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# A PURE DIRAC'S METHOD FOR YANG-MILLS EXPRESSED AS A CONSTRAINED BF-LIKE THEORY

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**Abstract:** A *pure Dirac's method* of Yang-Mills expressed as a constrained $BF$-like theory is performed. In this paper we study an action principle composed by the coupling of two topological $BF$-like theories, which at the Lagrangian level reproduces Yang-Mills equations. By a pure Dirac's method we mean that we consider all the variables that occur in the Lagrangian density as dynamical variables and not only those ones that involve temporal derivatives. The analysis in the complete phase space enable us to calculate the extended Hamiltonian, the extended action, the constraint algebra, the gauge transformations and then we carry out the counting of degrees of freedom. We show that the constrained $BF$-like theory correspond at classical level to Yang-Mills theory. From the results obtained, we discuss briefly the quantization of the theory. In addition we compare our results with alternatives models that have been reported in the literature.

## 1. Introduction

Nowadays, the study of topological field theories is a topic of great interest in physics. The importance for studying those theories lies in a closed relation

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with physical theories as for instance, Yang-Mills [YM] and General Relativity [1, 2]. Topological field theories are characterized by being devoid of local physical degrees of freedom[1] , they are background independent and diffeomorphisms covariant [3, 22]. Relevant examples of topological field theories are the so called $BF$ theories. $BF$ theories were introduced as generalizations of three dimensional Chern-Simons actions and in a certain sense this is the simplest possible gauge theory. It can be defined on spacetimes of any dimension. It is background free, meaning that to formulate it we do not need a pre-existing metric or any other such geometrical structure on spacetime [4, 5]. At classical level, the theory has no local degrees of freedom; all the interesting observables are global in nature and this seems to remain true upon quantization [6]. Thus $BF$ theory serves as a simple starting point for the studies of background free theories. In particular, general relativity in 3-dimensions is a special case of $BF$ theory, while general relativity in 4-dimensions can be viewed as a $BF$ theory with extra constraints [7]. Furthermore, we are able to find in the literature several examples where $BF$ theories come to be relevant models for instance, in alternative formulations of gravity such as the MacDowell-Mansouri approach [8]. MacDowell-Mansouri formulation of gravity consists in breaking down the internal symmetry group of a $BF$-theory from $SO(5)$ to $SO(4)$, to obtain Palatini's action plus a sum of the second Chern and Euler topological invariants. Because these terms have trivial local variations that do not contribute classically to the dynamics, one thus obtain essentially general relativity. On the other hand, within the framework of [YM] theories, we can find some cases where $BF$ theories have been relevant, an example of this is Martellini's model [9]. This model consists in to express [YM] theory as a $BF$-like theory in order to expose its relation with topological $BF$ theory. Thus, the first-order formulation (BF-YM) is equivalent on shell to the usual second-order formulation (YM). In fact, after a Wick rotation both formulations of the theory possess the same perturbative quantum properties; the Feynman rules, the structure of one loop divergent diagrams and renormalization has been studied founding that there exists an equivalence of the $uv$-behaviour for both approaches [10]. The main advantage of this formulation lies in the possibility to express some observables like the color magnetic operator in the continuum and express it as the dual 't Hooft observable employing the abelian projection gauge [11]. Nevertheless, the canonical quantization perspective is more subtle under Wick

[1] In the paper we refer as a topological theory, a classical theory lacking of local degrees of freedom, even though it does possess global physical degrees of freedom, which are characteristic by means of the topological properties either of the internal field space or of the base spacetime manifold.

rotations, because it shows that in order to make the theory euclidean, we must to complexify the Poisson algebra [12]. Recently other approaches use a well known duality between Maxwell-Chern Simons theory and a self dual massive model, this description has been extended to topological massive gauge theories providing a topological mechanism to generate mass for the bosonic $p$-tensor fields in any spacetime dimension [13, 14].

On the other side, we are able to find that Lisi in [18] and Smolin in [19] have worked with $BF$-like theories written as an extension of Plebanski's action, and in those works they got a consistent dynamics for any group $G$ containing the local Lorentz group, and then by using a simple mechanism which breaks down the symmetry, they obtain as resulting dynamics to [YM] coupled to general relativity plus corrections.

At the light of these facts, in this paper we analyze a $BF$-like action yielding [YM] equations of motion. We study the principal symmetries of that action by means of a detailed canonical analysis using a pure Dirac's method, and the quantization procedure is discussed. With the terminology *a pure Dirac's method* we mean that we shall consider in the Hamiltonian framework all the fields that define our theory are dynamical ones. Of course, our approach differs from the standard Dirac's analysis, because the standard analysis is developed on a smaller phase space by considering as dynamical variables only those variables with time derivative occurring explicitly in the Lagrangian. In this paper, our approach present clear advantages in respect to the standard one, namely; by working on the full phase space, we will able to know the full structure of the constrains and their algebra, the equations of motion obtained from the extended action and the full structure of the gauge transformations as well. The approach used in the present paper, has been performed to diffeomorphism covariant field theories [21, 20], showing results that are not obtained by means of a standard Dirac's analysis. In those works were reported the full structure of the constraints on the full phase space for the Second-Chern class and the latter for general relativity in the $G \to 0$ limit, being $G$ the gravitational coupling constant. The correct identification of the constraints is a very important step because are used to carry out the counting of the physical degrees of freedom and they let us to know the gauge transformations if there exist first class constraints. On the other hand, the constraints are the guideline to make the best progress for the quantization of the theory, therefore it is mandatory to know their full structure [22]. It is worthwhile to mention, that the constraints obtained by performing a pure Dirac's formalism, the algebra among them is closed, and is not necessary to fix by hand the constraints as in the case of Plebanski theory [23, 24], because the method itself provides us the required structure. One example of ambiguities found by developing the hamiltonian analysis on a reduced phase space, is presented in three dimensional tetrad gravity, in despite of the existence of several articles performing the hamiltonian analysis, in some papers it is written that the gauge symmetry is Poincare symmetry [25], in others that is Lorentz symmetry plus diffeomorphisms [26], or that there exist various ways to define the constraints leading to different gauge transformations. We think that the complete Hamiltonian method ( a pure Dirac's formalism) is the best tool for solving those problems.

Finally we show that the action analyzed in this paper is the coupling of topological theories namely; the $BF$-like action studied here can be split in two terms lacking of physical degrees of freedom, the complete action, however, does has physical degrees of freedom, the [YM] degrees of freedom.

