\ &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Sx, Ty, t) \\ &\quad * M(Ty, By, t) * M(Ax, Ty, t) \\ &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t) \end{aligned}$$

and hence, from Theorem 3.1, A, B, S and T have a unique fixed point in X. □

Corollary 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying:*

(i) $A(X) \subset T(X), B(X) \subset S(X)$.

(ii) *There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t).$$

(iii) *If one of $A(X), B(X), S(X)$, or $T(X)$ is complete subspace of X, then:*

(a) *A and S have a coincidence.*

(b) *B and T have a coincidence.*

Further, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof. We have

$$\begin{aligned} M(Ax, By, qt) &\geq M(Sx, Ty, t) \\ &= M(Sx, Ty, t) * 1 \\ &\geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) \end{aligned}$$

$$* M(Ty, Ax, t)$$

and hence, from Corollary 3.2, A, B, S and T have a unique fixed point in X. □

Corollary 3.4. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying:*

(i) $A(X) \subset T(X), B(X) \subset S(X)$.

(ii) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t).$$

(iii) If one of $A(X), B(X), S(X)$, or $T(X)$ is complete subspace of X, then:

(a) A and S have a coincidence.

(b) B and T have a coincidence.

Further, if (A, S) and (B, T) are weakly compatible, then A, B, S and T have a unique common fixed point in X.

Proof. We have

$$\begin{aligned} M(Ax, By, qt) &\geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) \\ &= M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) * 1 \\ &\quad M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) * M(Sx, By, 2t) \\ &\quad * M(By, Ty, t) * M(Ty, Sx, t) \\ &\geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t) * M(Sx, By, 2t) \\ &\quad * M(By, Ty, t) \end{aligned}$$

and hence, from Corollary 3.2, A, B, S and T have a unique fixed point in X. □

Theorem 3.5. *Let $(X, M, *)$ be a fuzzy metric space. Then self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping A of X such that the following conditions are satisfied:*

(i) $A(X) \subset T(X) \cap S(X)$.

(ii) If one of $A(X), S(X)$, or $T(X)$ is complete subspace of X.

(iii) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t).$$

(iv). The pairs (A, S) and (A, T) are weakly compatible.

Then A, S and T have a unique common fixed point in X.

Proof. Since S and T have a common fixed point in X, say z. Then

$Sz = z = Tz$. Let $Ax = z$ for all $x \in X$. Then we have $A(X) \subset T(X) \cap S(X)$ and given that (A, S) and (A, T) are weakly compatible, (i) and (ii) and (iv) are satisfied. For some $q \in (0, 1)$, we get

$$M(Ax, Ay, qt) = 1 \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$$

for every $x, y \in X$ and $t > 0$ and hence the condition (iii) is satisfied. Now, for the sufficiency of the conditions, let $A = B$ in Theorem 3.1. Then A, S and T have a unique common fixed point in X. □

Corollary 3.6. *Let $(X, M, *)$ be a complete fuzzy metric space. Then self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping A of X satisfying $(i), (ii)$ and (iv) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Sx, 2t) * M(Ax, Ty, t).$$

Then A, S and T have a unique common fixed point in X.

Proof. The proof follows from Corollary 3.2 and Theorem 3.5. □

Corollary 3.7. *Let $(X, M, *)$ be a fuzzy metric space. Then self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping A of X satisfying $(i), (ii)$ and (iv) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t).$$

Then A, S and T have a unique common fixed point in X.

Proof. The proof follows from Corollary 3.3 and Theorem 3.5. □

Corollary 3.8. *Let $(X, M, *)$ be a fuzzy metric space. Then continuous self mappings S and T of X have a common fixed point in X if and only if there exists a self mapping A of X satisfying $(i), (ii)$ and (iv) of Theorem 3.5 and there exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$,*

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t).$$

Then A, S and T have a unique common fixed point in X.

Proof. The proof follows from Corollary 3.4 and Theorem 3.5. □

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