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SOME NON-LINEAR RESONANCE OSCILLATIONS OF DUMBELL SATELLITE IN ELLIPTICAL ORBIT

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Abstract: Some non-linear oscillations of Dumbell satellite in elliptical orbit in the central gravitational field of force under the combined influence of Earth magnetic field, oblateness of the Earth and some external periodic forces of general in nature has been studied. The system comprises of two charged material particles connected by a light flexible and inextensible cable, moves with taut cable like a dumbbell satellite, in elliptical orbit around the Earth. The central gravitational field of force is the main force governing the motion and various perturbing forces influencing the system are disturbing in nature. Non-linear oscillations of dumbbell satellite about the equilibrium position in the neighbourhood of the main resonance ω=1, has been studied, exploiting the asymptotic method due to Bogoliubov, Krilov and Metropolosky, considering 'e'to be a small parameter. The analysis of stability of the system has been discussed due to Poincare method.

AMS Subject Classification: 70F15
Key Words: non-linear oscillations, dynamical system, perturbing forces, cable connected satellite

1. Introduction

The present paper deals with non-linear resonance oscillations of cable con-

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nected satellite system connected by a light flexible and inextensible cable in the gravitational field of the Earth. The spacecraft studied in the present research works is also subject to the influence of the three different perturbations (i) the small eccentricity of the elliptical orbit (ii)a magnetic field of the Earth due to the interaction between the Earth's magnetic field and the magnetic moment of the spacecraft (iii) the oblateness of the Earth and (iv) the small external periodic forces of the general in nature. The satellites which are connected by non conducting cable are considered to be a charged material particles and the motion of one of the particles is studied relative to their centre of mass, under the assumption that later moves along the elliptical orbit. The cable connecting the two satellites is considered to taut and non elastic in nature, so that the system moves like a dumbell satellite. Many space configurations have been proposed and discussed the stability of the system in elliptical orbit under the various perturbing forces. The simplest space configuration is dumbell satellite system; when the two satellites are connected by a rod [4], two or more satellites are connected by a tether Krupa et al [6, 7], Beletsky and Levin [2], Mishra and Modi [9], and the two satellites are connected by spring [13]. All these authors have mentioned numerous important applications of the system and stability of relative equilibrium of the system moves in a circular and elliptical orbit. Beletsky [1], Beletsky and Novikova [3] studied the motion of a system of two satellites connected by a light, flexible and inextensible string in the central gravitational field of force relative to the centre of mass of the system, which is itself assume to move along a Keplerian elliptical orbit, under the assumption that the two satellites are moving in the plane of motion of centre of mass. Singh [16, 17]dealt the some problem in its general form. He further investigated by considering the analysis of relative motion of the system for the elliptical orbit of the centre of mass in two dimensional as well as three dimensional cases. Narayan and Singh [10, 11, 12]studied non-linear oscillations due to the solar radiation pressure, provided the centre of mass of the system moves along an elliptical orbit. Singh et al [18, 19] studied the non-linear effects of the Earth's oblateness in the motion and stability of cable connected satellites system in elliptical orbit. Das et al [5] and Narayan et al [23] studied the non-linear effects of Earth's magnetic field in the stability of cable connected system in inclined and equatorial orbit. The present paper is devoted to the analysis of the combined effects of the magnetic field of the Earth, Oblateness of the Earth and the external periodic forces of general nature on non-linear resonance oscillations of cable connected satellites system in elliptical orbit. The perturbing forces due to the Earth's magnetic field results from the interaction between spacecraft's residual magnetic field and the geomagnetic field. The perturbing force is arising due to magnetic moments, eddy current and hysteresis, out of these the spacecraft magnetic moment is usually the dominant sources of disturbing effects. Nevertheless a distant satellite beyond gravitational field of the Earth, in addition to above mentioned forces(non uniform gravitational field and magnetic field of the Earth) it could still expected to be affected by general nature of external forces could arise due to dissipation of energy generated on account of friction of bodies in the atmosphere by tidal forces, gravitational radiation these forces though small can significantly affect the oscillations of the system under considerations. These forces could be modeled as frictional forces with small dissipation coefficient. Further more the forces generated by the multipole moments and absorption of gravitational waves at resonance frequency could be characterized as external periodic forces having a slowly varying frequency and these forces could be estimated by certain model assumption, see [20]. Thus, in order to study non-linear oscillations of dumbell satellite in elliptical orbit on realistic basis, it is essential to consider the combined influences of the Earth's magnetic field, oblateness of the Earth and the periodic force of general nature.