The paper is organized as follows: In Section 2 we show that [YM] action differ from the $BF$-like action studied here, because it is expressed as the coupling of two topological terms just as General Relativity in Plebanski's formulation [24]. By a pure Dirac's method of the $BF$-like theory, one realize that the couplet terms incorporate reducibility conditions in the constraints, thus, the topological invariance of the full action will be broken emerging degrees of freedom. In Section 3, a complete canonical analysis analysis of Martellini's model is performed, this exercise has not reported in the literature, then we compare the results of this section with those obtained in previous sections. We finish with some remarks about our results.

## 2. A pure Dirac's Method for [YM] Theory Expressed as a Constrained Bf-Like Theory

The action of our interest is given by

$$S[A,B] = \int_M *B^I \wedge B^I - 2B^I \wedge *F^I, \tag{1}$$

where $F^I$ corresponds to the curvature of the connection one-form $A^I$ valued on the algebra of $SU(N)$. In this manner, the action (1) takes the form

$$S[B,A] = \int_M \frac{1}{4} B^I_{\mu\nu} B^{\mu\nu I} - \frac{1}{2} B^{\mu\nu I} \left( \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + f^{IJK} A_\mu A_\nu \right), \tag{2}$$

the equations of motion obtained from (2) are given by

$$B^I_{\mu\nu} = F^I_{\mu\nu}, \qquad D_\mu B^{\mu\nu I} = 0, \tag{3}$$

which correspond to [YM] equations of motion. By substituting the former equations of motion in the action we recover the [YM] action

$$S[A] = -\int_M \frac{1}{4} F^I_{\alpha\beta} F^{\alpha\beta}_I d^4x, \tag{4}$$

where $F^I_{\alpha\beta} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + f^I{}_{JK} A^J_\mu A^K_\nu$, is the curvature tensor valued on a Lie algebra. It is important to remark, that the role of the dynamical variables occurring in the actions (2) and (4) is quite different. For the former, $A_\mu$ and $B_{\mu\alpha}$ both are determined by the dynamics. For the later, $A_\mu$ is determined by the dynamics and $F_{\mu\alpha}$ is not a dynamical variable anymore, it is a label.

In order to procedure with our analysis, we are able to observe that if we split the action (2) in two parts, say

$$S_1[B] = \int_M \frac{1}{4} B^I_{\mu\nu} B^{\mu\nu}_I, \tag{5}$$

and

$$S_2[A,B] = \int_M \frac{1}{2} B^{\mu\nu}_I \left( \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + f^I_{JK} A^J_\mu A^K_\nu \right), \tag{6}$$

we obtain two topological field theories. In fact, we can see immediately that $S_1[B]$ is topological since does not have dynamical variables occurring in the action. On the other hand, we will show below that the action (6) is a topological one as well. May be for the lector this part is not relevant, however, we need to remember that topological field theories are characterized by being devoid of local degrees of freedom. That is, the theories are susceptible only to global degrees of freedom associated with non-trivial topologies of the manifold in which they are defined and topologies of the gauge bundle [27, 28]. Thus, it is mandatory to perform the canonical analysis of the action (6) because there is a gauge group.

So, by performing the 3+1 decomposition in (6) we obtain

$$S_2[A,B] = \int \int_\Sigma \left[ B^{0iI} (\dot{A}^I_i - \partial_i A^I_0 + f^{IJK} A^J_0 A^K_i) + \frac{1}{2} B^{ijI} F^I_{ij} \right], \tag{7}$$

where $F^I_{ij} = \partial_i A^I_j - \partial_j A^I_i + f^I{}_{JK} A^J_i A^K_j$.

Dirac's method calls for the definition of the momenta $(\Pi^{\alpha I}, \Pi^{\alpha\beta I})$ canonically conjugate to the dynamical variables $(A^I_\alpha, B^I_{\alpha\beta})$

$$\Pi^{\alpha I} = \frac{\delta \mathcal{L}}{\delta \dot{A}^I_\alpha}, \qquad \Pi^{\alpha\beta I} = \frac{\delta \mathcal{L}}{\delta \dot{B}^I_{\alpha\beta}}, \tag{8}$$

On the other hand, the matrix elements of the Hessian

$$\frac{\partial^2\mathcal{L}}{\partial(\partial_\mu A^I_\alpha)\partial(\partial_\mu A^J_\rho)}, \quad \frac{\partial^2\mathcal{L}}{\partial(\partial_\mu B^I_{\alpha\beta})\partial(\partial_\mu A^J_\rho)}, \quad \frac{\partial^2\mathcal{L}}{\partial(\partial_\mu B^I_{\alpha\beta})\partial(\partial_\mu B^J_{\rho\gamma})}, \tag{9}$$

are identically zero, thus the rank of the Hessian is zero. Therefore, we expect $10(N^2-1)$ primary constraints. From the definition of the momenta we identify the following 10 primary constraints

$$\begin{aligned} \phi^{0I} &: \Pi^{0I} \approx 0, \\ \phi^{iI} &: \Pi^{iI} - B^{0iI} \approx 0, \\ \phi^{0i}_I &: \Pi^{0i}_I \approx 0, \\ \phi^{ij}_I &: \Pi^{ij}_I \approx 0, \end{aligned} \tag{10}$$

The canonical Hamiltonian density for the system has the following form

$$\begin{aligned} \mathcal{H}_c &= \dot{A}^I_\mu \Pi^\mu_I + \dot{B}^I_{0i}\Pi^{0i}_I + \dot{B}^I_{ij}\Pi^{ij}_I - \mathcal{L} \\ &= -A^I_0 D_i \Pi^{0i}_I - \frac{1}{2} B^{ij}_I F^I_{ij}, \end{aligned} \tag{11}$$

Thus, by taking in to account the primary constraints, we can identify the primary Hamiltonian given by

$$H_P = H_c + \int d^3x \left[\lambda^I_0 \phi^0_I + \lambda^I_i \phi^i_I + \lambda^I_{0i}\phi^{0i}_I + \lambda^I_{ij}\phi^{ij}_I\right], \tag{12}$$

where $\lambda^I_0, \lambda^I_i, \lambda^I_{0i}, \lambda^I_{ij}$ are Lagrange multipliers enforcing the constraints. The fundamental Poisson brackets for our theory are given by

$$\begin{aligned} \{A^I_\alpha(x), \Pi^{\mu I}(y)\} &= \delta^\mu_\alpha \delta^{IJ}\delta^3(x-y), \\ \{B^I_{\alpha\beta}(x), \Pi^{\mu\nu I}(y)\} &= \frac{1}{2}\left(\delta^\mu_\alpha\delta^\nu_\beta - \delta^\mu_\beta\delta^\nu_\alpha\right)\delta^3(x-y). \end{aligned} \tag{13}$$

