

ON EFFECTS OF A TRANSVERSE MAGNETIC FIELD ON
AN UCM FLUID OVER A STRETCHING SHEET

Ch. Mamaloukas^{1 §}, M. Subhas Abel², J.V. Tawade³, U.S. Mahabaleswar⁴

¹Department of Informatics
Athens University of Economics and Business
76, Patision Str., Athens, 10434, GREECE
e-mail: mamkris@aueb.gr

^{2,3}Department of Mathematics
Gulbarga University
Gulbarga, Kamataka, 585 106, INDIA

²e-mail: msabel2001@yahoo.co.uk
³e-mail: jagadish_maths@yahoo.co.in

⁴Department of Mathematics
Bangalore University
Mangalagangothri, Bangalore, 574 199, INDIA
e-mail: usmahabalesh@yahoo.co.in

Abstract: The effects of a transverse magnetic field within a boundary layer flow on an upper-convected Maxwell (UCM) fluid over a stretching sheet is examined in this paper. Similarity transformations are used to non-dimensionalize the governing boundary layer equations of motion. The resulting ordinary differential equations are then solved numerically by using efficient numerical shooting technique with fourth order Runge–Kutta algorithm [11]. The objective of the present work is to investigate the effect of elastic parameter β , and magnetic parameter Mn on the temperature field above the sheet. We compare the results obtained with the results of Hayat et al [8] by several graphs.

AMS Subject Classification: 35Q30, 35A35

Key Words: UCM fluid, boundary layer, stretching sheet, similarity transformation, magnetic parameter, shooting technique

Received: May 3, 2010

© 2011 Academic Publications

[§]Correspondence author

1. Introduction

The study of non-Newtonian fluid flows over a stretching sheet find applications in many manufacturing processes such as extrusion of molten polymers through a slit die for the production of plastic sheets, wire and fibre coating, foodstuff processing, etc. In a typical sheet production process, the extrudate starts to solidify as soon as it exits from the die. The quality of the final product depends on the rate of heat transfer and therefore cooling procedure has to be controlled effectively.

Sarpakaya (see [6]) was the first researcher who studied the MHD flow of a non-Newtonian fluid. Prandtl's boundary layer theory proved to be of great use in Newtonian fluids as Navier-Stokes equations can be converted into much simplified boundary layer equation which is easier to handle.

Crane [10] was the first among others who considered the steady of two-dimensional flow of a Newtonian fluid driven by a stretching elastic flat sheet which moves in its own plane with a velocity varying linearly with the distance from a fixed point.

For theoretical results of any industrial significance, realistic viscoelastic fluid models such as upper-convected Maxwell model or Oldroyd-B model are invoked in the analysis. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids (see [7]-[13]).

The focal point in the present work is to investigate numerically the MHD flow and heat transfer of a UCM fluid above a semi-infinite stretching sheet. To achieve this goal, use will be made of a recent analysis carried out by Hayat et al [8] in which the velocity field above the sheet was calculated for MHD flow of a Maxwell fluid with no heat transfer involved using homotopy analysis method (HAM).

2. Formulation of the Problem

For an MHD flow of an incompressible UCM fluid resting above a stretching sheet, which is stretched linearly, the equations governing transport of heat and momentum can be written as (see [8], [15])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

β	Hayat et. al (Porous sheet)		Present Results (Non-porous sheet)	
	$M=0.0$	$M=0.2$	$M=0.0$	$M=0.2$
0.0	-1.90250	-1.94211	-0.999962	-1.095445
0.4	-2.19206	-2.23023	-1.101850	-1.188270
0.8	-2.50598	-2.55180	-1.196692	-1.275878
1.2	-2.89841	-2.96086	-1.285257	-1.358733
1.6	-3.42262	-3.51050	-1.368641	-1.437369
2.0	-4.13099	-4.25324	-1.447617	-1.512280

Table 1: Comparison of values of skin friction coefficient $f''(0)$ for different elastic parameters with $M = 0.0$ and $M = 0.2$

$$u + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \lambda \left[u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right] - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2}, \quad (3)$$

where B_0 , is the strength of the magnetic field, ν is the kinematic viscosity of the fluid and λ is the relaxation time parameter of the fluid.