2. Equation of Motion

The combined influence of the geomagnetic field and oblateness of the Earth and external periodic forces of general nature on the motion and stability of a cable connected satellites system in the central gravitational field of Earth in elliptical orbit has been considered. The equation of motion of satellite in the central gravitational field of Earth under the combined influence of Earth's magnetic field, oblateness of the earth is given by

$$\begin{aligned}(1+e\cos v)\,\psi'' - 2e\psi' \sin v + 3\sin\psi\cdot\cos\psi + 5A\,(1+e\cos v)^2\sin\psi\cdot\cos\psi \\ = B\cos\delta\,(1+e\cos v)\cdot\sin\psi - B\cos\delta\cdot\sin v\cdot\cos\psi + 2e\sin v \quad (2.1)\end{aligned}$$

The non-linear oscillations described by (2.1) take place as long as inequality given below is satisfies

$$\begin{aligned}(1+e\cos v)^4\left(\psi'+1\right)^2 + (1+e\cos v)^3\left(3\cos^2\psi - 1\right) \\ - B\cos\delta\,(1+e\cos v)^3\left(\cos\psi + e\cos(\psi+v)\right) \\ - A\,(1+e\cos v)^3\left(4\cos^2\psi - \sin^2\psi\right) \geq 0, \quad (2.2)\end{aligned}$$

where v and e are respectively true anomaly and eccentricity of the orbit of centre of mass of the system. The prime denotes differentiation with respect to true anomaly v, whereψ is the angular derivative of the line joining the centre of mass and the cable connecting the satellite with the stable position of equilibrium. A is the oblateness due to the Earth and B is the magnetic field of the Earth. The equation (2.1) represents the oscillations of the system about the stable position of equilibrium in which the system lies wholly along the radius vector joining the centre of mass and the centre of force Narayan et al Substituting$2\psi = \eta$, the equation can be expressed as follows

$$\begin{aligned}\eta'' + 3\sin\eta =& 4e\sin v + e\eta'\sin v \\ & - \eta'' e\cos v - 5A\left(1 + e\cos v\right)^2 \sin\eta + 2B\cos\delta\sin\frac{\eta}{2} \\ & + 2eB\cos\delta\sin\left(\eta - \frac{v}{2}\right) + E\sin\nu v + \gamma\eta'. \qquad (2.3)\end{aligned}$$

Equation (2.3) describes the non-linear oscillations of the dumbbell satellite in elliptical orbit, in central gravitational field of oblate Earth's together with the magnetic field of the Earth, friction force and periodic force, where γ and E are some phenomenological parameter characterizing the tidal and periodic forces acting on the system and have been assumed to be the order of "e". Here v is the frequency of the external periodic force. However, these parameters can be determined from specific model assumptions concerning these problems

3. Non-Linear Resonance Oscillations of Dumbell Satellite System about the Position of Equilibrium for Small Eccentricity