In this manner, by using the fundamental Poisson brackets for our theory we find that the following $10(N^2-1)\times 10(N^2-1)$ matrix whose entries are the Poisson brackets among the primary constraints

$$\begin{aligned} \{\phi^0_I(x), \phi^0_J(y)\} &= 0, \qquad \{\phi^0_I(x), \phi^i_J(y)\} = 0, \\ \{\phi^0_I(x), \phi^{0i}_J(y)\} &= 0, \qquad \{\phi^0_I(x), \phi^{ij}_J(y)\} = 0, \\ \{\phi^i_I(x), \phi^j_J(y)\} &= 0, \qquad \{\phi^i_I(x), \phi^{0j}_J(y)\} = \frac{1}{2}\delta i_j \delta_{IJ}\delta^3(x-y), \end{aligned}$$

$$\{\phi^i_I(x), \phi^{jk}_J(y)\} \;=\; 0, \qquad \{\phi^{ij}_I(x), \phi^{kl}_J(y)\} = 0,$$

has rank $6(N^2-1)$ and $(4(N^2-1))$ null vectors. This means that we expect $4(N^2-1)$ secondary constraints

$$\begin{aligned} \dot{\phi}^0_I = \{\phi^0_I, H_P\} \approx 0 \;\;\Rightarrow\;\; \psi_I := D_i\Pi^i_I \approx 0, \\ \dot{\phi}^{ij}_I = \{\phi^{ij}_I, H_P\} \approx 0 \;\;\Rightarrow\;\; \psi^I_{ij} := \frac{1}{2}F^I_{ij} \approx 0, \end{aligned} \tag{14}$$

and the rank allows us to fix the following Lagrange multipliers

$$\begin{aligned} \dot{\phi^{0iI}} = \{\phi^{0iI}, H_P\} \approx 0 \;\;\Rightarrow\;\; \lambda^I_i = 0, \\ \dot{\phi^{ijI}} = \{\phi^{ijI}, H_P\} \approx 0 \;\;\Rightarrow\;\; \lambda^I_{0i} = 2D_jB^{ij}_I + 2f_I{}^{JK}A^J_0\Pi^{iK}. \end{aligned} \tag{15}$$

For this theory there are not, third constraints. In this manner, with all the constraints at hand, we need to identify those that are first and second class kind. For this purpose, we can observe that the $14(N^2-1)\times 14(N^2-1)$ matrix whose entries are the Poisson brackets among the primary and secondary constraints given by

$$\begin{aligned}
\{\phi^{0P}(x), \phi^{0I}(y)\} &= 0, \quad \{\phi^{0P}(x), \phi^{iI}(y)\} = 0, \\
\{\phi^{0P}(x), \phi^{0iI}(y)\} &= 0, \quad \{\phi^{0P}(x), \phi^{ijI}(y)\} = 0, \\
\{\phi^{lP}(x), \phi^{iI}(y)\} &= 0, \quad \{\phi^{lP}(x), \phi^{0iI}(y)\} = \frac{1}{2}\delta^i_l\delta^{PI}\delta^3(x-y), \\
\{\phi^{lP}(x), \phi^{ijI}(y)\} &= 0, \quad \{\phi^{lmP}(x), \phi^{ijI}(y)\} = 0, \\
\{\phi^{0P}(x), \psi^{I}(y)\} &= 0, \qquad \{\phi^{lP}(x), \psi^{I}(y)\} = f^{PIK}\Pi^{lK}\delta^3(x-y), \\
\{\phi^{0lP}(x), \psi^{I}(y)\} &= 0, \quad \{\phi^{lmP}(x), \psi^{I}(y)\} = 0, \\
\{\psi^{P}(x), \psi^{I}(y)\} &= f^{PIK}D_i\Pi^{iK}, \quad \{\psi^{lmP}(x), \psi^{I}(y)\} = 0, \\
\{\phi^{0P}(x), \psi^{ijI}(y)\} &= 0, \quad \{\phi^{lP}(x), \psi^{ijI}(y)\} = \frac{1}{2}\Big(-\delta^l_j\delta^{PI}\partial_i \\
&\qquad +\delta^l_i\delta^{PI}\partial_j + f^{PIK}(\delta^l_iA^K_j - \delta^l_jA^K_i)\Big)\,\delta^3(x-y), \\
\{\phi^{lmP}(x), \psi^{ijI}(y)\} &= 0, \quad \{\psi^{lmP}(x), \psi^{ijI}(y)\} = 0,
\end{aligned}$$

has rank$=6(N^2-1)$ and $8(N^2-1)$ null-vectors. From the null-vectors it is possible to identify the following $8(N^2-1)$ first class constraints

$$\begin{aligned}
\gamma^0_I &= \Pi^0_I \approx 0, \\
\gamma^{ij}_I &= \Pi^{ij}_I \approx 0, \\
\gamma^I &= D_i\Pi^{iI} + 2f^I{}_{JK}B^J_{0i}\Pi^{0iK}, \\
\gamma^I_{ij} &= \frac{1}{2}F^I_{ij} + \frac{1}{2}[D_i\Pi^0{}_j{}^I - D_j\Pi^0{}_i{}^I].
\end{aligned} \tag{16}$$

On the other side, the rank allows us to identify the next $6(N^2-1)$ second class constraints

$$\begin{aligned} \chi^{0i}_I &= \Pi^{0i}_I \approx 0, \\ \chi^{i}_I &= \Pi^{i}_I - B^{0i}_I \approx 0. \end{aligned} \tag{17}$$

However, we can observe in (16) that the fourth constraint can be written as

$$\Upsilon^I_i \equiv \eta_i{}^{jk}\gamma^I_{jk} = \frac{1}{2}\eta_i{}^{jk}F^I_{jk} + 2\eta_i{}^{jk}D_j\Pi^0{}_k{}^I, \tag{18}$$

thus, $D_i\Upsilon^{iI} - \eta_k{}^{ij}f^I{}_{JK}F^J_{ij}\Pi^{0kK} = 0$ because of Bianchi's identity $\eta^{ijk}D_iF^I_{jk} = 0$. In this way, the former relation represents a reducibility condition. This means that the number of independent first class constraints corresponds to $[8-1](N^2-1) = 7(N^2-1)$. Therefore, the counting of degrees of freedom can be carry out as follows: There are $20(N^2-1)$ canonical variables, $7(N^2-1)$ independent first class constraints and $6(N^2-1)$ independent second class constraints. Therefore, the system expressed by the action principle (2) is devoid of physical degrees of freedom and corresponds to be a topological theory. In this manner, the separated actions (5) and (6) are topological field theories.