As to the boundary conditions, we have adopted the following two kinds of boundary heating:

$$\left. \begin{array}{l} \text{(i) Prescribed Power-Law Surface Temperature (PST)} \\ u = Bx; \quad v = 0; \quad T = T_w(x) = T_0 - T_s \left(\frac{x}{L}\right)^2 \quad \text{at } y = 0. \\ \quad \quad \quad u \rightarrow 0; \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty; \\ \text{(ii) Prescribed Power-Law Heat Flux (PHF)} \\ u = Bx; \quad q_w = -k \left(\frac{\partial T}{\partial y}\right)_w = b \left(\frac{x}{L}\right)^2 \quad \text{at } y = 0. \\ \quad \quad \quad u \rightarrow 0; \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty, \end{array} \right\} \quad (4)$$

where T_w is the wall temperature, T_∞ is the temperature far away from the sheet, T_0 is the melt temperature at the die exit and $T - T_s$ is the melt solidification temperature ($T < T_s < T_\infty$). It is to be noted that in the above relationships, L is the distance between the die exit and the point at which the melt solidifies, k is the thermal conductivity of the fluid and b is a constant whose value also depends on the fluid.

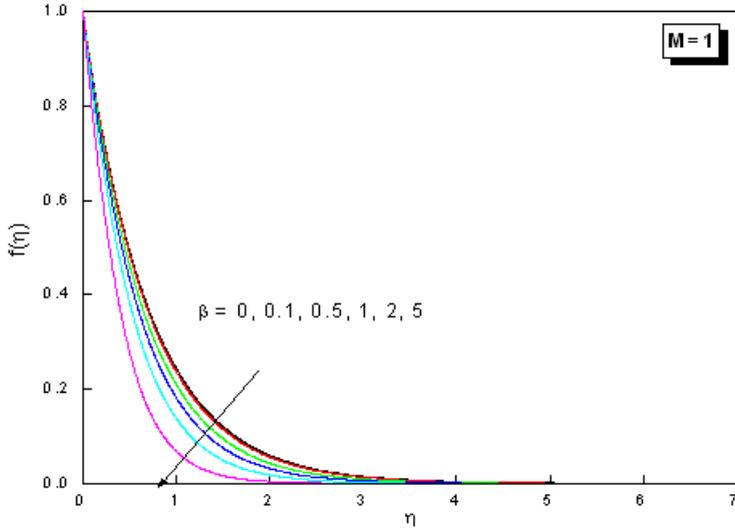


Figure 1: The effect of elastic parameter on u -velocity component

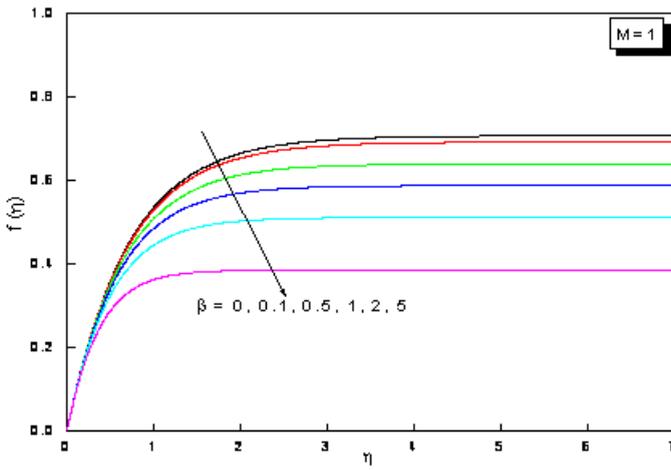


Figure 2: The effect of elastic parameter on v -velocity component

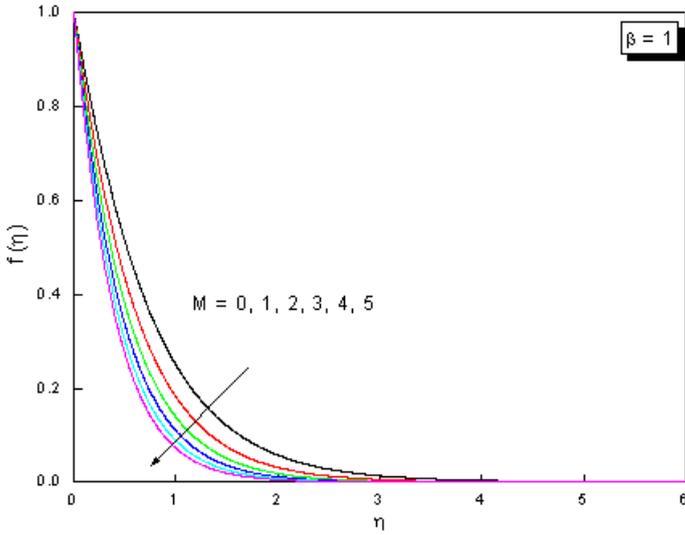


Figure 3: The effect of magnetic parameter on u -velocity component

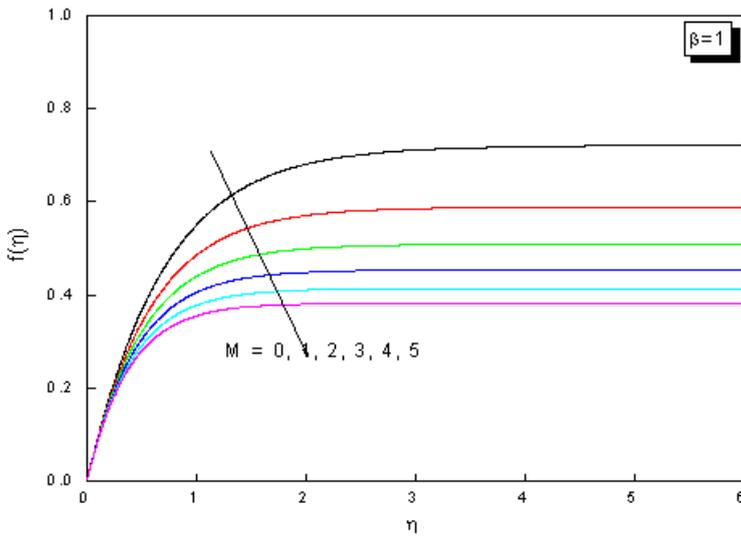


Figure 4: The effect of magnetic parameter on v -velocity component

3. Method of Solution

Introducing dimensionless variables and the similarity variable (see [10], [9]) as

$$\left. \begin{aligned} u &= Bx f'(\eta), \quad v = \sqrt{vB} f(\eta), \quad \eta = \left(\frac{B}{v}\right)^{\frac{1}{2}} y, \\ \theta(\eta) &= \frac{T-T_\infty}{T_w-T_\infty}, \quad g(\eta) = \frac{T-T_\infty}{b\left(\frac{x}{l}\right)^2 \frac{1}{k} \sqrt{\frac{x}{b}}}, \end{aligned} \right\} \quad (5)$$

the governing equations (1)-(3) can be transformed exactly into a set of ordinary differential equations:

$$f''' - Mf' - (f')^2 + f f'' + \beta (2f f' f'' - f^2 f''') = 0, \quad (6)$$

$$\text{Pr} [2f'\theta - \theta'f] = \theta'', \quad (7)$$

$$\text{Pr} [2f'g - g'f] = g'', \quad (8)$$

and their associated boundary conditions:

