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CONSTRUCTION AND SELECTION OF REPETITIVE DEFERRED SAMPLING (RDS) PLAN THROUGH ACCEPTABLE AND LIMITING QUALITY LEVELS

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Abstract: This paper mainly relates to the designing of RDS plan through AQL and LQL with their relative slopes on the OC curve and has been compared with RGS plan with respect to ratio of relative slopes. Tables are also provided with numerical illustrations.

Key Words: acceptable quality level, limiting quality level, acceptance sampling, consumer's risk, producer's risk, and repetitive deferred sampling plan

1. Introduction

Sampling is widely used in industry and government for controlling the quality of shipment of components, supplies and final products. In general, the number marking the boundary between "a few" and "too many" defectives, (the maximum acceptable number of defectives) varies depending on the situation. This number depends on the lot size, cost of inspecting and testing a part and an assembly, cost of dismantling and repairing an assembly, loss associated with the possible failure to meet customer requirements, etc.

In order to control the quality of purchased lot, two major alternatives are open to a buyer. One, complete inspection: every single item in the lot is inspected and tested. Two, partial inspection: a sample of items is taken, the sampled items are inspected and tested, and the lot as a whole is accepted or

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rejected depending on whether few or many defective items which are found in the sample. This type of sampling, one of many used to control the quality of manufacturing processes or lots, is known as acceptance sampling

2. Review of Repetitive Deferred Sampling Plan

The RDS plan has been developed by Sankar and Mahopatra (1991) and this plan is essentially an extension of the multiple deferred sampling plan MDS - (c_1, c_2) which was proposed by Rambert Vaerst (1980).

In this plan the acceptance or rejection of a lot in deferred state is dependent on the inspection results of the preceding or succeeding lots under repetitive group sampling (RGS) inspection. RGS is the particular case of RDS plan. Wortham and Baker (1976) have developed multiple deferred state sampling (MDS) plans and also provided tables for construction of plans. Suresh (1992) has proposed procedures to select deferred state sampling plan indexed through AQL and LQL. Suresh (1993) has proposed procedures to select Multiple Deferred State plan of type MDS and MDS - 1 indexed through producer and consumer quality levels considering filter and incentive effects.

Vedaldi (1986) has studied the two principal effects of sampling inspection which are filter and incentive effect for attribute Single Sampling plan and also proposed a new criterion based on the (AQL, $1-\alpha$) point of the OC curve and an incentive index.

Lilly Christina (1995) has given the procedure for the selection of RDS plan with given acceptable quality levels and also compared RDS plan with RGS plan with respect to operating ratio (OR) and ASN curve. Santhiya (2004) has given the procedure for the selection of RDS plan indexed with MAAOQ.

3. Conditions for the Application of RDS Plan

1. Production is steady so that result of past, current and future lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of their production.
3. A fixed sample size, n from each lot is assumed.
4. Inspection by attributes with quality defined as fraction non-conforming.

4. Operating Procedure for RDS Plan

1. Draw a random sample of size n from the lot and determine the number of defectives (d) found therein.

2. Accept the lot if $d \leq c1$. Reject the lot if $d > c_2$.

3. If $c_1 < d < c_2$, accept the lot provided "i" proceeding or succeeding lots are accepted under RGS inspection plan, otherwise reject the lot.

Hear c_1 and c_2 are acceptance number such that $c_1 < c_2$. When $i = 1$ this plan reduces to RGS plan.

The operating characteristic function $P_a(p)$ of RDS plan is derived by Sanker and Mahopatra (1991) using the Poisson model as

$$P_a(p) = \frac{P_a(1-P_c)^i + P_c P_a^i}{(1-P_c)^i},$$

where

$$p_a = P[d \leq c_1] = \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!},$$

$$p_c = P[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x} x^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x} x^r}{r!},$$

where $x = np$.

5. Designing of RDS Plan for Given Acceptable Quality Level

Hamaker (1950a) considered two important features of the OC curves namely the place where the OC curve shows its steeper descent and the degree of its steepness, as the basis for two indices namely, the IQL (p_0) and the relative slope of the OC curve at $(p_0, 0.05)$ denoted as h_0, which may be used to design any sampling plan. Hamaker (1950b) has given simple empirical relations existing between the sample size and the acceptance number; between the parameters p0 and h0 under the conditions for application of Poisson, Binomial and Hypergeometric models for single sampling attributes plan. Soundararajan and Muthuraj (1985) have given procedures and tables for designing single sampling attribute plans for specified p_0 and h_0.

The proportion nonconforming corresponding to the inflection point of the OC curve as denoted by p_* and interpreted as Maximum Allowable Percent Defective (MAPD) by Mayer (1967) used as the quality standard along with some other condition for the selection of sampling plans. The relative slope of the OC curve at this point denoted as h_*, also be used to fix the discrimination of the OC curve for any sampling plan.

