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AN ANALYTICAL ARL OF BINOMIAL DOUBLE MOVING AVERAGE CHART

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Abstract: The objective of this paper is to study Statistical Process Control (SPC) with Double Moving Average control chart (DMA) chart. The characteristic of control chart is Average Run Length (ARL) which is the average number of samples taken before an action signal is given. The explicit formula for when observations are Binomial distribution is presented. This formula is easy to implement and useful to practitioners. In addition, the comparisons of performance of DMA, Moving Average control chart(MA), Exponentially weighted Moving Average chart (EWMA) and np chart are considered. The numerical results found that the performance of the EWMA chart is superior to DMA and MA charts for small shifts, DMA performs appreciably better than EWMA and MA for moderate shifts but that MA performs better than EWMA and DMA for large shifts.

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Key Words: double moving average chart, average run length, binomial distribution, stopping times

1. Introduction

A control chart is an effective tool in SPC for detecting changes in a process. It is also useful for measuring, controlling and improving quality not only in fields of industrial statistics and manufacturing (Mason and Antony [10]) but also in computer science and telecommunications (Mazalov and Zhuravlev [11]; Ye et al. [21]), in finance and economics (Ergashev [6]; Golosnoy and Schmid [8])

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epidemiology (Sitter et al. [19]; Frisen [7]) and in other fields of applications.

All popular charts such as Shewhart, Exponentially Weighted Moving Average (EWMA) and Cumulative Sum (CUSUM) charts have been developed for detecting changes in process means and variances. The traditional Shewhart chart, first introduced by Shewhart [17], is still widely used in many applications as a common tool for detecting large changes in a process mean. However, it is necessary to use attribute control charts when a quality characteristic cannot be measured on a continuous scale, for example, if the number of defective products or the number of nonconformities is being counted. Traditionally, attribute control charts such as p and np charts are mainly used for detecting changes in parameters of binomial distributions and c and u charts are used for Poisson distributions (for further detail see Montgomery [14] and Alwan [4]).

In the past few decades, CUSUM and EWMA charts have been proposed as good alternatives to the Shewhart chart for detecting small and moderate shifts. The CUSUM chart was initially presented by Page [15]. It has been shown that CUSUM charts are asymptotically optimal under minimax type criteria (for details see Lorden [9]; Shiryayev [18]). The EWMA chart was initially introduced by Roberts [16]. It is a very flexible and effective chart for detecting small changes and has the advantage of showing robustness to non-normality (Borror et al. [5]; Stoumbos and Reynolds [20].) Recently the MA chart was introduced (see Montgomery [14] and Alwan [4].) Khoo [12] studied the MA chart for monitoring the fraction of non-conforming observations and showed that an MA chart is more efficient than a p chart. Later, Khoo and Wong [13] proposed the use of a DMA chart for observations from a normal distribution. They used numerical simulations to show that the DMA chart performed better than the MA chart for small and moderate shifts.

Common characteristics used for comparing the performance of control charts are first an in-control Average Run Length (ARL_0) and secondly an Average of Delay time (ARL_1). ARL_0 is defined as the expected number of observations taken from an in-control process until the control chart falsely signals out-of-control. ARL_0 is regarded as acceptable if it is large enough to keep the level of false alarms at an acceptable level. ARL_1 is defined as the expected number of observations taken from an out-of-control process until the control chart correctly signals that the process is out-of-control. Ideally, the ARL_1 time should be small as possible.

In this paper we derive analytical formula for calculating ARL for detection of a change in number of defects for a DMA chart when observations are binomially distributed. We compare our results with MA, EWMA and np charts to show the efficiency of the proposed chart.

2. Methodology

In order to monitor the number of defective products we consider SPC charts under the assumption that sequential observations $X_1, X_2, ...$ are independent random variables with a binomial distribution function $F(X; n, p)$. The process is assumed to be in an "in-control" state when the parameter has the value $p = p_0$. We also assume that there is a change-point time $\theta \leq \infty$ ($\theta = \infty$ means that the process is always "in-control"). After the change-point time θ, the parameter p_0 is assumed to change to p_1, where $p_1 > p_0$, and the process is then in an "out-of-control" state.

All popular charts such as Shewhart, Cumulative Sum (CUSUM) and EWMA charts (see e.g. [17, 15, 16] are based on use of a stopping time (τ). The typical condition on choice of the stopping time (τ) is the following:

$$E_\theta(\tau) = T, \quad \theta = \infty(in - control\ state), \tag{1}$$

where T is given (usually large). Let $E_\infty(.)$ denote that the expectation under distribution $F(X; n, p_0)$ (for in-control state) that the change-point occurs at point θ (where $\theta \leq \infty$). In literatures on quality control the quantity $E_\infty(\tau)$ is called as Average Run Length (ARL_0) of the algorithm. Then, by definition, $ARL_0 = E_\infty(\tau)$ and the typical practical constraint is $ARL_0 = T$.