$$f = 0; \quad f' = 1; \quad \theta = 1; \quad g' = -1, \quad \text{at} \quad \eta = 0, \quad (9)$$

$$f' = 0; \quad \theta = 0; \quad g = 0, \quad \text{as} \quad \eta \rightarrow \infty, \quad (10)$$

where $M = \frac{\sigma B_0^2}{\rho B}$, is a magnetic parameter and $\beta = 2B$, is the elastic parameter.

The non-linear differential equations (6), (7) and (8) with appropriate boundary conditions given in (9) and (10) are first decomposed into a system of first order differential equations. The resulting initial value problem (IVP) then can be solved numerically by the most efficient numerical shooting method technique. The convergence criterion largely depends on fairly good guesses of the initial conditions in the shooting technique. Once the convergence is achieved we integrate the resultant ordinary differential equations using standard fourth order Runge-Kutta method with the given set of parameters to obtain the required solution (see [11]).

4. Results and Discussion

The numerical results obtained in the present work are compared with some of the earlier published results in some limiting cases and are shown in Table 1.

Furthermore Figures 1 and 2 show the effect of magnetic parameter, M , in the absence of Elastic parameter on the velocity profile above the sheet. An increase in the magnetic parameter leads in decrease of both u - and v -velocity components at any given point above the sheet. This is due to the fact that applied transverse magnetic field produces a drag in the form of Lorentz force

thereby decreasing the magnitude of velocity. The drop in horizontal velocity as a consequence of increase in the strength of magnetic field is observed.

Moreover, Figures 3 and 4 show the effect of magnetic parameter on the temperature profiles above the sheet for both PST and PHF cases. An increase in the magnetic parameter is seen to increase the fluid temperature above the sheet. That is, the thermal boundary layer becomes thicker as larger the magnetic parameter.

In conclusion, we observe that, when the magnetic parameter increases, the velocity decreases, also, for increase in elastic parameter, results in decrease of velocity. The effect of magnetic field and elastic parameter on the UCM fluid above the stretching sheet is to suppress the velocity field, which in turn causes the enhancement of the temperature field.

