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DIFFERENTIAL GAMES DESCRIBED BY INFINITE SYSTEM OF DIFFERENTIAL EQUATIONS OF SECOND ORDER. THE CASE OF NEGATIVE COEFFICIENTS

Abbas Badakaya Ja'afaru[1 §], Gafurjan I. Ibragimov[2]

[1,2]Department of Mathematics
Faculty of Science
Institute for Mathematical Research
Universiti Putra
Malaysia, 43400 UPM Serdang, Selangor, MALAYSIA

Abstract: We study pursuit and evasion differential game problems for an infinite system of differential equations of second order. Control functions are subject to integral constraints.The pursuit is completed if $z(\tau) = \dot{z}(\tau) = 0$ at some $\tau > 0$, where $z(t)$ is the state of the system. The pursuer tries to complete the pursuit and the evader exactly tries to avoid this. A sufficient condition is obtained for completing the pursuit in the differential game when control recourse of the pursuer greater than that of the evader. In the case where the control recourse of the evader not less than that of the pursuer we study an evasion problem.

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1. Introduction

The study of two person zero-sum differential games was initiated by Isaac [10]. Since then many works with various approaches have been done in developing the theory of differential games (see, for example, [5], [10], [11], [12], [14], and [15]). Control problems in systems with distributed parameters were studied extensively in the literature, because of its considerable meaning both in theory and applications (see e.g. [1]- [4], [9] and [13]). As to the differential game problems in systems with distributed parameters, there is only a few literature

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§Correspondence author

(see e.g. [6]-[8] and [16] - [18]).

In the paper [17], differential game problem defined by the second-order time evolution equation:

$$\ddot{z}(t) + Az(t) = -u(t) + v(t), \quad z(0) = z^0, \ \dot{z}(t), \ 0 < t \leq T,$$

was reduced to the one described by the following system

$$\ddot{z}_k(t) + \mu_k z_k(t) = -u_k(t) + v_k(t), \quad 0 < t \leq T, \ k = 1, 2, \dots, \tag{1}$$

where the numbers μ_k are generalized eigenvalues of an elliptic operator of the form

$$Az = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial z}{\partial x_j}\right)$$

that satisfy the conditions

$$0 < \mu_1 \leq \mu_2 \leq \dots \to \infty,$$

$u_k, k = 1, 2, \dots,$ are control parameters of the pursuer, and $v_k, k = 1, 2, \dots,$ are that of the evader.

The work [7] proposed studying the differential game problems described by the infinite system of differential equations (1) in one frame separately from that described by partial differential equations. In that work, the numbers μ_k are assumed to be any positive numbers, and the control functions of the players are subject to integral constraints.

The paper [8] concerned with the differential games described by infinite systems of first order differential equations. Control functions are subjected to integral constraints. To solve pursuit and evasion differential games some conditions were obtained.

The main purpose of the present paper is to investigate pursuit and evasion differential game problems described by the system (1) in case of negative coefficients μ_k.

2. Statement of the Problem

Let $\lambda_1, \lambda_2, \dots$ be a bounded sequence of negative numbers and r be a real number. We introduce the space

$$l_r^2 = \{\alpha = (\alpha_1, \alpha_2, \dots) : \sum_{k=1}^{\infty} |\lambda_k|^r \alpha_k^2 < \infty\},$$

with inner product and norm

$$\langle \alpha, \beta \rangle_r = \sum_{k=1}^{\infty} |\lambda_k|^r \alpha_k \beta_k, \ \alpha, \ \beta \in l_r^2, \quad ||\alpha||_r = \left(\sum_{k=1}^{\infty} |\lambda_k|^r \alpha_k^2 \right)^{1/2}.$$

Let

$$L_2(t_0, \ T, \ l_r^2) = \{ w(t) = (w_1(t), w_2(t), \dots) : \\ \left. \sum_{k=1}^{\infty} |\lambda_k|^r \int_{t_0}^{T} w_k^2(t) dt < \infty, \ w_k(\cdot) \in L_2(t_0, \ T) \right\},$$

$$\|w(\cdot)\|_{L_2(0, \ T, \ l_r^2)} = \left(\sum_{k=1}^{\infty} |\lambda_k|^r \int_{t_0}^{T} w_k^2(s) ds \right)^{1/2},$$

where $T, \ T > t_0$, is a given number.