Another typical constraint consists in minimizing of the quantity

$$Q(p) = \sup_\theta E_{\theta(\tau-\theta+1|\tau\geq\theta)}, \tag{2}$$

where $E_\infty(.)$ is the expectation under distribution $F(X; n, p_1)$ (for out-of-control state) and p_1 is the value of parameter after the change-point. In this paper, we restrict on the special case, usually $\theta = 1$. The quantity $E_1(\tau)$ is called as Average Delay Time (ARL_1) and one could expect that a sequential chart has a near optimal performance if (ARL_1) is closed to a minimal value.

They are many other criteria in practice for optimality of SPC, however, ARL_0 and ARL_1 remain the most popular characteristics which are convenient to use for comparisons of the performance for different charts. For np chart, we have observations $X_1, X_2, ..., X_m$ where X_i is the number of non-conforming items. The values of X_i are plotted at 3σ control limits at

$$n\bar{p} \pm 3\sqrt{n\bar{p}(1-\bar{p})}$$

where $\bar{p}$ is estimated by

$$\bar{p} = \frac{\sum_{i=1}^{m} X_i}{n}$$

The Moving Average control chart is defined by the following statistics

$$M_i = \frac{X_i + X_{i-1} + ... + X_{i-w+1}}{w} = \frac{\sum\limits_{j=i-w+1}^{i} X_j}{w}, \quad i \geq w.$$

For moving average of width when periods $i < w$, we do not have w observations to calculate.

For period $i \geq w$, the 3σ control limits are given by

$$UCL/LCL = np \pm 3\sqrt{\frac{np(1-p)}{w}}.$$

For periods $i < w$, $\sqrt{\frac{np(1-p)}{w}}$ is replaced with $\sqrt{\frac{np(1-p)}{i}}$.

The alarm time for this type of procedure is the following:

$$\tau = \inf\{t > 0 : M_t > UCL \quad \text{or} \quad M_t < LCL\}.$$

The EWMA control chart is based on a weighted average Z_t of current and previous data which is defined through the recurrence relation:

$$Z_t = (1-\lambda)Z_{t-1} + \lambda X_t, \quad t = 1, 2, ..., \tag{3}$$

where X_t is a sequence of independent identically distributed random variables and λ is a weighting constant with $0 < \lambda < 1$. The mean of the data distribution is $\hat{E}_\infty(X_t)$. Typically, the target mean is defined to be $\hat{E}_\infty(X_t) = p_0$ and the initial value Z_0 is usually chosen to be this target mean, i.e. $Z_0 = p_0$. The control limits of the EWMA chart are the following:

$$UCL/LCL = p_0 \pm L\sigma\sqrt{\frac{\lambda}{2-\lambda}},$$

where L is a constant to be chosen.

The Double Moving Average procedure was initially proposed by Michael B.C. Khoo and Wong V.H. [13]. The DMA statistic is as the following form

$$DMA_i = \frac{MA_i + MA_{i-1} + ... + MA_{i-w+1}}{w}.$$

The control limits of DMA for $w > 2$ are

$$UCL/LCL = \begin{cases} np_0 \pm H\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}}, \quad i \le w \\ np_0 \pm H\sqrt{\frac{np_0(1-p_0)\sum_{j=i-w+1}^{w-1}\frac{1}{j}+(i-w+1)(\frac{1}{w})}{w^2}}, \quad w < i < 2w-1 \\ np_0 \pm H\sqrt{\frac{np_0(1-p_0)}{w^2}}, \quad i \ge 2w-1. \end{cases} \tag{4}$$

Note that for $w = 2$ we consider only the first and third lines of Eq. 4 and H is control limit.

3. The Explicit Formula for Evaluating ARL for DMA Chart

The ARL values of Double Moving Average control chart can be derived. Let $ARL = n$, then

$$\begin{aligned} \frac{1}{ARL} &= (\frac{1}{n})P(\text{o.o.c signal at time} \quad i < w) \\ &\quad + [\frac{n-(2w-1)}{n}]P(\text{o.o.c signal at time} \quad i \ge w) \\ &= \frac{1}{n}\{\sum_{i=1}^{w}[P(\frac{\sum_{j=1}^{i} np_j}{i} > UCL_i) + P(\frac{\sum_{j=1}^{i} np_j}{i} < LCL_i)]\} \\ &\quad + [\frac{n-(2w-1)}{n}][P(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j > UCL_w) \\ &\quad + P(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j < LCL_w)] \\ &= \frac{1}{n}\{\sum_{i=1}^{w}[P(\sum_{j=1}^{i} np_j - np_0 + 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}})] \\ &\quad + [P(\sum_{j=1}^{i} np_j < np_0 - 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}})]\} \end{aligned} \tag{5}$$