Now we will perform a pure Dirac's analysis for the coupled action (2) and we will show that will be not topological anymore. For our aims, we perform the $3+1$ decomposition of (2) obtaining

$$\begin{aligned} S[A,B] = \int\int_\Sigma \Big[ &\frac{1}{2}B^I_{0i}B^{0iI} + \frac{1}{4}B^I_{ij}B^{ijI} \\ &-B^{0iI}(\dot{A}^I_i - \partial_i A^I_0 + f^{ijk}A^J_0A^K_i) - \frac{1}{2}B^{ijI}F^I_{ij}\Big]\, d^3x dt, \end{aligned} \tag{19}$$

thus, to perform a pure Dirac's method we need the definition of the momenta $(\Pi^{\alpha I}, \Pi^{\alpha\beta I})$ canonically conjugate to $(A^I_\alpha, B^I_{\alpha\beta})$

$$\Pi^{\alpha I} = \frac{\delta\mathcal{L}}{\delta\dot{A}^I_\alpha}, \qquad \Pi^{\alpha\beta I} = \frac{\delta\mathcal{L}}{\delta\dot{B}^I_{\alpha\beta}}, \tag{20}$$

on the other hand, the matrix elements of the Hessian

$$\frac{\partial^2\mathcal{L}}{\partial\dot{A}^I_\alpha\partial\dot{A}^J_\rho}, \quad \frac{\partial^2\mathcal{L}}{\partial\dot{B}^I_{\alpha\beta}\partial\dot{A}^J_\rho}, \quad \frac{\partial^2\mathcal{L}}{\partial\dot{B}^I_{\alpha\beta}\partial\dot{B}^J_{\rho\gamma}}, \tag{21}$$

are identically zero, the rank of the Hessian is zero. Thus, we expect $10(N^2-1)$ primary constraints. From the definition of the momenta (20), we identify the next $10(N^2-1)$ primary constraints

$$\begin{aligned} \phi^{0I} &: \Pi^{0I} \approx 0, \\ \phi^{iI} &: \Pi^{iI} + B^{0iI} \approx 0, \\ \phi^{0iI} &: \Pi^{0iI} \approx 0, \\ \phi^{ijI} &: \Pi^{ijI} \approx 0. \end{aligned} \tag{22}$$

The canonical Hamiltonian density for the system has the next form

$$\begin{aligned} \mathcal{H}_c &= \dot{A}^I_\mu \Pi^{\mu I} + \dot{B}^I_{\mu\nu}\Pi^{\mu\nu I} - \mathcal{L} \\ &= \frac{1}{2}\Pi^{iI}\Pi^I_i - \frac{1}{4}B^I_{ij}B^{ijI} - A^I_0 D_i \Pi^{iI} + \frac{1}{2}B^{ijI}F^I_{ij}. \end{aligned} \tag{23}$$

Thus the primary Hamiltonian is given by

$$H_P = H_c + \int d^3x \left[\lambda^I_0 \phi^{0I} + \lambda^I_i \phi^{iI} + \lambda^I_{0i}\phi^{0iI} + \lambda^I_{ij}\phi^{ijI}\right], \tag{24}$$

where $\lambda^I_0, \lambda^I_i, \lambda^I_{0i}, \lambda^I_{ij}$ are Lagrange multipliers enforcing the constraints. The fundamental Poisson brackets for our theory are given by

$$\begin{aligned} \{A^I_\alpha(x), \Pi^{\mu J}(y)\} &= \delta^\mu_\alpha \delta^{IJ}\delta^3(x-y), \\ \{B^I_{\alpha\beta}(x), \Pi^{\mu\nu J}(y)\} &= \frac{1}{2}\left(\delta^\mu_\alpha\delta^\nu_\beta - \delta^\mu_\beta\delta^\nu_\alpha\right)\delta^{IJ}\delta^3(x-y). \end{aligned} \tag{25}$$

The $10(N^2-1)\times 10(N^2-1)$ matrix whose entries are the Poisson brackets among the primary constraints are given by

$$\begin{aligned} \{\phi^{0P}(x), \phi^{0I}(y)\} &= 0, \qquad \{\phi^{0P}(x), \phi^{iI}(y)\} = 0, \\ \{\phi^{0P}(x), \phi^{0iI}(y)\} &= 0, \qquad \{\phi^{0P}(x), \phi^{ijI}(y)\} = 0, \\ \{\phi^{lP}(x), \phi^{iI}(y)\} &= 0, \qquad \{\phi^{lP}(x), \phi^{0iI}(y)\} = -\frac{1}{2}\delta^l_i\delta^{PI}\delta^3(x-y), \\ \{\phi^{lP}(x), \phi^{ijI}(y)\} &= 0, \qquad \{\phi^{0lP}(x), \phi^{0iI}(y)\} = 0, \\ \{\phi^{0lP}(x), \phi^{ijI}(y)\} &= 0, \qquad \{\phi^{lmP}(x), \phi^{ijI}(y)\} = 0, \end{aligned}$$

has rank $6(N^2-1)$ and $4(N^2-1)$ null vectors. Thus by using the null vectors, consistency conditions yield the following $4(N^2-1)$ secondary constraints

$$\begin{aligned} \dot{\phi}^{0I} = \{\phi^{0I}, H_P\} \approx 0 \;\;&\Rightarrow\;\; \psi^I := D_i\Pi^{iI} \approx 0, \\ \dot{\phi}^{ijI} = \{\phi^{ijI}, H_P\} \approx 0 \;\;&\Rightarrow\;\; \psi^{ijI} := B^{ijI} - F^{ijI} \approx 0, \end{aligned} \tag{26}$$

and the rank yields fix the following values for the Lagrange multipliers

$$\begin{aligned}\dot{\phi}^{0iI} &= \{\phi^{0iI}, H_P\} \approx 0 \;\;\Rightarrow\;\; \lambda_i^I = 0,\\ \dot{\phi}^{ijI} &= \{\phi^{ijI}, H_P\} \approx 0 \;\;\Rightarrow\;\; \lambda_{0i}^I = 2D_j B^{jiI} - 2f^{IJK} A_0^J \Pi^{iK}.\end{aligned} \tag{27}$$