We examine the differential game problems described by the following infinite system of differential equations

$$\ddot{z}_k(t) + \lambda_k z(t) = -u_k(t) + v_k(t), \ z_k(0) = z_k^0, \ \dot{z}_k(0) = z_k^1, \ k = 1, 2, \dots \ , \tag{2}$$

where

$$z_k, u_k, v_k \in R^1, \ k = 1, 2, \dots, \ z_0 = (z_1^0, \ z_2^0, \dots) \in l_{r+1}^2, \ z_1 = (z_1^1, \ z_2^1, \dots) \in l_r^2,$$

$u = (u_1, u_2, \dots)$ is the control parameter of the Pursuer, $v = (v_1, v_2, \dots)$ is that of the Evader.

Let $\rho_0, \ \rho$ and σ be given positive numbers.

Definition 1. A function $w(\cdot)$, $w : [0, \ T] \longrightarrow l_r^2$, with measurable coordinates $w_k(t)$, $0 \leq t \leq T, \ k = 1, 2, \dots$, subject to

$$\sum_{k=1}^{\infty} |\lambda_k|^r \int_{0}^{T} w_k^2(s) ds \leq \rho_0^2$$

is referred to as the admissible control.

We denote the set of all admissible controls by $S(\rho_0)$.

Definition 2. A function $u(\cdot) \in S(\rho)$ (respectively $v(\cdot) \in S(\sigma)$) is referred to as the admissible control of the Pursuer (the Evader).

We call the numbers ρ and σ resources of controls of the Pursuer and the Evader, respectively. It should be noted that the system (2) is considered on the interval $[0, T]$, since the existence-uniqueness theorem for this system was proved for the finite interval. At the same time T can be any fixed positive number.

Definition 3. Let $u(\cdot)$ and $v(\cdot)$ are admissible controls of the pursuers and the evader, respectively. A function $z(t) = (z_1(t), z_2(t), \dots),\ 0 \le t \le T$, is called the solution of the equation (2) if each coordinate $z_k(t)$ of that:

1) is continuously differentiable on $(0,\ T)$, and satisfies the initial conditions $z_k(0) = z_k^0,\ \dot{z}_k = z_k^1$;

2) has the second derivative $\ddot{z}_k(t)$ almost everywhere on $(0,\ T)$ that satisfies the equation

$$\ddot{z}_k(t) + \lambda_k z_k(t) = -u_k(t) + v_k(t),\ k = 1, 2, \dots,$$

almost everywhere on $(0,\ T)$.

Definition 4. A function $u(t,\ v) = (u_1(t,\ v),\ u_2(t,\ v), \dots),\ u : [0,\ T] \times l_r^2 \longrightarrow l_r^2$, with components of the form $u_k(t, v) = w_k(t) + v_k(t), k = 1, 2, \dots,$ where $w(\cdot) = (w_1(\cdot), w_2(\cdot), \dots) \in S(\rho - \sigma)$, and $v(\cdot) = (v_1(\cdot), v_2(\cdot), \dots)$ is any admissible control of the Evader, is called a strategy of the Pursuer.

Definition 5. If there exists a strategy $u(\cdot)$ of the Pursuer such that for any admissible control of the Evader $z(\tau) = 0,\ \dot{z}(\tau) = 0$ at some $\tau,\ 0 \le \tau \le \theta$, then we say that pursuit can be completed for the time θ in the differential game (2) (of course $\theta \le T$).

