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A MODEL FOR CO-DISCOVERY IN SCIENCE BASED ON THE SYNCHRONIZATION OF GAUSS MAPS

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Abstract: In this article we present an explanation for co-discovery in science. Our core proposal is that co-discovery i.e. the synchronization of different people offering the same (or very similar) ideas, takes place when a common external action is provided by an external and common information flux. In our mathematical model this external action takes the form of a chaotic or a noisy signal. From our model we obtain an estimation for the number of co-discoveries expected for the 21$^{\text{th}}$ century which will be in the order of thousands.

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1. Introduction

Co-discovery in science, i.e. discoveries taking place approximately at the same

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time carried out by independent people, is an important scientific and social phenomenon which has not yet received enough attention. Nor to our knowledge has ever been modeled. Even worst, there is not an official world repository of this information. In this article we present an explanation of co-discovery in science. Our core proposal is that co-discovery, i.e. the synchronization of different people offering the same (or very similar) ideas, takes place when a common external action is provided by an external and common information flux. In our mathematical model this external action takes the form of a chaotic or a noisy signal. In a fundamental article Robert Merton [1] based on Bacon *Novum Organum*, discusses a sociological theory of scientific discovery where he deals with "the strategic fact of the multiple and independent appearance of the same scientific discovery", which for convenience he describe as a "multiple". He presents abundant examples and references to point out the fact that despite the many occasions on which the theory of multiples was published, it has periodically emerged as an idea new to many observers who worked it out for themselves for about three centuries. Merton [1], writes, "multiple discoveries in science continues to be regarded by some, including minds of a high order, as something surpassing strange and almost unexplainable". It is here where we believe that our work is relevant since we present a mathematical model of co-discovery in science i.e. discoveries taking place simultaneously carried out by independent people -which is the basic component of a multiple, such as a doublet, triplet and so on- based on synchronization ideas and models able to provide some qualitative and quantitative explanations. We believe that this may also be relevant to support and develop theoretical basis such as the one presented by Small [2], for scientometrics and citationology, and also in order to enlighten Matthew Effect as explained by Merton [3]. It is known that studies and observation of synchronization are not new. Engelberth Kaempfer observed synchronization in living systems (glowworms) during his observations in his voyage to Siam in 1680. This was the first reported observation of synchronization in a large population of oscillating systems. Besides in 1665 Christian Huygens discovered that a couple of pendulum clocks hanging from a common support synchronized. Latter in the middle of XIX century William Strutt discovered the phenomenon of synchronization in acoustical systems and in the early XX century Edward Appleton and Balthasar van der Pol showed that the frequency of an electric generator can be synchronized by a weak external signal as shown by Pikovsky et al. [4]. Also, it was shown by Maritan et al. [5] that synchronization may be induced by a periodic external action or by noise in systems which may also be chaotic. As explained by Toral et al. [6] and Guan et al. [7], noise-induced synchronization refers to the phe-

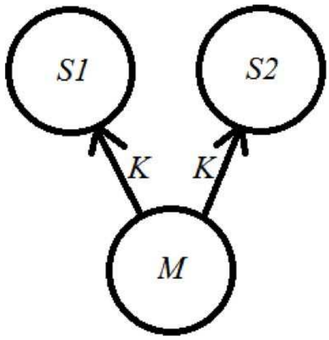

Figure 1: Proposed basic star topology network to study synchronization between slave systems ($S1$ and $S2$) coupled to a master system (M) through a coupling strength (K).

nomenon where two uncoupled, independent nonlinear oscillators can achieve synchronization through a "common" noisy forcing. What now seems clear is that synchronization in physical, biological or social systems may have some common description that obeys some universal laws. Due to the complexity of real world problems, simple mathematical models and vast simplifications are often used in all areas of science. Galileo frictionless kinematics as well as quantum harmonic interatomic potentials are good examples of simplifications in science. As it is known, nowadays social and biological sciences are also being studied through mathematical models and computer simulations which also involve small or large simplifications.