(*cont.*)

$$+ [\frac{n-(2w-1)}{n}][P(\frac{1}{w}\sum_{j=i-w+1}^{i} np_j > np_0 + 3\sqrt{\frac{np_0(1-p_0)}{w^2}})$$

$$
\begin{aligned}
&+ P(\sum_{j=i-w+1}^{i} np_j < np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w^2}})] \\
&= \frac{1}{n}\{\sum_{i=1}^{w}[P(Z > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}}) \\
&+ P(Z < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}})]\} \\
&+ [\frac{n-(2w-1)}{n}][P(Z > \frac{np + 3\sqrt{\frac{np(1-p)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}}) \\
&+ P(Z < \frac{np - 3\sqrt{\frac{np(1-p)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}})].
\end{aligned}
$$

Theorem 1. *The Average Run Length for the DMA chart with $w = 2$ is calculated as follows.*

$$
\begin{aligned}
ARL = (1 - \sum_{i=1}^{w}&[P(Z > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}}) \\
&+ P(Z < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}})]\} \\
&\times [P(Z > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}}) \\
&+ P(Z < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}})]^{-1} + (2w-1).
\end{aligned}
$$

Proof. In order to proof Theorem 1, we use Equation 5 as follows. Let

$$
\begin{aligned}
A &= \sum_{i=1}^{w}[P(Z > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}}) \\
&\quad +P(Z < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}})] \\
B &= [P(Z > \frac{np_0 + 3\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}}) + P(Z < \frac{np_0 - 3\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}})]
\end{aligned}
$$

and substitute A and B into Equation 5. Then we get as following

$$
\begin{aligned}
\frac{1}{n} &= \frac{1}{n}A + [\frac{n-(2w-1)}{n}]B \\
n &= (1-A)B^{-1} + (2w-1).
\end{aligned}
$$

Finally, we obtain the explicit formula for the average run length of DMA chart as Theorem 1. □

For general width of control limit it can be found for desired ARL. The formula for ARL can be written as

$$
\begin{aligned}
ARL = (1 - \sum_{i=1}^{w}[P(Z &> \frac{np_0 + H\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}}) \\
&+ P(Z < \frac{np_0 - H\sqrt{\frac{np_0(1-p_0)\sum_{j=1}^{i}(\frac{1}{j})}{i^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{i^2}}})]\} \\
&\times [P(Z > \frac{np_0 + H\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}}) \\
&+ P(Z < \frac{np_0 - H\sqrt{\frac{np_0(1-p_0)}{w^2}} - np_1}{\sqrt{\frac{np_1(1-p_1)}{w^2}}})]^{-1} + (2w-1),
\end{aligned} \tag{6}
$$

where H is width of control limit.

4. Numerical Results

In this section, we compare the numerical results for ARL_0 and ARL_1 from Eq. 6 with np, MA and EWMA charts as shown in Table 1. The parameter values for np, MA, DMA and EWMA charts were chosen by given desired $ARL_0 = 370$, in-control parameter $p_0 = 0.02$ and out-of-control parameter $p_1 \subseteq [0.025, 0.1]$. For the EWMA procedure, the parameter values $\lambda = 0.01$ and $L = 2.1278$ and 3.157 for $n = 100$ and 150 respectively. For DMA chart, the boundary value $H = 2.9984$ is used for responding those given parameters.

The numerical results shown that DMA chart has a better performance when parameter p has been changed in moderate shifts. For example, DMA chart with $w = 2$ when $0.03 < p < 0.035$ (moderate shifts) shows the best performance (minimum ARL_1), otherwise, MA chart with $w = 2$ and 3 is superior to DMA chart when $p > 0.035$. Besides, the performance of EWMA chart is superior to np DMA and MA charts when $p < 0.03$ (small shifts).

Note that the calculations from exact formula (Eq. 6) is much faster, simple to program and computational time based on the proposed technique takes computational time less than 1 second. In addition, Table 2 is presented in the same manner of Table 1 when fixed $ARL_0 = 500$ by proposed formula from Eq. 6 and compared the numerical results with np, MA and EWMA charts. For the EWMA procedure, we used parameter values $\lambda = 0.01$ and $L = 2.147$ and 3.181 for $n = 100$ and 150, respectively. For MA and DMA charts, the boundary value $H = 3.0905$. The results are in good agreement when given $ARL_0 = 370$, but that DMA performs better than EWMA np and MA for moderate shifts. Consequently, uses of the proposed explicit formulas for ARL_0 and ARL_1 can greatly reduce the computation times, easy to implement and useful to practitioners as shown in Table 1 and 2.