Definition 6. A function of the form

$$v(t) = \begin{cases} 0, & 0 \le t \le \varepsilon, \\ u(t - \varepsilon), & \varepsilon < t \le T. \end{cases}$$

where ε is a positive number, $u(t),\ 0 \le t \le T$, is an admissible control of the Pursuer, is called a strategy of the Evader.

Definition 7. We say that evasion is possible in the game (2) if there is an $\varepsilon > 0$ such that for any admissible control of the Pursuer and corresponding to it strategy of the Evader

$$||z(t)||_{l_{r+1}^2} + ||\dot{z}(t)||_{l_r^2} \neq 0,\ 0 \le t \le T.$$

The problems are to find conditions
1) for completing pursuit for a finite time in the differential game (2),
2) for evasion to be possible in the game (2).

3. Problem Analysis

First we consider the following system

$$\ddot{z}_k(t) + \lambda_k z(t) = w_k(t),\ z_k(0) = z_k^0,\ \dot{z}_k(0) = z_k^1,\ k = 1, 2, \dots, \tag{3}$$

where $w(\cdot) = (w_1(\cdot),\ w_2(\cdot), \dots) \in L_2(0,\ T;\ l_r^2)$ is a control.

It is not difficult to verify that the kth equation in (3) has the unique solution

$$z_k(t) = z_k^0 \cosh(\alpha_k t) + z_k^1 \frac{\sinh(\alpha_k t)}{\alpha_k} + \int_0^t w_k(\tau) \frac{\sinh(\alpha_k(t-s))}{\alpha_k} ds, \tag{4}$$

where $\alpha_k = \sqrt{-\lambda_k}$. It's derivative is

$$\dot{z}_k(t) = \alpha_k z_k^0 \sinh(\alpha_k t) + z_k^1 \cosh(\alpha_k t) + \int_0^t w_k(\tau) \cosh(\alpha_k(t-s)) ds. \tag{5}$$

Let $C(t_0, T, l_r^2)$ be the space of continuous functions $z(t) = (z_1(t),\ z_2(t), \dots)$, $0 \le t \le T$, with values in l_r^2. The following assertion is true [1]

Assertion 1. If $\lambda_k < 0,\ k = 1, 2, \dots,$ is a bounded below sequence, then the functions $z(t) = (z_1(t),\ z_2(t), \dots)$, and $\dot{z}(t) = (\dot{z}_1(t),\ \dot{z}_2(t), \dots)$ defined by (4) and (5) belong to the spaces $C(0,\ T;\ l_{r+1}^2)$ and $C(0,\ T;\ l_r^2)$, respectively.

In (4) and (5), letting

$$\bar{x}_k(t) = \alpha_k z_k(t),\ x_{k0} = \alpha_k z_k^0,\ \bar{y}_k(t) = \dot{z}_k(t),\ y_{k0} = z_k^1 \tag{6}$$

we obtain

$$\begin{cases} \bar{x}_k(t) \doteq x_{k0} \cosh(\alpha_k t) + y_{k0} \sinh(\alpha_k t) + \int_0^t \sinh(\alpha_k(t-s)) w_k(s) ds, \\ \bar{y}_k(t) \doteq x_{k0} \sinh(\alpha_k t) + y_{k0} \cosh(\alpha_k t) + \int_0^t \cosh(\alpha_k(t-s)) w_k(s) ds. \end{cases}$$

It is clear that $z_k(t) = 0,\ \dot{z}_k(t) = 0$ if and only if $\bar{x}_k(t) = 0,\ \bar{y}_k(t) = 0$. The latter equalities can be written as

$$A_k(t) \begin{bmatrix} x_{k0} \\ y_{k0} \end{bmatrix} + \int_0^t A_k(t) \begin{bmatrix} -\sinh(\alpha_k s) \\ \cosh(\alpha_k s) \end{bmatrix} w_k(s) ds = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{7}$$

where

$$A_k(t) = \begin{bmatrix} \cosh(\alpha_k t) & \sinh(\alpha_k t) \\ \sinh(\alpha_k t) & \cosh(\alpha_k t) \end{bmatrix}.$$