The desirability of developing a set of sampling plans indexed through p_* has been explained by Mandelson (1962) and Soundararajan (1971). Sampling plan can be selected for given p_* and k ($k = p_T/p_*, p_T$ is the point at which the tangent line at the inflection point of the OC curve cuts the p - axis) the measure of discrimination or with h_*, the relative slope of the OC curve at p_*. Soundararajan and Muthuraj (1985) have provided the procedures and tables for designing single sampling attributes plan for given p_* and h_*.

6. Examples

1. For given i, p_1 and h_1 value one gets RDS plan using table 1. For example, given $i = 1, p_1 = 0.046$ and $h_1 = 1.065$. Using table 1 under the column headed h_1, one gets a tabulated value of $h_1 = 1.0710$ which is closest to or just greater than desired value. Corresponding to this h_1 value one also gets $np_1 = 1.6535, c_1 = 1, c_2 = 4$. Now the sample size is obtained as $n = np_1/p_1 = 35.7 \approx 36$. Thus the plan is (36, 1, 4, 1)

2. For given i, p_1 and h_1 value one gets RDS plan using table 3. For example, given $i = 3, p_1 = 0.0256$ and $h_1 = 0.73$. Using table 3 under the column headed h_1, one gets a tabulated value of $h_1 = 0.7402$ which is closet to or just greater than desired value. Corresponding to this h_1 value one also gets $np_1 = 1.6785, c_1 = 2, c_2 = 4$. Now the sample size is obtained as $n = np_1/p_1 = 65.56 \approx 66$. Thus the plan is (66, 2, 4, 3)

7. Selection Through Ratio of Relative Slopes

Cameron (1952) had provided tables for Constructing and for Computing the Operating Characteristics for Single Sampling Plan

Table 4 is used select the RDS plan for specified AQL (or LQL) with h_2/h_1. For example, given $p_1 = 0.042, h_1 = 3.1956$ and $h_2 = 4.3368$, one finds that $h_2/h_1 = 1.3574$. By using table 4, under the column headed by h_2/h_1, one can locate the value, which is equal to or just greater than the desired ratio. Corresponding to the located value, one finds the other values $np_1 = 3.1440, c_1 = 3, c_2 = 7$ and $i = 4$. Now the sample size is obtained as $n = np_1/p_1 = 74.86 \approx 75$. Thus the plan is (75, 3, 7, 4). If more than one plan is found for the ratio h_2/h_1, choice among these can be made on the desirability of either economy or higher discrimination.

8. Comparison Between RDS Plan with RGS Plan

The standard method of comparison between any two lot inspection plans is to compare with their operating characteristic curves. An approach to this is presented by means of their ratio of relative slopes, For two plans that are well matched, will have ratio of relative slopes that are very nearly the same and identical.

However, it should be noted that exact matching may not be possible due to the basically different nature of the sampling procedures. Two plans will be considered matched if their OC curves are identical. Here to compare the plans one employs a measure called ratio of relative slopes.

Values of h_1 and h_2 as well as h_2/h_1, for a wide range of RDS $(n; c_1, c_2; i)$ plans are presented in tables 1 to 5. Now the RDS $(n; c_1, c_2; i)$ plan is compared with RGS plan. The Table 1 gives RGS plan parameters and Tables 2 to 5 gives RDS plan parameters.

An RDS plan with $h_1 = 0.5557$ and $h_2 = 4.4125$ and $i = 2$ is given. Compute $h_2/h_1 = 7.9404$. From Table 2 the h_2/h_1 value closest to 7.9404 is 7.9150. The corresponding c_1, c_2 and np_1 values are $c_1 = 3, c_2 = 5$ and $np_1 = 2.3840$ respectively. If p_1 is given to be 0.029 hence the sample size is $n = np_1/p_1 = 82$.

For given $h_2/h_1 = 7.9150$, the h_2/h_1 of RGS plan is obtained from Table 1 as 7.9675, which corresponds to $c_1 = 4, c_2 = 5$ and $np_1 = 2.5650$. The sample size is then obtained as $n = np_1/p_1 = 89$. Thus for given h_1 and h_2 the RDS plan has the sample size smaller than the matched RGS plan. Thus the RGS plan (89, 4, 5) and the RDS plan (82, 3, 5, 2) are matched with h_1 and h_2.