The non-linear oscillations of the dumbell satellite under the influence of above mentioned forces described by (2.3), will be investigated for resonance case on the assumption that γ and E are of order of e. This is justified on account of the fact that these estimates are always concerned with a certain model assumptions. Hence setting $E = eE_1$ and $\lambda = e\lambda_1$ and $B\cos\delta = eB\cos\delta$, these equation (2.3) can be put in the form:

$$\begin{aligned}\eta'' + \omega^2\eta =& e\Big[\beta\left(\eta - \sin\eta\right) + 2\eta'\sin v + 4\sin v - \eta''\cos v \\ & + E_1\sin\nu v + \gamma'\eta' + 2B\cos\delta\sin\frac{\eta}{2} - 5A\sin\eta\Big] \\ & + e^2\left[10A\cos v\sin\eta + 2B\cos\delta\sin\big(v - \frac{\eta}{2}\big)\right]. \qquad (3.1)\end{aligned}$$

Here $\omega^2 = 3$ and $\beta = \dfrac{\omega^2}{e}$. Moreover the non-linearity term $(\eta - \sin\eta)$ will be assumed to be the order of e.

The system described by the equation (3.1) moves under the forced vibration due to the presence of the magnetic field of the Earth, oblateness of the Earth and the external periodic forces of general nature on the right hand side of the equation.

This periodic sine force of perturbative nature as long as the period of oscillations of the system is different from the period of sine force for which solution is obtain [25]. As the period of sine force is always changing, it may become equal to the sine force, in that case the periodic sine force plays vital role in the oscillatory motion of the system. While examining the non resonance case Narayan and M. D. Pandey [25], we conclude that the system experience resonance behavior at and near $\omega = 1$ and hence the non-resonance solution fails. We are benefitted by the smallness of the eccentricity "e" in equation (3.1) and hence the solution of the differential equation may be obtained by exploiting the Bogoliubov, Krilov and Metropolosky method [21].

Now we construct the asymptotic solutions of the system representing (3.1) in the most general case, which is valid at and near the main resonance $\omega = 1$ exploiting the well known Bogoliubov, Krilov and Metropoloskey method, see [21]. The solution of equation (3.1)in the first approximation will be sought in the form:

$$\eta = acos\left(v+\theta\right), \tag{3.2}$$

$$\frac{da}{dv} = eA_1\left(a,\theta\right), \tag{3.3}$$

$$\frac{d\theta}{dv} = \left(\omega-1\right)+eB_1\left(a,\theta\right), \tag{3.4}$$

where $A_1\left(a,\theta\right)$ and $B_1\left(a,\theta\right)$ are particular solution periodic with respect to θ of the system.

$$\left(\omega-1\right)\frac{\partial A_1}{\partial\theta}-2a\omega B_1 = \frac{1}{2\pi^2}\sum_{\sigma=-\infty}^{\sigma=+\infty}\int_0^{2\pi}\int_0^{2\pi} e^{-i\sigma\theta} f_0\left(a,\eta,\eta',\eta''\right)e^{-i\sigma\theta'}\cos k\,dv\,dk,$$

$$a\left(\omega-1\right)\frac{\partial B_1}{\partial\theta}+2\omega A_1 = -\frac{1}{2\pi^2}\sum_{\sigma=-\infty}^{\sigma=+\infty}e^{-i\sigma\theta}\int_0^{2\pi}\int_0^{2\pi} f_0\left(a,\eta,\eta',\eta''\right)e^{-i\sigma\theta}\sin k\,dv\,dk, \tag{3.5}$$

where $\theta = k - v$ and $f(a, \eta, \eta', \eta'')$ is the coefficient of e on the right hand side of equation (3.1).