Figure 1 shows our model to study synchronization and co-discovery in science. It is based on a star topology network between slaves systems ($S1$ and $S2$) and a master system (M) coupled through a coupling strength (K) to the slaves. Slaves may represent people or identical systems, and master represents a system providing a unidirectional and external action or information flux over the slaves through a coupling strength K. Clark et al. [8] stated that the human and chimp sequences differ by only 1.2 percent in terms of single-nucleotide changes to the genetic code, due to this affirmation here we assume that all human beings are essentially identical in their DNA, and even though every one lives in a different place on the Earth, some of them are influenced by the same, or very similar, external signals provided by: radio, television, internet, books, journals, newspapers and scientific articles, among others. Due to the increase of information in the world and the number of people having access to it, it should be expected that the occurrence of co-discovery in science will also increase in the future. As time passes by, this phenomenon will be more and more important. Counting some of the best known and important cases of co-discovery in science, such as Calculus by Newton and Leibniz, or the Laser by Townes, Basov and Prokhorov, and arbitrarily neglecting other less known cases due to the lack of an official list of world co-discoveries; a basic estimate of some of the most important co-discoveries in science from the 16th till the 20th centuries is shown in the following data compiled by the authors and shown in Appendix I: 2 in the 16th, 5 in the 17th, 2 in the 18th, 19 in the 19th, 42 in the 20th century. Fitting the above data to an exponential function $Y = \exp(a + bx + cx^2)$, we obtain the following parameters: a =176.41405, b =-0.02013 and c =5.74534 E-5. Using this function we find that 1,148 is an estimate of the number of co-discoveries expected to occur during the 21th century. Certainly this model suggests that this value will be in the order of thousands. This data should also be seen in parallel to the contemporary information explosion. Some authors, such as Wurman [9], have pointed out that the information contained in a daily edition of the New York Times exceeds the one received by an average 17th century English person during all of his life, and that during the last thirty years more information has been produced than in the last five thousand years. The study "How much information" from the University of California in Berkeley [10] is an attempt to estimate how much new information is created each year. It concludes that from 1999 till 2002, the amount of new information stored on paper, film, magnetic, and optical media has about doubled in the last three years. The worldwide production of original information, if stored digitally, in terabytes circa 2002 has grown at a 74.5 % annual rate. Clearly for co-discovery occurring it is important not only the world growth of total information but also the access of this information to more population. Literacy is essential to access information, and even though it is difficult to obtain long term world literacy statistics and precise figures are open to debate, the conventional (usually census-based) literacy data according to the Institute of Statistics of UNESCO [11] reports that world illiteracy halved between 1970 and 2005. The broad access to information is a fact for a large and growing percentage of world population. As discussed by Stromquist [12], it is generally accepted that literacy skills are fundamental to informed decision-making, personal empowerment, active and passive participation in local and global social community as well as creative thinking. It has been pointed out by Smyth [13], that it is difficult to provide a systematic evidence-based account of the effects of literacy due to a relatively small number of studies on this topic. It is also difficult to separate the effects of literacy programs from schooling. According to Robinson-Pant [14], the tendency is "to conflate schooling, education, literacy and knowledge". Even though it is complicated to positively prove the hypothesis that links the increase in literacy and world knowledge with co-discovery in science, we believe that it is a reasonable one. The structure of this article is the following: Section 2 provides a "smooth" introduction to the basic concepts of maps for non-specialists which are useful to follow the rest of the article. In particular some of the most important ideas are taken from the so-called logistic map. Section 3 describes the synchronization of two Gauss map slaves systems as the basic unit for our model for co-discovery in science. Based on the results of Section 3; Section 4 describes our model for co-discovery which is a generalization of the basic unit model including several thousand slaves in the network. It is shown that as a consequence of the increase in population and in the coupling strength K, an exponential grow in co-discovery is predicted. Finally, Section 5 presents our main conclusions.