5. Conclusion

We propose the explicit formula for ARL of Double Moving Average control chart for the case of binomial distribution. We have shown that proposed formula is very accurate, easy to calculate and program. The performance comparison of the control charts has been based on Average Run Length (ARL_0) and Average of Delay Time (ARL_1) criteria. For Binomial distribution when

N	p	ARL					
		np	MA		DMA		EWMA
			w=2	w=3	w=1	w=2	$\lambda = 0.01$
100	**0.02**	370.398	370.398	370.398	370.398	370.370	373.366
	0.025	98.0295	74.4752	59.6019	98.0295	50.6071	**31.0129**
	0.027	60.0202	41.1259	30.9959	60.0202	25.9213	**21.7259**
	0.03	31.7591	19.7234	14.3492	31.7591	**12.4553**	14.9244
	0.033	18.7653	11.1365	8.1893	18.7653	**7.6823**	11.4791
	0.035	13.9162	8.1780	6.1648	13.9162	**6.1321**	10.0070
	0.04	7.6008	4.5863	**3.7708**	7.6008	4.2788	7.5304
	0.05	3.4313	2.3952	**2.2968**	3.4313	3.0691	5.0471
	0.07	1.6046	**1.4332**	1.4864	1.6046	2.3980	3.1315
	0.10	1.1144	**1.1035**	1.1109	1.1144	2.1026	2.1498
N	**p**	**np**	**w=2**	**w=3**	**w=1**	**w=2**	$\lambda = 0.01$
150	**0.02**	370.398	370.398	370.398	370.398	370.370	369.787
	0.025	84.6748	59.3018	45.0696	84.6748	37.2232	**31.512**
	0.027	48.799	30.6212	22.0592	48.799	18.2984	**17.3311**
	0.03	24.2102	13.9137	9.9242	24.2102	**8.8998**	12.147
	0.033	13.7442	7.7259	5.7631	13.7442	**5.7707**	9.3516
	0.035	10.0219	5.6934	**4.4441**	10.0219	4.7728	8.1238
	0.04	5.3708	3.3010	**2.9045**	5.3708	3.5669	6.2043
	0.05	2.4701	**1.8802**	1.9035	2.4701	2.7304	4.1455
	0.07	1.2910	**1.2349**	1.2612	1.2910	2.2272	2.6417
	0.10	1.032	**1.0310**	1.0317	1.032	2.0310	1.8806

Table 1: Comparison of ARL_0 and ARL_1 for binomial DMA chart by explicit formula with other control charts given $ARL_0 = 370$

given $(ARL_0) = 370.4$ and 500, we have shown that the performance of DMA chart is superior to np, MA and EWMA charts for moderate shifts.

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N	p	ARL					
		np	MA		DMA		EWMA
			w=2	w=3	w=1	w=2	$\lambda = 0.01$
100	**0.020**	500.451	500.451	500.451	500.451	500.356	500.535
	0.025	123.190	92.455	73.2677	123.190	61.439	**35.4382**
	0.027	73.5919	49.6351	36.9327	73.5919	30.3472	**24.7692**
	0.030	37.7704	22.9943	16.4475	37.7704	**13.9391**	17.1623
	0.033	21.7781	12.6203	9.0999	21.7781	**8.3149**	13.1803
	0.035	15.9301	9.1291	6.7363	15.9301	**6.5306**	11.3621
	0.040	8.4625	4.9653	**3.9987**	8.4625	4.4457	8.5165
	0.050	3.6801	2.5026	**2.3719**	3.6801	3.1303	5.7671
	0.070	1.6551	**1.4608**	1.5152	1.6551	2.4206	3.538
	0.100	1.1241	**1.1115**	1.1241	1.1241	2.1104	2.380
N	**p**	**np chart**	**w=2**	**w=3**	**w=1**	**w=2**	$\lambda = 0.01$
150	**0.02**	500.451	500.451	500.451	500.451	500.451	500.255
	0.025	105.737	72.9582	54.7943	105.737	44.5681	**28.9317**
	0.027	59.3521	36.5482	25.9202	59.3521	21.0416	**20.2946**
	0.030	28.5144	16.002	11.1791	28.5144	**9.7539**	13.8498
	0.033	15.7796	8.6263	6.2849	15.7796	**6.1274**	10.6722
	0.035	11.3427	6.2544	**4.7682**	11.3427	4.9995	9.2439
	0.040	5.9062	3.5177	**3.0370**	5.9062	3.668	6.9449
	0.050	2.6147	**1.9428**	1.9546	2.6147	2.7717	4.7704
	0.070	1.3168	**1.2519**	1.2803	1.3168	2.2429	3.0527
	0.100	1.0353	**1.0341**	1.0353	1.0353	2.0341	2.1015

Table 2: Comparison of ARL_0 and ARL_1 for binomial DMA chart by explicit formula with other charts given $ARL_0 = 500$

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