Simple integration, gives us:

$$\begin{aligned}(\omega-1)\frac{\partial A_1}{\partial\theta} &- 2a\omega B_1 \\ &= \beta\left(a - 2J_1(a) - 4\sin\theta - 10AJ_1(a) + 2B\cos\delta J_1\left(\frac{a}{2}\right)\right), \\ a(\omega-1)\frac{\partial B_1}{\partial\theta} &- 2\omega A_1 = \gamma_1 a - 4\cos\theta, \end{aligned} \tag{3.6}$$

where $J_1(a)$ is the Bessel's function of the first order:

$$\sin(a\cos\theta) = 2\sum_{k=0}^{\infty}(-1)^k J_{2k+1}(a)\cdot\cos(2k+1)\theta,$$

$$\cos(a\cos\theta) = J_0(a) + 2\sum_{k=0}^{\infty}(-1)^k J_{2k}(a)\cdot\cos 2k\theta. \tag{3.7}$$

Here J_k, $k = 0, 1, 2, 3....$ stands for Bessel's function. The periodic solution of the system given by equations (3.6) can obtained as:

$$\begin{aligned} A_1 &= \left[\frac{\gamma_1 a}{2\omega} - \frac{4\cos\theta}{(\omega+1)}\right], \\ B_1 &= \left\{-\frac{\beta_1}{2a\omega}(a - 2J_1(a))\right\} + \frac{4\sin\theta}{a(\omega+1)} + \frac{10AJ_1(a)}{2a\omega} \\ &\quad - \frac{2B\cos\delta}{2a\omega}J_1\left(\frac{a}{2}\right), \end{aligned} \tag{3.8}$$

where the amplitude "a" and phase "θ" are the given by the system of differential equations:

$$\frac{da}{dv} = \frac{\gamma a}{2\omega} - \frac{4e\cos\theta}{(\omega+1)};$$

$$\begin{aligned}\frac{d\theta}{dv} = (\omega-1) - \frac{1}{2a\omega}(a - 2J_1(a)) + \frac{4esin\theta}{a(\omega+1)} - \frac{2eB\cos\delta}{2a\omega}J_1\left(\frac{a}{2}\right) \\ + \frac{10eAJ_1\left(\frac{a}{2}\right)}{2a\omega}.\end{aligned} \tag{3.9}$$

It is clear from (3.9) that there is absence of external periodic force$Esinvv$ on the amplitude and phase of the oscillatory system in first approximation at this overtone. But the presence of dissipative force and the magnetic field and oblateness of the Earth introduces a connection in the amplitude of the system. The set of equations (3.9) cannot be integrated in a closed form due to dependence of right hand side on a and θ. However the qualitative aspects of the solution can be examines with the help of Poincare theory, see [22].

$$\begin{aligned} \frac{da}{dv} &= a\delta_e\left(a\right) - \frac{4ecos\theta}{\left(\omega+1\right)}, \\ \frac{d\theta}{dv} &= \omega_e - 1 + \frac{4esin\theta}{a\left(\omega+1\right)}, \end{aligned} \tag{3.10}$$

where $\delta_e\left(a\right) = \frac{\gamma}{2}$ and

$$\omega_e = \omega - \left(\frac{a^2}{16\omega} - \frac{Bcos\delta a^2}{128} - \frac{5ea^2A}{16\omega}\right). \tag{3.11}$$

The parameters $\delta_e\left(a\right)$ and ω_e introduced here respectively, are the equivalent damping decrement and equivalent frequency of non linear oscillation of the dumbbell satellite system where impressed force is absent. We now examine the stationary regime of oscillation of the system in the first approximation.

The stationary state of oscillation is defined by

$$\frac{da}{dv} = 0, \quad \frac{d\theta}{dv} = 0.$$

Hence, from the set of equation (3.9) retaining up to the second order termed in the amplitude, we obtain:

$$\begin{aligned} \gamma_1 a - 4ecos\theta &= 0, \\ (\omega_e^2 - 1)a + 4esin\theta &= 0, \end{aligned} \tag{3.12}$$

where $\gamma_1 = \frac{\gamma(\omega+1)}{2\omega}$.

Eliminating the phase θ in (3.12) we obtain

$$\left[\omega_e^2 - 1\right]^2 = \left[\frac{16e^2}{a^2} - \gamma_1^2\right]. \tag{3.13}$$

In order to obtain this relation in the neighborhood of the resonance frequency, substituting

$$\omega = 1 + \delta.$$

Here δ is a small quantity which shows the variation of the natural frequency of the system about the resonance frequency.