9. Conversion of Parameters

One likes to convert a given set of parameters into another familiar set, to get information on related parameters. For example when $p_1 = 0.01$ and $h_1 = 0.6092$ are specified the other equivalent set of parameters are found using Tables 1 to 5. Corresponding to $h_1 = 0.6092$, one finds using these Tables $c_1 = 5, c_2 = 7, np_1 = 3.6870, np_2 = 9.2810, np_0 = 6.0970, np_* = 5.6043, nAOQL = 3.8377, np_m = 4.6180, h_0 = 2.7456, h_* = 2.1775$ and $h_2 = 5.0182$. Dividing np1 by p_1, one gets $n = 369$ with $p_2 = 0.025, p_0 = 0.017, p_* = 0.015$, AOQL = 0.01. Thus when $p_1 = 0.01$ and $h_1 = 0.6092$ the other set of parameters are,

1. $p_1 = 0.01$ $(\alpha = 0.05)$; $p_2 = 0.025$ $(\beta = 0.10)$,
2. $p_1 = 0.01$ $(\alpha = 0.05)$; AOQL $= 0.010$,

3. $p_0 = 0.017$; $h_0 = 2.7456$,
4. $p_* = 0.015$; $h_* = 2.1775$,
5. $p_1 = 0.025$ $(\beta = 0.10)$; $h_1 = 5.0182$.

A similar procedure is used for converting parameters when the values of p_2 and h_2 are specified.

10. Construction of Tables

The operating characteristics function for the RDS plan under Poisson model is given as

$$P_a(p) = \frac{P_a(1-P_c)^i + P_c P_a^i}{(1-P_c)^i},$$

where

$$p_a = P[d \le c_1] = \sum_{r=0}^{c_1} \frac{e^{-x}x^r}{r!},$$

$$p_c = P[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x}x^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x}x^r}{r!},$$

where $x = np$.

The relative slope of the OC curve is given as

$$h = -\frac{p}{P_a(p)}\left[\frac{dP_a(p)}{dp}\right]$$

at $p = p_*$.

The values of relative slopes at AQL and LQL are h1 and h2 values, which are calculated using the np_1 and np_2 values in the formulas

$$h_1 = -\frac{p_1}{P_a(p_1)}\left[\frac{dP_a(p_1)}{dp}\right]$$

at $p = p_1$,

$$h_2 = -\frac{p_2}{P_a(p_2)}\left[\frac{dP_a(p_2)}{dp}\right]$$

at $p = p_2$.

11. Conclusion

Acceptance sampling is the technique, which deals with procedures in which decision to accept or reject lots or process based on the examination of samples. The present work mainly relates to the construction and selection of tables for Repetitive Deferred Sampling plan through relative slope on the OC curve at AQL and LQL. Conversions of parameters are also given for convenience. Matching of RDS plan with RGS plan is also provided. Tables are provided here which tailor-made, handy and ready-made use to the industrial shop-floor condition.

References

[1] J.M. Cameron, Tables for constructing and for computing the quality characteristics of single sampling plans, *Industrial Quality Control*, **9** (1952), 37-39.

[2] G. Sankar, B.N. Mahopatra, GERT analysis of repetitive deferred sampling plans, *IAPQR Transactions*, **16**, No. 2 (1991), 17-25.

[3] Hamakar, The theory of sampling inspection plans, *Philips Technical Review*, **14**, No. 9 (1950), 260-270.

[4] A. Lilly Christina, *Contribution to the Study of Design and Analysis of Suspension System and Some other Sampling Plans*, Ph.D. Thesis, Bharathiar University, Tamilnadu, India (1995).

[5] J. Mandelson, The statistician, the engineer and sampling plans, *Industrial Quality Control*, **19** (1962), 12-15.

[6] P.L. Mayer, A note on Poisson probabilities and an application, *Annals of Institute of Statistical Mathematics*, **19** (1967), 537-542.

[7] R.E. Sherman, Design and evaluation of repetitive group sampling plan, *Techno Metrics*, **17**, No. 1 (1965), 11-21.

[8] K.S. Stephens, H.F. Dodge, Comparison of chain sampling plans with single and double sampling plans, *Journal of Quality Technology*, **8**, No. 1 (1976), 24-33.

[9] K.K. Suresh, Designing deferred state sampling plan indexed by AQL and LQL, *Unpublished* (1992).

[10] K.K. Suresh, *A Study on Acceptance Sampling using Acceptable and Limiting Quality Levels*, Ph.D. Thesis, Bharathiar University, Tamilnadu (1993).

[11] Rembert, Vaerst, A procedure to construct multiple deferred state sampling plan, *Method of Operation Research*, **37** (1981), 477-485.

[12] R. Vedaldi, A new criterion for the construction of single sampling inspection plans by attributes, *Rivista di Statistica Applicata*, **19**, No. 3 (1986), 235-244.

[13] A.W. Wortham, R.C. Baker, Deferred state sampling procedures, *Annals of Assurance Science* (1971), 65-70.

[14] A.W. Wortham, R.C. Baker, Multiple deferred state sampling inspection, *The International Journal of Production Research*, **14**, No. 6 (1976), 719-731.