2. Introduction to Maps

In simple terms, a map is any mathematical algorithm able to produce a sequence of numbers. A simple example is: $x_{n+1} = 2x_n + 1$. Arbitrarily starting with the initial value $x_0 = 3$, we obtain the following sequence: $x_1 = 7$, $x_2 = 15$, $x_3 = 31$, etcetera. As any search in internet or elsewhere may show, there are many well known maps $x_{n+1} = f(x_n)$. They are very important in the study of chaos and dynamic systems and are applied in a wide range of fields such as chemistry, biology, medicine, economics, ecology, neuroscience, laser physics and many others. It is no doubt fascinating to see that very similar mathematical models can be applied to so many different areas. An interesting example is the so-called logistic map which is described by the map: $x_{n+1} = \rho x_n(1 - x_n)$, where ρ is a constant and ρ and x_n are in the ranges; $1 < \rho \leq 4$ and; $0 \leq x \leq 1$. Depending on ρ value the logistic map may describe a very rich dynamics including fixed points, period doubling and chaos. For example, when $\rho = 2$ the following map is obtained: $x_{n+1} = 2x_n(1 - x_n)$. In this case it is easy to show that after a short transient depending on the initial value x_0 a fixed point is obtained, e.g. starting with $x_0 = 0.25$, we obtain: $x_1 = 0.375$, $x_2 = 0.46875$, $x_3 = 0.498$, $x_4 = 0.4999$ and $x_n \to 0.5$ (for n larger than 5). In a similar way, when $\rho = 3.2$, the map $x_{n+1} = 3.2x_n(1 - x_n)$ will produce the following values after a short transient which depends on the initial value x_0: $x_0 = 0.6$, $x_1 = 0.511$, $x_2 = 0.7996$, $x_3 = 0.5127$, $x_4 = 0.7994$, $x_5 = 0.5131$, $x_6 = 0.7994$, $x_7 = 0.5131$. As we may see the obtained values for x_n alternate between the fixed values 0.5131 and 0.7994. The full richness of the logistic map is better

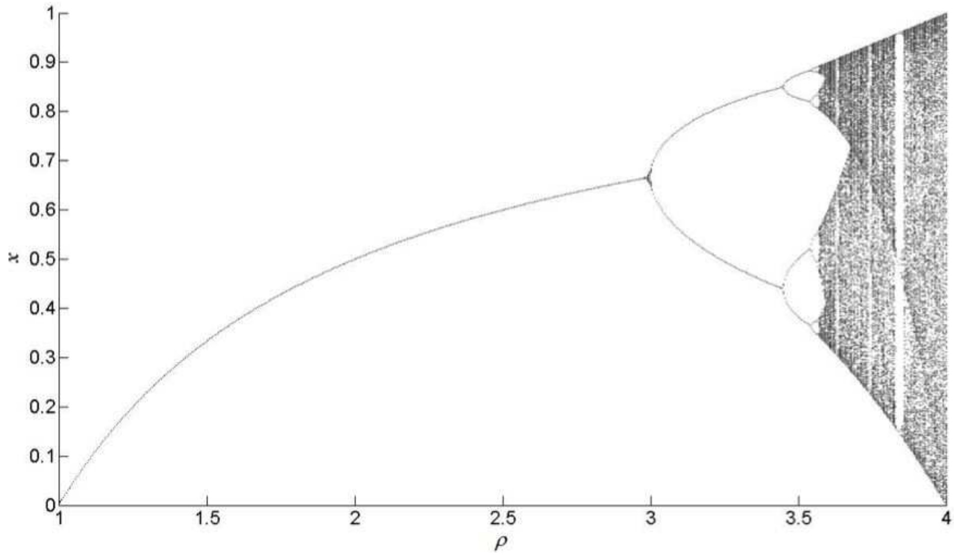

Figure 2: Bifurcation diagram for the logistic map.

shown in a bifurcation diagram. This is shown in Figure 2. We can see that depending on α value we may obtain a stable fixed point ($1 < \rho < 3$). Also at $\rho = 3$ starts period-2, and at $\rho = 3.449$ starts period-4. Finally at $\rho = 3.57$ a chaos regions starts.

3. Basic Model for Co-Discovery

In our basic network, shown in Figure 1, slave oscillators, $S1$ and $S2$, are described by Gauss maps which due to different initial conditions have a different temporal behavior, whereas the master oscillator, M, is described by a chaotic Gauss map. The general iterative Gauss map is shown in equation (1)

$$x_{n+1} = e^{-\alpha x_n^2} + \beta. \tag{1}$$

The parameter values used for slave systems were $\alpha = 5.9$ and $\beta = -0.5$. These values guarantee that systems $S1$ and $S2$ will operate in a chaotic region of the Gauss map as shown in Figure 3.