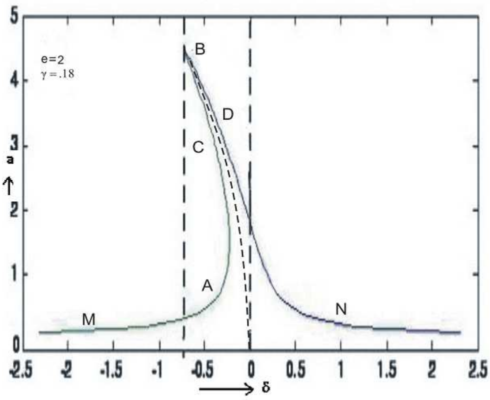

Figure 1: Behaviour of the system under perturbing forces

We obtain the relation (3.12) in a more convenient form:

$$\delta = La^2 \pm \frac{1}{2}\sqrt{\frac{16e^2}{a^2} - \gamma^2}, \tag{3.14}$$

where

$$L = \left[\frac{1}{16} - \frac{Bcos\delta}{128} - \frac{5eA}{16}\right].$$

This is the relation between the amplitude of the stationary oscillations and frequency of the system. Equation (3.14) can be written in the form:

$$a^2\left[4(\delta - La^2)^2 + \gamma^2\right] = 16e^2. \tag{3.15}$$

A schematic representation of behaviors of the relation (3.14) in the range of the parameter $\gamma$is given in Figure 1. The dotted line in the figure represents the skeleton curve $\omega = 1$This after using the relation $\omega = 1 + \delta$, takes the form:

$$\delta = La^2.$$

We notice here that as δ increases the amplitude of the oscillation increases along MA but at A in increased discontinuously from A to B and further decreased along B with increase in δ. On the other hand if δ, decreases the amplitude of the forced oscillation increases along ND but at D there is a discontinuity in the amplitude which falls abruptly to the value corresponding to C. Thus the section BC corresponds to the unstable amplitude of the oscillation while remaining portion of the response curve corresponds to the stable amplitude. The points A and C corresponds to the jump and break in the amplitude of the stationary oscillations. The specific property of the curve is the fact that the three stationary amplitude of the oscillations situated in the region ACBD corresponds to the same frequency of the external force over some frequency range when the parameter γ and e are connected by certain relationship. Two of the stationary amplitude of the oscillation are stable while the third stationary amplitude which corresponds to the section BC of the curve is unstable.

We shall determine the relation that must exist between the parameter of the system for the effect under consideration to occur. The limits of this range are determine by the condition: $\dfrac{da}{d\delta} = \infty$ which holds at the points C and B. Proceedings with the relation (3.15) differentiating further with respect to δ, we obtain:

$$\frac{da}{d\delta} = \frac{a(La^2 - \delta)}{[3L^2a^4 - 4\delta La^2 + (\gamma^2 + \delta^2)]}.$$

Hence C and B are determined by the simultaneous solution: if $\dfrac{da}{d\delta} = \infty$

$$3L^2a^4 - 4\delta La^2 + (\gamma^2 + \delta^2) = 0. \tag{3.16}$$

We observed from the equation (3.16) that both the roots of are positive that is the effect under study is always possible at a frequency less than the resonance frequency of the system. Hence it is necessary to determine a relation between the parameter γ, e, A and B which are responsible for the existence of resonance behavior in the oscillations of the system near the resonance frequency. The figure shows that the resonance will not occur when the section AC reduce to a point of inflexion that is the two roots of the quadratic equation (3.16) coincide.