For this theory there are not third constraints, instead we obtain the following Lagrange multipliers.

$$\dot{\psi}^{lmP} = \{\psi^{lmP}, H_P\} \approx 0 \;\;\Rightarrow$$
$$\alpha_{lm}^P = 0, \quad \lambda_{lm}^P = D_l\Pi^{mP} - D_m\Pi^{lP} - f^{PKI} F_{lm}^K A_0^I \tag{28}$$

In this manner, with all the constraints at hand , we need identify those that are first and second class kind. For this purpose, we can observe that the $14(N^2-1)\times 14(N^2-1)$ matrix whose entries are the Poisson's brackets among the primary and secondary constraints are given by

$$\begin{aligned}
\{\phi^{0P}(x), \phi^{0I}(y)\} &= 0, \qquad \{\phi^{0P}(x), \phi^{iI}(y)\} = 0,\\
\{\phi^{0P}(x), \phi^{0iI}(y)\} &= 0, \qquad \{\phi^{0P}(x), \phi^{ijI}(y)\} = 0,\\
\{\phi^{lP}(x), \phi^{iI}(y)\} &= 0, \qquad \{\phi^{lP}(x), \phi^{0iI}(y)\} = -\frac{1}{2}\delta_l^i \delta^{PI}\delta^3(x-y),\\
\{\phi^{lP}(x), \phi^{ijI}(y)\} &= 0, \qquad \{\phi^{lmP}(x), \phi^{ijI}(y)\} = 0,\\
\{\phi^{0P}(x), \psi^{I}(y)\} &= 0, \qquad \{\phi^{lP}(x), \psi^{I}(y)\} = f^{PIK}\Pi^{lK}\delta^3(x-y),\\
\{\phi^{0lP}(x), \psi^{I}(y)\} &= 0, \qquad \{\phi^{lmP}(x), \psi^{I}(y)\} = 0,\\
\{\psi^{P}(x), \psi^{ijI}(y)\} &= -f^{PIM}F_{ij}^M, \qquad \{\psi^{P}(x), \psi^{I}(y)\} = f^{PIK}\psi^K = 0\\
\{\phi^{0P}(x), \psi^{ijI}(y)\} &= 0, \qquad \{\phi^{lP}(x), \psi^{ijI}(y)\} = \Big(\delta_j^l\delta^{PI}\partial_i - \delta_i^l\delta^{PI}\partial_j\\
&\qquad\qquad + f^{IPK}(\delta_i^l A_j^K + \delta_j^l A_i^K)\Big)\,\delta^3(x-y),\\
\{\psi^{lmP}(x), \phi^{ijI}(y)\} &= (\delta_l^i\delta_m^j - \delta_m^i\delta_l^j)\delta^{PI}\delta^3(x-y), \qquad \{\phi^{0lP}(x), \psi^{ijI}(y)\} = 0,\\
\{\psi^{lmP}(x), \phi^{0I}(y)\} &= 0,
\end{aligned}$$

has rank=$12(N^2-1)$ and $2(N^2-1)$ null-vectors. From the null-vectors we identify the following $2(N^2-1)$ first class constraints

$$\begin{aligned}\gamma^{0I} &= \Pi^{0I} \approx 0,\\ \gamma^{I} &= D_i\Pi^{iI} + 2f^{IJK}B_{0i}^J\Pi^{0iK} + f^{IJK}B_{ij}^J\Pi^{ijK} \approx 0.\end{aligned} \tag{29}$$

In particular, we would like to stress that the full structure of the Gauss constraint in (29) has not been reported in the literature. On the other hand, that Gauss constraint is the full generator of $SU(N)$ transformations of the theory under study. Furthermore, the rank allow us to identify the following $12(N^2-1)$ second class constraints

$$\begin{aligned} \chi^{iI} &= \Pi^{iI} + B^{0iI} \approx 0, \\ \chi^{0iI} &= \Pi^{0iI} \approx 0, \\ \chi^{ijI} &= \Pi^{ijI} \approx 0, \\ \phi^{ijI} &= (B^{ijI} - F^{ijI}) \approx 0. \end{aligned} \tag{30}$$

Therefore, the counting of degrees of freedom is performed as follows. There are $20(N^2-1)$ phase space variables, $2(N^2-1)$ independent first class constraints and $12(N^2-1)$ second class constraints, thus the theory given in (2) has $2(N^2-1)$ degrees of freedom just like [YM] theory.

The algebra among the constraints (29) and (30) is given by

$$\begin{aligned}
\{\gamma^{0P}(x),\gamma^{0I}(y)\} &= 0, \qquad \{\chi^{lP}(x),\gamma^{I}(y)\} = f^{PIK}\chi^{lK} = 0,\\
\{\gamma^{0P}(x),\chi^{iI}(y)\} &= 0, \qquad \{\chi^{0lP}(x),\gamma^{I}(y)\} = f^{PIK}\chi^{0lP} = 0,\\
\{\gamma^{0P}(x),\chi^{0iI}(y)\} &= 0, \qquad \{\chi^{lmP}(x),\gamma^{I}(y)\} = f^{PIK}\chi^{lmK} = 0,\\
\{\gamma^{0P}(x),\chi^{ijI}(y)\} &= 0, \qquad \{\phi^{lmP}(x),\gamma^{I}(y)\} = f^{PIK}\phi^{lmK} = 0,\\
\{\gamma^{0P}(x),\phi^{ijI}(y)\} &= 0, \qquad \{\gamma^{P}(x),\gamma^{I}(y)\} = f^{PIK}\gamma^{K} = 0,\\
\{\gamma^{0P}(x),\gamma^{I}(y)\} &= 0, \qquad \{\chi^{lP}(x),\chi^{iI}(y)\} = 0,\\
\{\chi^{lP}(x),\chi^{0iI}(y)\} &= -\frac{1}{2}\delta^{i}_{l}\delta^{PI}\delta^{3}(x-y), \qquad \{\chi^{lmP}(x),\chi^{ijI}(y)\} = 0\\
\{\chi^{lP}(x),\chi^{ijI}(y)\} &= 0, \qquad \{\chi^{lP}(x),\phi^{ijI}(y)\} = \Big(\delta^{l}_{j}\delta^{PI}\partial_i - \delta^{l}_{i}\delta^{PI}\partial_j\\
&\qquad\qquad + f^{PIK}(\delta^{l}_{j}A^{K}_{i} - \delta^{l}_{i}A^{K}_{j}\Big)\,\delta^{3}(x-y),\\
\{\chi^{lP}(x),\phi^{ijI}(y)\} &= 0, \qquad \{\chi^{0lP}(x),\phi^{ijI}(y)\} = 0,\\
\{\chi^{0lP}(x),\chi^{0iI}(y)\} &= 0, \qquad \{\chi^{lmP}(x),\phi^{ijI}(y)\} = -\frac{1}{2}\Big(\delta^{l}_{i}\delta^{m}_{j}\\
&\qquad\qquad -\delta^{l}_{j}\delta^{m}_{i}\Big)\,\delta^{PI}\delta^{3}(x-y),\\
\{\phi^{lmP}(x),\phi^{ijI}(y)\} &= 0.
\end{aligned}$$