A drop in skin friction as investigated in this paper has an important implication that in free coating operations, elastic properties of the coating formulations may be beneficial for the whole process. This means that less force may be needed to pull a moving sheet at a given withdrawal velocity or equivalently higher withdrawal speeds can be achieved for a given driving force resulting in, increase in the rate of production, see [2]. A drop in skin friction with increase in elastic parameter as observed in Table 1 gives the comparison of present results with that of Hayat et al [8]. Without any doubt, from this table, we can claim that our results are in excellent agreement with that of Hayat et al [8].

Acknowledgments

The authors are thankful to DST-New Delhi for financial support to pursue this work under DST-Major Research Project (Grant No. SR/S4/MS: 198/03).

References

- [1] V. Aliakbar, A. Alizadeh-Pahlavan, K. Sadeghy, The influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets, *Communications in Nonlinear Science and Numerical Simulation*, **14**, No. 3 (2009), 779-794.
- [2] K.R. Rajgopal, *Boundary Layers in Non-Newtonian Fluids* (Ed-s: M.D.P. Montiero Marques, J.F. Rodrigues).

- [3] A. Alizadeh-Pahlavan, V. Aliakbar, F. Vakili-Farahani, K. Sadeghy, MHD flows of UCM fluids above porous stretching sheets using two-axillary-parameter homotopy analysis method, *Commun. Nonlinear Sci. Numer. Simulat.*, **14** (2009), 473-488.
- [4] A. Alizadeh-Pahlavan, K. Sadeghy, On the use of homotopy analysis method for solving unsteady MHD flow of Maxwellian fluids above impulsively stretching sheets, *Communications in Nonlinear Science and Numerical Simulation*, **14**, No. 4 (2009), 1355-1365.
- [5] H.I. Andersson, K.H. Bech, B.S. Dandapat, Magneto hydrodynamic flow of a power-law fluid over a stretching sheet, *Int. J. Non-linear Mech.*, **27** (1992), 929-936.
- [6] L.J. Crane, flow past a stretching plate, *Z. Angrew. Math. Phys.*, **21** (1970), 645-647.
- [7] V.G. Fox, L.E. Erickson, L.T. Fan, The laminar boundary layer on a moving continuous flat sheet immersed in a non-Newtonian fluid, *AIChE J.*, **15**, No. 3 (1969), 327-333.
- [8] T. Hayat, Z. Abbas, M. Sajid, Series solution for the upper-convected Maxwell fluid over a porous stretching plate, *Phys. Lett A*, **358** (2006), 396-403.
- [9] Ch. Mamaloukas, S. Spartalis, An exact solution of a viscous incompressible electrically conducting fluid over an elastic-stretching sheet, *International Conference on Numerical Analysis and Applied Mathematics 2005*, (ICNAAM), WILEY-VCH Verlag GmbH & Co. KGaA (2005), 366-368.
- [10] C. Mamaloukas, St. Spartalis, Z. Manoussaridis, Similarity approach to the problem of second grade fluid flows over a stretching sheet, *Applied Mathematical Sciences*, **1**, No. 7 (2007), 327-338.
- [11] Ch. Mamaloukas, M. Subhas Abel, J.V. Tawade, U.S. Mahabaleswar, Some effects of viscous dissipation and heat source/sink on MHD heat transfer over an unsteady stretching sheet, Presentation in the *International HERCMA 2009 Conference*; Published on-line: <http://www.aueb.gr/pympe/hercma/proceedings2009/H09-full-papers-1/Mamaloukas-Subhas%20Abel-Tawade-Mahabaleswar-Abstract-1.doc>
- [12] I.J. Rao, K.R. Rajgopal, On a new interpretation of the classical Maxwell model, *Mechanics Research Communications*, **34** (2007), 509-514.

- [13] M. Renardy, High Weissenberg number boundary layers for the upper convected Maxwell fluid”, *J. Non-Newtonian Fluid Mech.*, **68**, No. 1 (1997), 125-132.
- [14] K. Sadeghy, Hadi Hajibeygi, Seyed-Mahammad Taghavi, Stagnation point flow of upper-convected Maxwell fluids, *I.J. Non-Linear Mechanics*, **41** (2006), 1242-1247.
- [15] K. Sadeghy, A.H. Najafi, M. Saffaripour, Sakiadis flow of an upper convected Maxwell fluid”, *Int. J. Non-Linear Mech.*, **40**, No. 9 (2005), 1220-1228.
- [16] B.Siddappa, S. Abel., Non-Newtonian flow past a stretching plate, *Z. Angew. Math. Phys*, **36** (1985), 890-892.

10