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SPACE CURVES IN THE RANGE C IN POSITIVE CHARACTERISTIC

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Abstract: In this note we prove the upper bound for the genus of space curves not contained in a surface of degree $< s$ in the range C in characteristic $p \neq 2$.

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We continue our attempts to extend to the positive characteristic case the projective geometry of embedded curves. In this note we prove the following result.

Theorem 1. *Assume $p \neq 2$. Fix integers d, s such that $s \geq 2$ and $d > (s-1)^2 + 1$. If $p < s$, then assume $d > 2(s-1)^2$. Let e be the only integer such that $0 \leq e < s$ and $d + e \equiv 0 \pmod{s}$. Let $C \subset \mathbb{P}^3$ be a smooth curve such that $h^0(\mathcal{I}_C(s-1)) = 0$. Then $g(C) \leq 1 + d(d + (d/s) - 4) - e(s-e)(s-1)/2s$ and equality holds if and only if C is linked by two surfaces, one of degree s*

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and one of degree $(d+e)/s$, to a plane curve D of degree e (with the convention $D = \emptyset$ if $e = 0$).

Proof. Let $H \subset \mathbb{P}^3$ be a general plane. Set $S := C \cap H$. By [1], Theorem 1, the numerical character of S has no gaps and S is in uniform position. A general curve $D \subset H$ with minimal degree containing S is irreducible ([2] or use that S is in uniform position). Since $h^0(\mathcal{I}_C(s-1)) = 0$, our assumptions on d, s, p gives $h^0(H, \mathcal{I}_S(s-1)) = 0$ ([3], Theorem 1.2). Since the numerical character of S is connected, the upper bound for $g(C)$ follows for the usual Castelnuovo method and the numerical computations in [6], pp. 44–45. The Hilbert function of a plane curve of degree e depends only from e. Linkage theory gives $g(C) = 1 + d(d + (d/s) - 4) - e(s-e)(s-1)/2s$ if C is linked to a degree e plane curve by a surface of degree s and a surface of degree $(d+e)/s$. Now assume $g(C) = 1 + d(d + (d/s) - 4) - e(s-e)(s-1)/2s$. The proofs in [6], pp. 44–45, give the Hilbert function of S and that S is linked by a plane curve of degree s and a plane curve of degree $(d+e)/s$ to a degree e scheme contained in a line. By [6], Lemma 3.5, C is arithmetically normal. Hence we may lift these two curves to surfaces of $\mathbb{P}^3$ containing C and get that the linked curve is a plane curve of degree e. □

Question 1. 1) Is Theorem 1 true if $p = 2$?

2) Is it possible to extend [5] and [4] to the case of positive characteristic?

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