where we can appreciate that the algebra is closed.

The identification of the constraints will allow us to identify the extended action. By using the first class constraints (29), the second class constraints (30), and the Lagrange multipliers we find that the extended action takes the form

$$S_E[A^I_\mu, \Pi^{\mu I}, B^I_{\mu\nu}, \Pi^{\mu\nu I}, \lambda^I_0, \lambda^I, u^I_i, u^I_{0i}, u^I_{ij}, v^I_{ij}] = \int d^4x(\dot{A}^I_\mu \Pi^{\mu I}$$

$$
\begin{aligned}
&+ \dot{B}^I_{\mu\nu}\Pi^{\mu\nu I} - \frac{1}{2}\Pi^{iI}\Pi^I_i + \frac{1}{4}B^I_{ij}B^{ijI} \\
&+ A^I_0 D_i\Pi^{iI} - \frac{1}{2}B^I_{ij}F^I_{ij} - 2D_iB^{ijI}\Pi^{0jI} + 2f^{PKI}\Pi^{lK}A^I_0\Pi^{0lK} \\
&- 2D_l\Pi^{mP}\Pi^{lmP} + f^{PKI}F^K_{lm}A^I_0\Pi^{lmP} \\
-\lambda^I_0\gamma^{0I} - \lambda^I\gamma^I - u^I_i\chi^{iI} - u^I_{0i}\chi^{0iI} - u^I_{ij}\chi^{ijI} - v^I_{ij}\phi^{ijI}). \qquad (31)
\end{aligned}
$$

From the extended action we can identify the extended Hamiltonian given by

$$
H_E = H + \lambda^I_0\gamma^{0I} + \lambda^I\gamma^I, \qquad (32)
$$

where $H$ is given by

$$
\begin{aligned}
H = \frac{1}{2}\Pi^{iI}\Pi^I_i + \frac{1}{4}B^I_{ij}B^{ijI} + A^I_0D_i\Pi^{iI} - \frac{1}{2}B^I_{ij}F^I_{ij} - 2D_iB^{ijI}\Pi^{0jI} \\
+ 2f^{PKI}\Pi^{lK}A^I_0\Pi^{0lK} - 2D_l\Pi^{mP}\Pi^{lmP} + f^{PKI}F^K_{lm}A^I_0\Pi^{lmP}. \qquad (33)
\end{aligned}
$$

We will continue this section by computing the equations of motion obtained from the extended action, which are expressed by

$$
\begin{aligned}
\delta A^P_0 &: \dot{\Pi}^{0P} = D_l\Pi^{lP} + 2f^{JKP}\left(\Pi^{lK}\Pi^{0J} + F^K_{lm}\Pi^{lmJ}\right), \\
\delta\Pi^{0P} &: \dot{A}^P_0 = \lambda^P_0, \qquad (34) \\
\delta A^P_l &: \dot{\Pi}^{lP} = f^{IPK}A^I_0\Pi^{lK} + D_iB^P_{il} - 2f^{KPJ}B^{ljJ}\Pi^{0jK} - 2f^{IPK}\Pi^{jK}\Pi^{ljI} \\
&\quad - 2D_i\left(f^{KPI}A^I_0\Pi^{ilK}\right) + f^{IPK}\Pi^{lK}\lambda^I + 2D_iv^{ilP}, \\
\delta\Pi^{lP} &: \dot{A}^P_l = \Pi^{lP} - D_lA^P_0 - 2f^{KPI}A^I_0\Pi^{0lK} + u^{lP} \\
\delta B^P_{0l} &: \dot{\Pi}^{0lP} = f^{PIK}\Pi^{0lK}\lambda^I - \frac{1}{2}u^{lP} \\
\delta B^P_{lm} &: \dot{\Pi}^{lmP} = 2D_l\Pi^{0mP} + f^{PIK}\Pi^{lmK}\lambda^I - \frac{1}{2}u^P_{lm} \\
\delta\Pi^{0lP} &: \dot{B}^P_{0l} = D_iB^{ilP} - f^{PKI}\Pi^{lK}A^I_0 + \frac{1}{2}u^P_{0l} \\
\delta\Pi^{lmP} &: \dot{B}^P_{lm} = 2D_l\Pi^{mP} - f^{PKI}F^K_{lm}A^I_0 + f^{IJP}\lambda^IB^J_{lm} + u^P_{lm} \\
\delta\lambda^I_0 &: \gamma^{0I} = 0 \\
\delta\lambda^I &: \gamma^I = 0 \\
\delta u^I_i &: \chi^I_i = 0 \\
\delta u^I_{0i} &: \chi^I_{0i} = 0 \\
\delta u^I_{ij} &: \chi^I_{ij} = 0
\end{aligned}
$$

$$\delta v_i^I : \phi_i^I = 0$$

By following with our analysis, we need to know the gauge transformations on the phase space of the theory under study. For this step, we shall use Castellani's formalism which allow us to define the following gauge generator in terms of the first class constraints (29)

$$G = \int_{\Sigma} \left[ D_0 \epsilon_0^I \gamma^{0I} + \epsilon^I \gamma^I \right] d^3x, \tag{35}$$

thus, we find that the gauge transformations on the phase space are given by

$$\begin{aligned}
\delta_0 A_0^P &= D_0 \epsilon_0^P, \\
\delta_0 A_i^P &= -D_i \epsilon^P, \\
\delta_0 \Pi^{0P} &= -f^{PKI} \epsilon_0^K \Pi^{0I}, \\
\delta_0 \Pi^{iP} &= f^{PIK} \Pi^{iK} \epsilon^I, \\
\delta_0 B_{0i}^P &= f^{PIJ} \epsilon^I B_{0i}^J, \\
\delta_0 \Pi^{0i} &= f^{PIK} \epsilon^I \Pi^{0iK}, \\
\delta_0 B_{ij}^P &= f^{PIJ} \epsilon^I B_{ij}^J, \\
\delta_0 \Pi^{ijP} &= f^{PIK} \epsilon^I \Pi^{ijK}.
\end{aligned} \tag{36}$$