The complete model described by Gauss maps is given by:

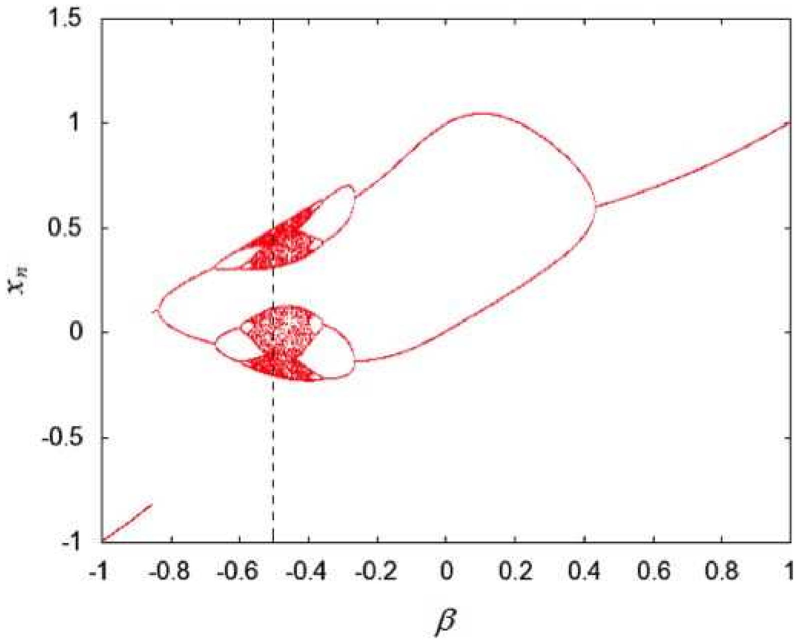

Figure 3: Bifurcation diagram of a Gauss map for = 5.9. It should be noticed that for = - 0.5 the system is in chaotic regime.

$$M_{n+1} = \exp(-\alpha_M M_n^2) + \beta_M, \tag{2a}$$

$$S2_{n+1} = \exp(-\alpha_{S2}(S2_n - K(M_n - S2_n))^2) + \beta_{S2}, \tag{2b}$$

$$S1_{n+1} = \exp(-\alpha_{S1}(S1_n - K(M_n - S1_n))^2) + \beta_{S1}. \tag{2c}$$

We may see that slave systems are coupled to the master system through the coupling strength parameter K. A resume of our main simulation results for the synchronization between slaves in the network are the following:

Case I. Identical slave and master systems.

In this case slave and master are described by chaotic Gauss maps where we used the following parameters; $\alpha_{S1} = \alpha_{S2} = \alpha_M = 5.9$, $\beta_{S1} = \beta_{S2} = \beta_M = -0.5$ and different initial conditions ($S1_0 = 0.6$, $S2_0 = 0.3$, $M_0 = 0.9$). Figures 4a) and 4b) show the temporal slaves behavior. Figure 4c) shows the difference between slaves in time, clearly if this difference is zero, slaves have achieved complete synchronization and they behave in identical manner. For a

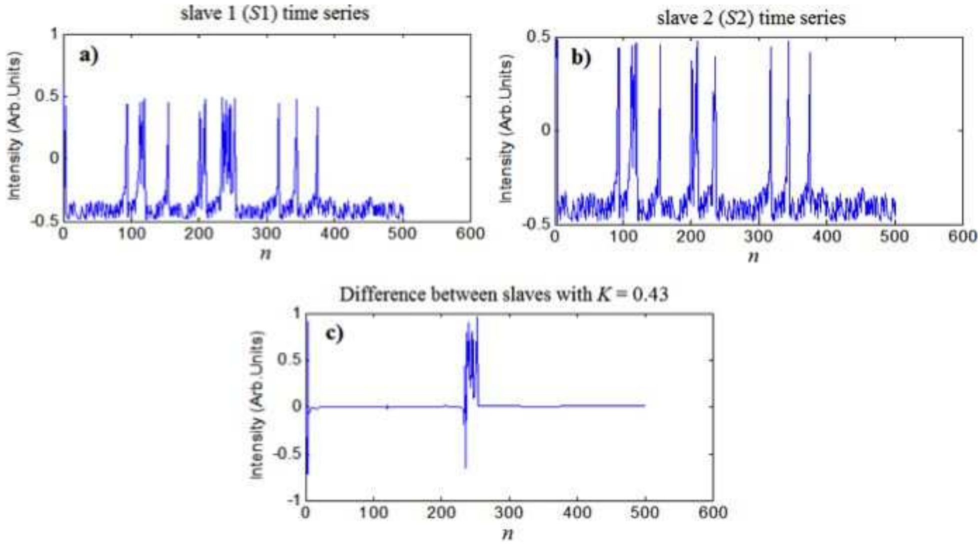

Figure 4: Case 1. Results for three identical Systems with $K = 0.43$. a) and b) shows temporal behavior for slaves $S1$ and $S2$ and c) shows the difference between them.

coupling strength parameter K =0.43 we achieve full synchronization through intermittency. For lower values of K no synchronization was observed.