The maximum value of the amplitude is defined by the condition $\dfrac{da}{d\delta} = \infty$. Thus we obtain $a_{max} = \frac{4e}{\gamma}$, also we obtain from (3.16) the critical value γ_k of γ of the dissipative force

$$\gamma_k^3 = 16e^2L. \tag{3.17}$$

The effects under study that there is the break and jump in the amplitude of the oscillations near the resonance frequency $\omega = 1$, are possible only for those values of γ which are less than the critical value γ_k. The curves for different value of γ are shown, for a fixed value of e, B, δ and A, in figure ($\gamma > \gamma_k$) and $\gamma \leq \gamma_k$.

Hence we conclude that resonance occurs in oscillations of the system in the neighborhood of the frequency $\omega = 1$for the dissipative coefficient γ which are less than the critical value γ_k given in the relation (3.17). The break and jump in the amplitude of the system are possible at an frequency greater than

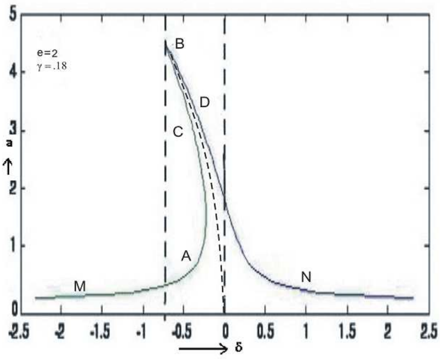

Figure 2: Behaviour of the system under perturbing forces

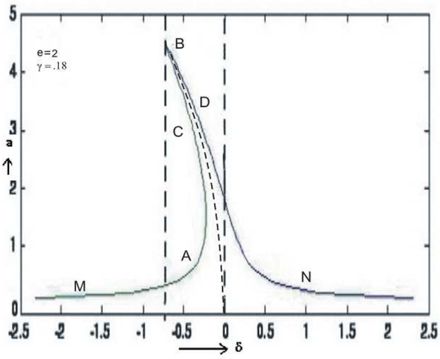

Figure 3: Behaviour of the system under perturbing forces

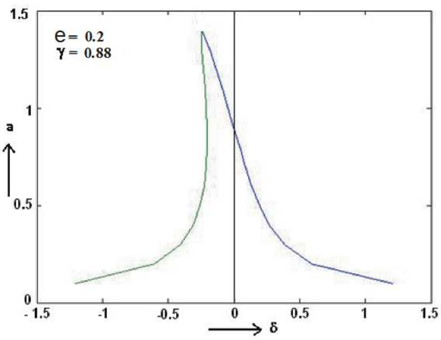

Figure 4: Behaviour of the system under perturbing forces

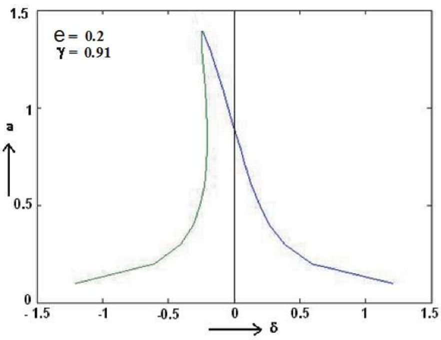

Figure 5: Behaviour of the system under perturbing forces

the resonance frequency $\omega = 1$. Thus the discontinuous changes in the orbit parameter take place when a small dissipative force oblateness of the Earth and magnetic field of the Earth are present.

4. Discussion and Conclusion

The non-linear resonance oscillations of dumbbell satellite in elliptical orbit in the central gravitational field of force under the combined influence of the Earth magnetic field, oblateness of the Earth and some external periodic force of general nature has been discussed. Non-linear oscillations of dumbbell satellites about the equilibrium position in the neighborhood of the main resonance$\omega = 1$