Multiplying (7) by $A_k^{-1}(t)$, yields

$$\begin{cases} x_{k0} = \int_0^t \sinh(\alpha_k s) w_k(s) ds, \\ -y_{k0} = \int_0^t \cosh(\alpha_k s) w_k(s) ds. \end{cases} \tag{8}$$

Our task is now to find a control $w(\cdot) = (w_1(\cdot), w_2(\cdot), \dots)$ that satisfy (8) for all $k = 1, 2, \dots$. To this end we define the following set

$$X_k(\theta,\ \sigma) = \Bigg\{ (x_k,\ y_k) : x_k = \int_0^\theta \sinh(\alpha_k s) w_k(s) ds, \\ y_k = \int_0^\theta \cosh(\alpha_k s) w_k(s) ds,\ w_k(\cdot) \in S(\theta,\ \sigma) \Bigg\},$$

where

$$S(\theta,\ \sigma) = \left\{ w_k(\cdot) : \int_0^\theta w_k^2(s) ds \le \sigma^2,\ w_k(\cdot) \in L_2(0,\ \theta) \right\}.$$

It is clear that $(x_{k0},\ -y_{k0}) \in X_k(\theta;\ \sigma)$ if and only if there exists $w_k(\cdot) \in S(\theta,\ \sigma)$ to satisfy (8).

4. Pursuit Differential Game

Let

$$X(\theta,\ \sigma) = \bigcup_{(\sigma_1, \sigma_2, \dots)} \prod_{k=1}^{\infty} X_k(\theta,\ \sigma_k),$$

where $\prod$ is cartesian product of the sets $X_k(\theta,\ \sigma_k)$, and the union is taken over all the sequences $\sigma_1,\ \sigma_2, \cdots$ such that

$$\sum_{k=1}^{\infty} \sigma_k^2 = (\rho - \sigma)^2,\ \sigma_k \ge 0.$$

Theorem 1. *If $\rho > \sigma$ and $\bar{z}_0 \in X(\theta,\ \rho - \sigma)$, where*

$$\bar{z}_0 = \left((\alpha_1 z_1^0,\ -z_1^1),\ (\alpha_2 z_2^0,\ -z_2^1), \dots \right),$$

then the differential game (2) can be completed for the time θ.

Proof. As

$$X(\theta,\ \rho-\sigma)=\bigcup_{(\sigma_1,\ \sigma_2,\dots),}\prod_{k=1}^{\infty}X_k(\theta,\ \sigma_k),$$

$$\sum_{k=1}^{\infty}\sigma_k^2=(\rho-\sigma)^2,\ \sigma_k\geq 0,\ k=1,2,\dots\ ,$$

then $\bar{z}_0\in X(\theta,\ \rho-\sigma)$ implies that there exists a sequence $\sigma_1,\ \sigma_2,\dots,\ \sigma_k\geq 0,\ k=1,2,\dots\ ,\ \sum_{k=1}^{\infty}\sigma_k^2=(\rho-\sigma)^2$, such that $(\alpha_k z_k^0,\ -z_k^1)\in X_k(\theta,\ \sigma_k),\ k=1,2,\dots$.

This gives

$$(x_{k0},\ -y_{k0})\in X_k(\theta,\ \sigma_k),\ k=1,2,\dots\ .$$

This means that there exists a control $w^0(t)=(w_1^0(t),\ w_2^0(t),\dots),\ w_k^0(\cdot)\in S(\theta,\ \sigma_k),\ 0\leq t\leq\theta$, such that (see (8))

$$\begin{cases} x_k(\theta)=x_{k0}-\int\limits_0^\theta\sinh(\alpha_k s)w_k^0(s)ds=0,\\ y_k(\theta)=y_{k0}+\int\limits_0^\theta\cosh(\alpha_k s)w_k^0(s)ds=0,\ k=1,2,,\dots\ .\end{cases}\tag{9}$$

We construct the strategy of the pursuer as follows:

$$u_k(t)=-w_k^0(t)+v_k(t),\tag{10}$$

where $v_k(t)$ is a control of the evader.