We can observe that by redefining the gauge parameters $\epsilon_0^I = \epsilon^I$, the gauge transformations take the form

$$\begin{aligned}
A_\mu'^I &\to A_\mu^I - D_\mu \epsilon^I, \\
B_{\mu\nu}'^I &\to B_{\mu\nu}^I - f^I{}_{JK} \epsilon^J B_{\mu\nu}^K,
\end{aligned} \tag{37}$$

where the first one transformation corresponds to the usual gauge transformations for [YM] theory, and the later one by using the equations of motion gives us the transformation of a valued compact Lie algebra curvature tensor field. In order to obtain the path integral quantization of the theory and its *uv* behaviour, it is straightforward to perform Senjanovic's method [29] by taking into account the full constraint algebra obtained above to define the corresponding non-abelian measure. After some integration over the second class constraints, and by using the first class constraints to identify an appropriate gauge fixing [30], one finally gets the usual quantum effective action of the [YM] theory.

## 3. Martellini's Model

An interesting alternative model to express [YM] theory as a constrained $BF$-like theory has been reported by M. Martellini and M. Zeni [10]. Martellini's model is a deformation of a topological field theory, namely the pure $BF$ theory resulting in the first order formulation of [YM] theory. In this formulation, new non local observables can be introduced following the topological theory and giving an explicit realization of t'Hooft algebra, recovering at the end the standard $u$-$v$ behaviour of the theory [12]. So, the aim of this section is to perform the canonical analysis for Martellini's model on the full phase space context, which is absent in the literature, then we compare the results obtained with those found in former sections.

Let us start with the action proposed by Martellini et al [10]

$$S[A,B]=\int\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}B_{I\mu\nu}F^{I}{}_{\alpha\beta}+g^{2}\int B^{I}{}_{\mu\nu}B_{I}{}^{\mu\nu}. \tag{38}$$

The firs term in the r.h.s. of (38) is the usual $BF$ theory laking of local degrees of freedom, and has been analyzed within a smaller phase space context in [31], and by using a pure Dirac's analysis in [22]. As we shall see below, local degrees of freedom are restored by the $g^2B^{I}{}_{\mu\nu}B_{I}{}^{\mu\nu}$ term of (38), allowing an explicit breaking of the topological sector as long as $g\neq 0$. Therefore, in Martellini's formulation [YM] theory is expressed as a deformation of the topological $BF$ field theory.

We are able to observe that the actions (2) and (38) differ in the first term. In (2) neither is present the imaginary number that provides the euclidean feature nor the space time indices are contracted with the epsilon tensor. However, because the physical relation among [YM] and the action (38), in this section we are interested in develop a complete Hamiltonian framework of the action (38) because is absent in the literature.

By performing the $3+1$ decomposition of the action (38) we obtain

$$S[A,B]=\int\int_{\Sigma}dtd^{3}xg^{2}\left(2B^{I}_{0i}B^{0iI}+B^{I}_{ij}B^{ijI}\right)+i\eta^{ijk}\left(B^{I}_{0i}F^{I}_{jk}+B^{I}_{ij}F^{I}_{0k}\right), \tag{39}$$

hence, by following the procedure developed in above section, we find the following results; there are the following $2(N^2-1)$ first class constraints

$$\begin{aligned}\gamma^{0I}&=\Pi^{0I}\approx 0,\\ \gamma^{I}&=D_i\Pi^{iI}+2f^{IJK}B^{J}_{0i}\Pi^{0iK}+f^{IJK}B^{J}_{ij}\Pi^{ijK}\approx 0,\end{aligned} \tag{40}$$

and $12(N^2-1)$ second class constraints

$$\begin{aligned} \phi^{iI} &= \Pi^{iI} - i\eta^{ijk}B^I_{jk} \approx 0, \\ \phi^{0iI} &= \Pi^{0iI} \approx 0, \\ \phi^{ijI} &= \Pi^{ijI} \approx 0, \\ \psi^{0iI} &= 2g^2 B^{0iI} + \frac{i}{2}\eta^{ijk}F^I_{jk} \approx 0. \end{aligned} \tag{41}$$

Therefore, the counting of degrees of freedom is performed as follows. There are $20(N^2-1)$ phase space variables, $2(N^2-1)$ independent first class constraints and $12(N^2-1)$ second class constraints, thus the theory given in (38) has $2(N^2-1)$ degrees of freedom.
Now, we observe that the algebra of the constraints is given by