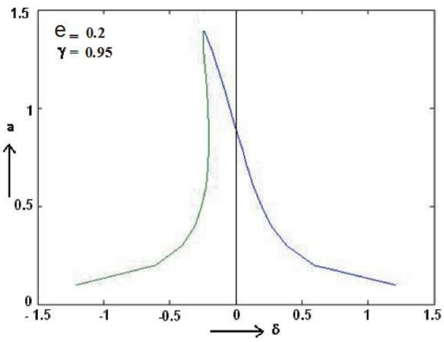

Figure 6: Behaviour of the system under perturbing forces

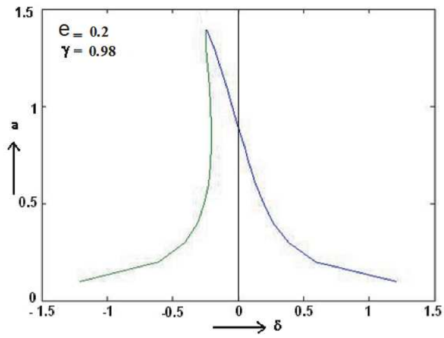

Figure 7: Behaviour of the system under perturbing forces

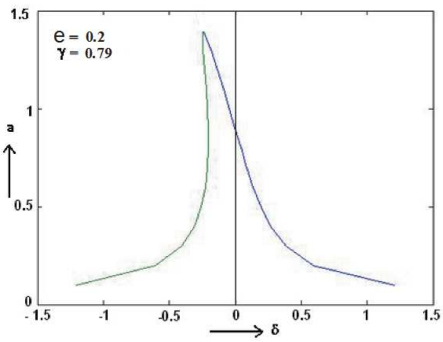

Figure 8: Behaviour of the system under perturbing forces

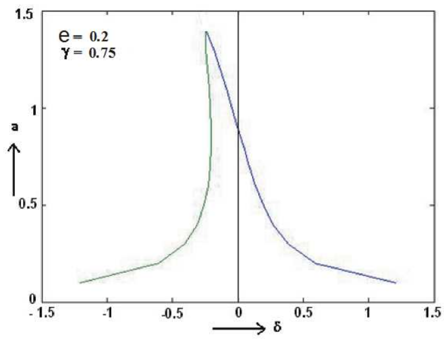

Figure 9: Behaviour of the system under perturbing forces

has been studied, exploiting the asymptotic method due to Bogoliubov, Krilov and Metropolosky method, considering e to be a small parameter. The stability of the system has been investigated using Poincare method.

The effect under study that there is the break and jump in the amplitude of oscillations near the resonance frequency $\omega = 1$ are possible only for the value of γ which is less than the critical valueγ_k taking into account of the magnetic field and oblateness of the Earth parameter. The break and jump in the amplitude of the system are possible at a frequency greater than the resonance frequency $\omega = 1$. Thus the discontinuous changes in the orbit parameter take place when a small dissipative force, oblateness of the Earth and magnetic field of the Earth are taken into account.

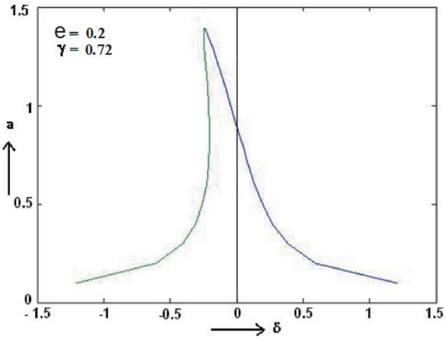

Figure 10: Behaviour of the system under perturbing forces

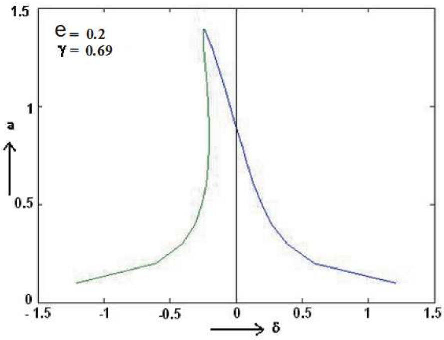

Figure 11: Behaviour of the system under perturbing forces

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