We show admissibility of the pursuer's strategy, i.e., we show that

$$\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta|u_k(t)|^2dt\right)^{1/2}\leq\rho.$$

We have

$$\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta|u_k(t)|^2dt\right)^{1/2}=\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta|-w_k^0(t)+v_k(t)|^2dt\right)^{1/2}$$

$$\leq\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta\left(|w_k^0(t)|+|v_k(t)|\right)^2dt\right)^{1/2}$$

$$\leq\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta|w_k^0(t)|^2dt\right)^{1/2}+\left(\sum_{k=1}^{\infty}|\lambda_k|^r\int_0^\theta|v_k(t)|^2dt\right)^{1/2}$$

$$\leq \rho - \sigma + \sigma = \rho.$$

We now show that the pursuit is completed for the time θ. According to (4) and (5) equation (2) has the solution

$$z_k(t) = z_k^0 \cosh(\alpha_k t) + z_k^1 \frac{\sinh(\alpha_k t)}{\alpha_k} + \int_0^t (-u_k(s) + v_k(s)) \frac{\sinh(\alpha_k(t-s))}{\alpha_k} ds,$$

and its derivative is

$$\dot{z}_k(t) = \alpha_k z_k^0 \sinh(\alpha_k t) + z_k^1 \cosh(\alpha_k t) + \int_0^t (-u_k(s) + v_k(s)) \cosh(\alpha_k(t-s)) ds.$$

As the system

$$z_k(\theta) = 0, \ \dot{z}_k(\theta) = 0, \ k = 1, 2, \dots,$$

is equivalent to the one

$$\begin{cases} x_{k0} - \int\limits_0^\theta \sinh(\alpha_k s)(-u_k(s) + v_k(s)) ds = 0, \\ y_{k0} + \int\limits_0^\theta \cosh(\alpha_k s)(-u_k(s) + v_k(s)) ds = 0, \ k = 1, 2, \cdots . \end{cases} \tag{11}$$

From here, in view of (10) we obtain the system (9). Therefore the strategy (10) ensures that pursuit is completed at θ. The proof is complete. □

5. Evasion Differential Game

In this section, we study an evasion differential game described by infinite system of differential equations (2). We formulate the main result.

Theorem 2. . *If $\sigma \geq \rho$, then from any initial position*

$$z^0 = (z_1^0, \ z_2^0, \cdots), \ z^1 = (z_1^1, \ z_2^1, \cdots), \ ||z^0||_{l^2_{r+1}} + ||z^1||_{l^2_r} \neq 0,$$

evasion is possible in the game (2).

Proof. As mentioned above system $z_k(t) = 0, \ \dot{z}_k(t) = 0, \ k = 1, 2, \cdots$, is equivalent to the system (11), where $x_0 = (x_{10}, \ x_{20}, \dots) \in l_r^2, \ y_0 = (y_{10}, \ y_{20}, \dots) \in l_r^2, \ z^0, \ z^1$ and $x_0, \ y_0$ are connected with (6). The condition

$$||z^0||_{l^2_{r+1}} + ||z^1||_{l^2_r} \neq 0,$$

implies that there exists a number $k = m$ such that $(x_{m0},\ y_{m0}) \neq (0,\ 0)$, say $x_{m0} \neq 0$ (the case $y_{m0} \neq 0$ is similar). We show that there exists admissible strategy of the evader such that $x_m(t) \neq 0,\ t \in [0,\ T]$. Let the pursuer use an admissible control $u(t),\ t \in [0,\ T]$. We construct the strategy of the evader as follows

$$\begin{aligned} v_m(t) &= \begin{cases} 0, & 0 \le t \le \varepsilon, \\ u_m(t-\varepsilon), & \varepsilon < t \le T, \end{cases} \\ v_i(t) &= 0,\ 0 \le t \le T,\ i = 1, \cdots, (m-1), (m+1), \cdots . \end{aligned} \tag{12}$$