Case II. Identical slaves and different master.

In this case slaves and master are still described using chaotic Gauss maps with the same initial conditions as in the previous Case 1, but the master has different parameters ($\alpha_M = -5.8$ and $\beta_M = -0.4$). In this case the master is located in a different chaos region, causing that slaves reach complete synchronization with a slightly smaller coupling strength constant value, $K = 0.41$, than in the previous case. Figures 5a) and 5b) show the temporal evolution of the slaves meanwhile Fig. 5c) shows their complete synchronization after a brief transient.

These two basic cases were presented in order to help the understanding of synchronization of Gauss systems in a network. In the next section this network will be extended to several thousands slaves which is the basis of our co-discovery model.

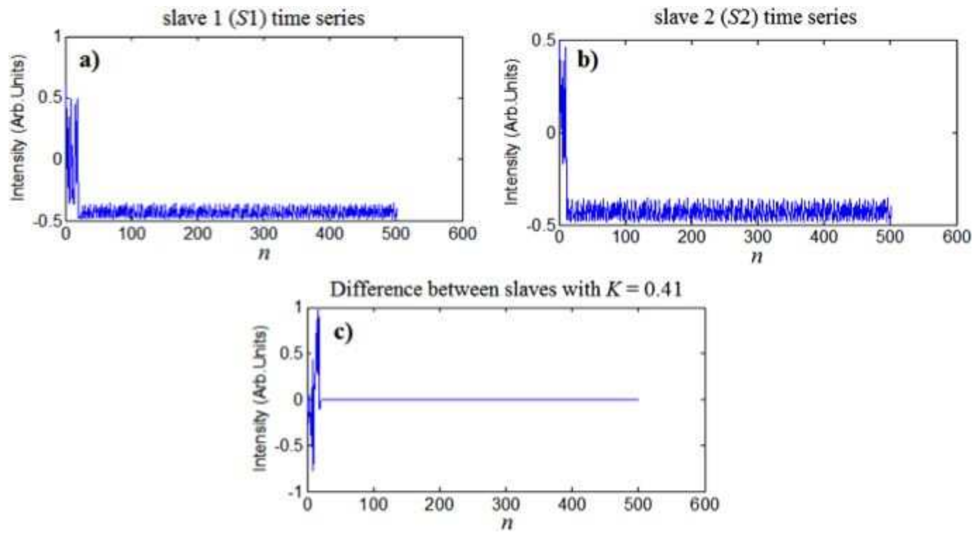

Figure 5: Results for master different from slaves with K =0.41. a) and b) shows temporal behavior for slaves $S1$ and $S2$ and c) shows the difference between them.

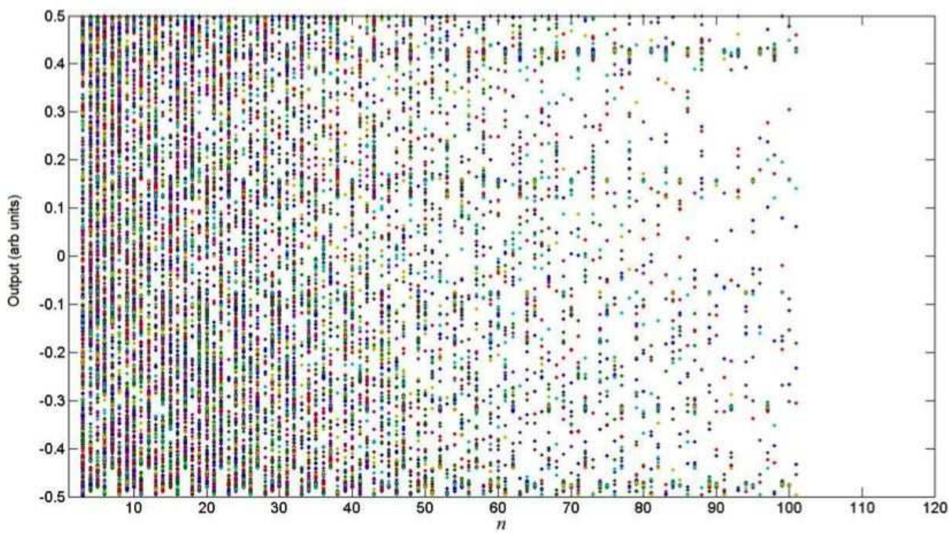

Figure 6: Oscillators synchronizing in groups. The starting population is 10,000 each with a different initial value, as time passes by, the oscillators formed 10 groups.