$$\begin{aligned}
\{\gamma^{0P}(x), \gamma^{0I}(y)\} &= 0, \quad \{\phi^{lP}(x), \gamma^I(y)\} = f^{PIK}\phi^{lK} = 0, \\
\{\gamma^{0P}(x), \phi^{iI}(y)\} &= 0, \quad \{\phi^{0lP}(x), \gamma^I(y)\} = f^{PIK}\phi^{0lP} = 0, \\
\{\gamma^{0P}(x), \phi^{0iI}(y)\} &= 0, \quad \{\phi^{lmP}(x), \gamma^I(y)\} = f^{PIK}\phi^{lmK} = 0, \\
\{\gamma^{0P}(x), \phi^{ijI}(y)\} &= 0, \quad \{\psi^{0lP}(x), \gamma^I(y)\} = f^{PIK}\psi^{0lK} = 0, \\
\{\gamma^{0P}(x), \psi^{0iI}(y)\} &= 0, \quad \{\gamma^{P}(x), \gamma^I(y)\} = f^{PIK}\gamma^{K} = 0, \\
\{\gamma^{0P}(x), \gamma^{I}(y)\} &= 0, \quad \{\phi^{lP}(x), \phi^{iI}(y)\} = 0, \\
\{\phi^{lP}(x), \phi^{0iI}(y)\} &= 0, \quad \{\phi^{lmP}(x), \phi^{ijI}(y)\} = 0 \\
\{\phi^{lP}(x), \phi^{ijI}(y)\} &= -i\eta^{lij}\delta^{PI}\delta^3(x-y), \quad \{\phi^{0lP}(x), \psi^{0iI}(y)\} \\
&\qquad = -g^2\delta^l_i\delta^{PI}\delta^3(x-y), \\
\{\phi^{lP}(x), \psi^{0iI}(y)\} &= i\eta^{ijl}(\delta^{PI}\partial_j + f^{PIK}A^K_j)\delta^3(x-y), \ \{\phi^{0lP}(x), \phi^{ijI}(y)\} = 0, \\
\{\phi^{0lP}(x), \phi^{0iI}(y)\} &= 0, \quad \{\psi^{lmP}(x), \psi^{ijI}(y)\} = 0,
\end{aligned}$$

where we can appreciate that the constraints form a set of first and second class constraints as is expected.

On the other hand, the identification of the constraints will allow us to identify the extended action. By using those results, we find the extended action given by

$$\begin{aligned} S_E[A^I_{mu}, \Pi^{\mu I}, B^I_{\mu\nu}, \Pi^{\mu\nu I}, \lambda^I_0, \lambda^I, u^I_i, u^I_{0i}, u^I_{ij}, v^I_{0i}] &= \int d^4x (\dot{A}^I_\mu \Pi^{\mu I} \\ + \dot{B}^I_{\mu\nu}\Pi^{\mu\nu I} - \frac{1}{2}\Pi^{iI}\Pi^I_i + g^2 2B^I_{0i}B^{0iI} \\ + A^I_0 D_i \Pi^{iI} + i\eta^{ijk}B^I_{0i}F^I_{jk} - \frac{1}{2g^2}\eta^{ijk}f^{PIJ}A^I_0 F^J_{jk}\Pi^{0iP} \end{aligned}$$

$$
\begin{aligned}
&+\frac{i}{2}\eta^{ijk}D_j\Pi^{kP}\Pi^{0iP}-2D_iB^I_{0j}\Pi^{ijI}\\
&-\frac{i}{2}\eta_{ijk}f^{PIK}A^I_0\Pi^{kK}\Pi^{ijP}-\lambda^I_0\gamma^{0I}-\lambda^I\gamma^I\\
&-u^I_i\phi^{iI}-u^I_{0i}\phi^{0iI}-u^I_{ij}\phi^{ijI}-v^I_{0i}\psi^{0iI}).
\end{aligned}
\tag{42}
$$

From the extended action we can identify the extended Hamiltonian given by

$$
H_E = H + \lambda^I_0\gamma^{0I} + \lambda^I\gamma^I, \tag{43}
$$

where $H$ has the following form

$$
\begin{aligned}
H =& \frac{1}{2}\Pi^{iI}\Pi^I_i - 2g^2B^I_{0i}B^{0iI} - A^I_0D_i\Pi^{iI} - i\eta^{ijk}B^I_{0i}F^I_{jk}\\
&+\frac{1}{2g^2}\eta^{ijk}f^{PIJ}A^I_0F^J_{jk}\Pi^{0iP} - \frac{i}{2}\eta^{ijk}D_j\Pi^{kP}\Pi^{0iP}\\
&+2D_iB^I_{0j}\Pi^{ijI} + \frac{i}{2}\eta_{ijk}f^{PIK}A^I_0\Pi^{kK}\Pi^{ijP}.
\end{aligned}
\tag{44}
$$

Hence, the following question rise; Are there differences among the action (1) and Martellini's propose?. The difference lies in the constraint algebra, in fact, we observe the algebra among the second class constraints for action (1) and Martellini's is different. Furthermore, in Martellini's theory the algebra among the constraints is defined over the complex numbers, consequence of a Wick rotation, and the definition of the momenta gives dual expressions of the constraints defined for the action (1). In particular note that in Martellini's model, $B$ is proportional to the field strength and satisfies the Bianchi identities on-shell. This is no longer true off-shell and this fact has been related to the presence of monopole charges in the vacuum [12] which should enter in the non perturbative sector of the theory. Moreover the action (38) has been used to define new non local observables related to the phase space structure of the theory [32]. On the other side, it is mandatory to investigate the quantum behavior of the action (1) at perturbative level for finding new local observables, and thus, compare with Martellini's model possibles advantages of the action (1); we remark that the action (1) and Martellini's model have different algebra among the second class constraints, and this fact will be important in the quantum treatment for instance, in the construction of Dirac's brakets. In this respect, the present letter has the necessary tools for studying these subjects in forthcoming works.

## 4. Conclusions and Prospects

In this paper, we have developed a consistent application of a pure Dirac's method for constrained systems. By working with the original phase space we performed a complete Hamiltonian dynamics for two $BF$-like theories. The first one, was related with [YM] theory, and the second action was associated with Martellini's model, which has been used in recently works for studying the non perturbative character of the QCD confinement. From the present analysis, we calculated for the theories under study, the extended action, the extended Hamiltonian and the full constraints program, which is considerably enlarged in comparison with the analysis performed on the reduce phase space. The correct identification of the constraints as first and second class, enabled us to carry out the counting of degrees of freedom, concluding that classically, the theories under study have the same number of degrees of freedom of [YM] theory. The full phase space framework, allowed us observe that the physical degrees of freedom emerge from the coupling of topological theories. The topological invariance is broken because there are not in the full action reducibility conditions among the constraints, which endow the theory with local dynamics. The nature of such conditions are closely related to the full phase space, and cannot be obtained from the reduce one. With regard to the quantum aspect, the application of the pure Dirac's procedure provides the full structure of the constraints, and this fact give us a complete gauge information of the theory. It is worth mentioning that once the full set of constraints is calculated, our procedure could shed light on the search of observables in the context of covariant field theories specifically in the case of strong-Dirac observables, which must be defined in the complete phase space. Finally, we observed that the action (1) and Martellini's model yield [YM] equations of motion, however, the algebra of their constraints is different, thus, we expect different quantum scenarios for these theories, all those ideas are in progress and will be reported in forthcoming works.

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