The admissibility of evader's strategy follows from the relation

$$\sum_{k=1}^{\infty} |\lambda_k|^r \int_0^T v_k^2(s)ds = |\lambda_k|^r \int_\varepsilon^T u_m^2(s-\varepsilon)ds = |\lambda_k|^r \int_0^{T-\varepsilon} u_m^2(s)ds$$

$$\le \sum_{k=1}^{\infty} |\lambda_k|^r \int_0^T u_k^2(s)ds \le \rho^2 \le \sigma^2.$$

For the possibility of evasion, we have

$$\begin{aligned} x_m(t) &= x_{m0} + \int_0^t \sinh(\alpha_m s) u_m(s)ds - \int_0^t \sinh(\alpha_m s) v_m(s)ds \\ &= x_{m0} + \int_0^\varepsilon \sinh(\alpha_m s) u_m(s)ds + \int_\varepsilon^t \sinh(\alpha_m s) u_m(s)ds \\ &\quad - \int_0^\varepsilon \sinh(\alpha_m s) v_m(s)ds - \int_\varepsilon^t \sinh(\alpha_m s) v_m(s)ds. \end{aligned} \tag{13}$$

Substituting (12) into (13) and transforming, we have

$$x_m(t) = x_{m0} + \int_0^\varepsilon [\sinh(\alpha_m s) - \sinh(\alpha_m(s+\varepsilon))] u_m(s)ds$$

$$+ \int_\varepsilon^{t-\varepsilon} [\sinh(\alpha_m s) - \sinh(\alpha_m(s+\varepsilon))] u_m(s)ds + \int_{t-\varepsilon}^t \sinh(\alpha_m s) u_m(s)ds.$$

If ε approaches zero, then $x_m(t) \to x_{m0} \neq 0$, since each integral approaches zero. For example, letting

$$I = \int\limits_0^{\varepsilon} [\sinh(\alpha_m s) - \sinh(\alpha_m (s+\varepsilon))] u_m(s) ds$$

then we have

$$I = (1 - \cosh(\alpha_m \varepsilon)) \int_0^{\varepsilon} \sinh(\alpha_m s) u_m(s) ds - \sinh(\alpha_m \varepsilon) \int_0^{\varepsilon} \cosh(\alpha_m s) u_m(s) ds.$$

The integral

$$\int_0^{\varepsilon} \sinh(\alpha_m s) u_m(s) ds$$

is bounded, since

$$\int_0^{\varepsilon} \sinh(\alpha_m s) u_m(s) ds \leq \left(\int_0^{\varepsilon} \sinh^2(\alpha_m s) ds \int_0^{\varepsilon} u_m^2(s) ds \right)^{1/2}$$

$$\leq \rho \left(\int_0^{\varepsilon} u_m^2(s) ds \right)^{1/2}.$$

Also $\int_0^{\varepsilon} \cosh(\alpha_m s) u_m(s) ds$ is bounded. Therefore, I approaches zero as ε approaches zero.

Consequently, one can choose $\varepsilon > 0$ to ensure $z(t) \neq 0,\ t \in [0,\ T]$. This means evasion is possible in the game (2). This completes the proof of the theorem. □

6. Conclusions

We have studied pursuit and evasion differential game problems for an infinite system of differential equations of second order, the control functions being subject to integral constraints. In the case where the control recourse of the evader less than that of the pursuer we have studied a pursuit problem. If this is not so, then we have solved an evasion problem. In solving the pursuit problem we used the solution of a control problem.

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