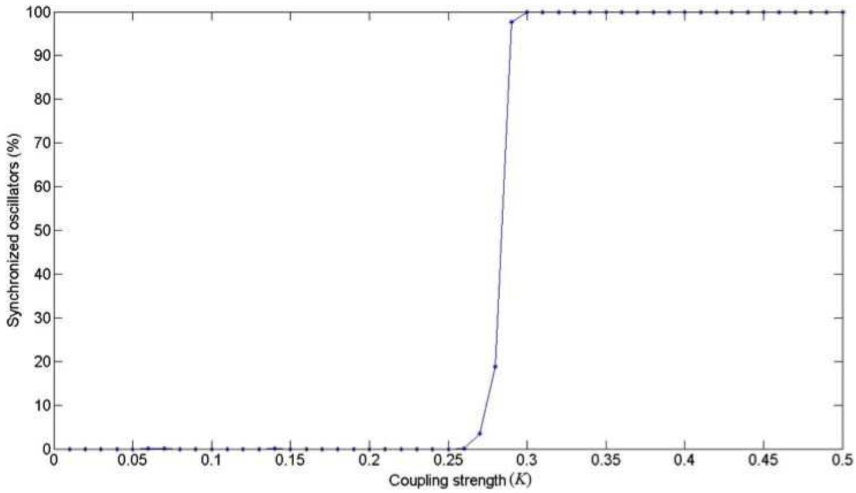

Figure 7: Synchronization state for a network composed by 125,000 oscillators.

4. Model Expansion: A Network for Co-Discovery Dynamics

The previous section explained, for two systems, the different ways to get synchronized by an external action (i.e. information). In this section the former star network instead of having two slaves will have several thousands. This will be used to study the dynamics of co-discovery in science modeled by a large number of Gauss slave systems when the master oscillator is also a Gauss map operating in a chaotic regime. This network is just an expansion of the previous model build with many more oscillators, representing people, unidirectionally linked to a master node, representing information, through a coupling strength constant representing information access. Figure 6 shows the network dynamics when it's composed by 10,000 slave oscillators with different initial conditions and with a coupling strength constant $K = 0.28$. The horizontal axis shows the temporal evolution of the network. Initially all the oscillators have different values. As time passes by, the oscillators start synchronizing among them forming groups. This can be observed in the extreme right of Figure 6 ; the oscillators formed ten groups just as it is expected to occur in real life (people search to be in contact with their similar).

When the slaves' population is increased to 125,000 oscillators in the network, we can observe that its synchronization state increases exponentially as shown in Figure 7. This dynamics qualitatively agree with the exponential

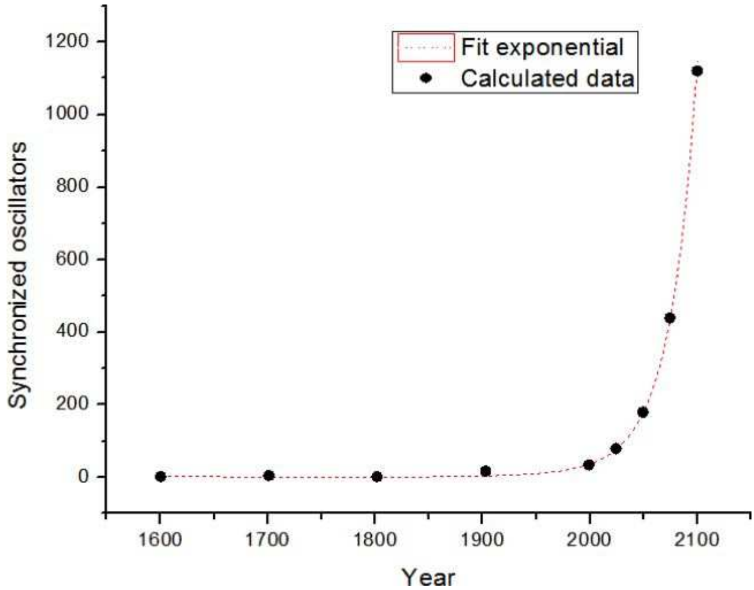

Figure 8: Co-discoveries reported and predicted (Fit exponential) versus synchronized oscillators in a network (Calculated data).

behavior of co-discovery discussed in our previous Section 1.

Adjusting the number of slave oscillators accordingly to the world's population since 1700 and predicted to 2100 we have found that the number of co-discoveries varies accordingly with the exponential prediction previously mentioned as can be seen in Figure 8.

The co-discoveries data was taken by centuries between 1600 and 2000. From 2000 till 2100 in order to get a higher reliability to our proposal, the estimations were done every quarter of a century. As can be seen, the exponential prediction for co-discoveries obtained directly from adjusting the available data shown in Section 1 is in good agreement with the estimation obtained with the full network dynamics. In fact, the exponential prediction for co-discovery is a result of the full network behavior.

5. Conclusions

A model to describe co-discovery in science is presented based on the synchronization of identical systems. Even though it is complicated to positively prove the hypothesis that links the increase in literacy and world knowledge, with co-discovery in science we believe that it is a reasonable one. It is shown that the synchronization of identical systems described by Gauss maps takes place when an external harmonic or chaotic signal drives them both, this is to our knowledge, a new result. It is suggested that the driving external excitation that causes synchronization of systems and therefore co-discovery in science is the information access and its broad distribution in modern society produced by radio, tv, internet, books, newspapers, scientific articles among other media. It is concluded that co-discovery in science is a phenomenon which will be every time more frequent. For our 21^{th} century we estimate that of the order of thousands co-discoveries will occur. The exponential prediction for co-discoveries obtained directly from adjusting the available data from 17^{th} till 20^{th} centuries is in good agreement with the estimation obtained with the full network model. In fact, it is found that the exponential prediction for co-discovery is a result of the proposed network dynamics.

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Appendix A: List of Significant Co-Discoveries Known by the Authors

16$^{\text{th}}$ Century

1520 - Scipione dal Ferro and Niccoló Tartaglia (1535): independently developed a method for solving cubic equations.

1585 - Simon Stevin and Galileo Galilei: heavy and light balls fall together (contra Aristotle).

17$^{\text{th}}$ Century

1610 - Thomas Harriot, Johannes and David Fabricius (1611), Galileo Galilei (1612), Christoph Scheiner (1612): Sunspots.

1614 - John Napier and Joost Bürgi (1618): Logarithms.
René Descartes and Pierre de Fermat: Analytic geometry.
Gottfried Wilhelm Leibniz and Seki Kõwa: Determinants.

1675 - Leibniz and Newton: Calculus.

18$^{\text{th}}$ Century

1778 - Antoine Lavoisier and Joseph Priestley: Co-discovery of oxygen.
Antonio de Ulloa and Charles Wood: Platinum.

19$^{\text{th}}$ Century

1817 - Friedrich Strohmeyer and K.S.L Hermann: Cadmium.

1828 - Friedrich Wöhler and A.A.B. Bussy: Beryllium.

1830 - Nikolai Ivanovich Lobachevsky and Jnos Bolyai (1832): Non-Euclidian geometry hyperbolic geometry.

1831 - Michael Faraday and Joseph Henry: Electromagnetic induction.

1831 - Samuel Guthrie, Eugène Soubeiran and Justus von Liebig: Chloroform.

Germinal Pierre Dandelin, Karl Heinrich Grffe and Nikolai Ivanovich Lobachevsky: Lobachevsky method, an algorithm for finding multiple roots of a polynomial.

1837 - Charles Wheatstone and Samuel F. B. Morse: Electrical Telegraph.

1843 - James Prescott Joule and Helmholtz (1947): Conservation of energy.

1846 - Urbain Le Verrier and John Couch Adams: Discovery of Neptune.

1858 - August Ferdinand Möbius and Johann Benedict Listing: Möbius strip.

1859 - Charles Darwin and Alfred R. Wallace: Evolution theory.

1862 - Lewis Swift and Horace Parnell Tuttle: The Perseid meteor shower.

1868 - Pierre Jansen and Norman Lockyer: Helium.

1869 - Dmitri I. Mendeleev and Julius L. Meyer: Periodic table of chemical elements.

1876 - Oskar Hertwig and Hermann Fol: Described the entry of sperm into the egg.

1876 - Alexander Graham Bell and Elisha Gray: Telephone.

1877 - Charles Cros and Thomas Edison (1878): Phonograph.

1886 - Charles Martin Hall and Paul Héroult: Hall-Héroult process for producing Aluminum.

1896 - Jacques Hadamard and Charles de la Vallée-Poussin : Asymptotic law of the distribution of prime numbers.

Filip F. Fortunatov and Ferdinand de Saussure: Saussure-Fortunatovs sound law.

20$^{\text{th}}$ Century

1900 - Henri Poincaré, Olinto de Pretto (1903), Albert Einstein (1905) and Paul Langevin (1906): $E = mc^2$.

1902 - Walter Sutton and Theodor Boveri: the hereditary information is carried in the chromosomes.

1902 - Richard Assman and Léon Teisserenc de Bort: Stratosphere's discovery.

1904 - Friedrich Stolz and Henry Drysdale Dakin: Epinephrine.

1907 - Georges Urbain and Carl Auer von Welsbach: Lutetium.

1907 - Frigyes Riesz and Maurice René Fréchet : Riesz representation theorem.

1908 - Jhoannes Stark and Albert Einstein: photoequivalence law.

1915 - Frederick Twort and Félix d'Hérelle (1917): Bacteriophages.

1915 - Albert Einstein and David Hilbert: General relativity theory.

1915 - Theo A. van Hengel with R.P.C. Spengler, Edward Hebern (1917), Arthur

Scherbius (1918), Hugo Koch (1919) and Arvin Damm (1919): Rotor cipher machines.

1922 - León Brillouin and Leonid Mandelstam: Inelastic dispersion of Light by acoustic phonons.

1922 - Joseph Tykocinski-Tykociner and Lee de Forest (1923): Sound film.

1924 - Alexander Oparin and J.B.S. Haldane: Primordial soup theory.

Kurt Gödel and Alfred Tarki: Indefinability theorem.

1934 - Gerhard Gentzen and Stanislaw Jaskowski: Natural deduction, an approach to proof theory in philosophical logic.

1934 - Aleksandr Gelfond and Theodor Schneider (1935): the Gelfond-Schneider theorem.

1936 - Alan Turing and Emil Post: Turing machine.

1939 - Hans von Ohain, Secondo Campini (1940) and Frank Whittle (1941): The jet engine.

1941 - Wlliam G. Templeman, Philip Nutman (1942), Franklin Jones (1942) and Ezra Kraus (1943): hormone herbicides.

1948 - S.I. Tomonaga and J. Schwinger: Renormalization.

1948 - Richard Feynman, Julian Schwinger, Sin-Itiro Tomonaga and Freeman Dyson: quantum chromodynamics.

1953 - Gell-Mann and Kazuhiko Nishijima: Gell-Mann-Nishijima Formula.

1957 - Robert Marshak with George Sudarshan, and, Richard Feynman with Murray Gell-Mann: vectorial structure of week interactions.

1961 - Gell-Mann and Yuval Neéman: Hadrons classification.

1961 - Charles Hard Townes, Nicolay Gennadiyevich Basov and Aleksandr Mikhailovich Prokhorov: Laser.

1962 - S. Okubo and Gell-Mann: Gell-Mann-Okubo Mass formula.

1964 - Peter Higgs, Robert Brout and Gerald Guralnik: Higgs Boson.

1964 - Gell-Mann and George Zweig: Quarks.

1965 - T. Kamasi, Daniel H. Younger (1967) and John Cocke (1970): The Cocke-Younger-Kamasi algorithm.

1965 - Ray Solomonoff, Andrey Kolmogorov and Gregory Chaitin: Kolmogorov-Chaitin complexity.

1966 F. Dyson with A. Lenard and E.H. Lieb with W. Thirring: Exclusion principle to prove stability of matter.

1970 - Howard Temin and David Baltimore: Reverse transcription enzymes.

1971 - Stephen Cook and Leonid Levin (1973): The Cook-Levin theorem.

1973 - Clifford Cocks and Ron Rivest (1977): RSA, an algorithm suitable for signing and encryption of data.

1973 - David Gross with Frank Wilczek and David Politzer: Asymptotic freedom in quarks.

1974 - B. Richter and S. Ting: Discovery of the J/ψ particle.

1975 - Endorphins were discovered independently in Scotland and America.

1985 - Neal Koblitz and Victor S. Miller: Elliptic curve cryptography.

1987 - Neil Immerman and Róbert Szelepcsényi: The Immerman-Szelepcsényi theorem.

1989 - Thomas R. Cech and Sidney Altman: Discovery of Ribozymes

1993 - Donald S. Bethune and Sumio Lijima: Discovery of single-wall carbon nanotubes.

1997 - Steven Chu, Claude Cohen-Tannoudji and William D. Phillips: Cooling of atoms using laser.