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EFFECT OF PRELIMINARY UNIT ROOT TESTS ON PREDICTOR OF $AR(p)$ MODEL

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Abstract: This paper presents a method to estimate the predictor and the scaled prediction mean squares error of an AR(p) model after preliminary unit root tests by using Augmented Dickey-Fuller, Phillips Perron, KPSS and DF-GLS unit root tests. Monte Carlo simulation results are given to compare the relative efficiencies of one-step-ahead prediction using the scaled prediction mean squares error for an AR(2) model with a linear trend. All preliminary unit root tests considered here perform well to improve the predictors from trending AR(2) process when the root near unit root. Moreover, the preliminary unit root tests of KPSS and DF-GLS are slightly superior to other unit root tests.

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Key Words: preliminary unit root tests, scaled prediction mean square error, AR(p) Model

1. Introduction

Prediction is an important task in forecasting time series data. Many economic

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and financial time series data exhibit trending behavior of non-stationary in the mean. For examples, they are asset prices, exchange rates, stock price and the levels of macroeconomic aggregates like real GDP see e.g. Nelson and Plosser [1], Hamilton [2] and Deibold and Kilian [3]. Nelson and Plosser [1] examined whether macroeconomic time series are better characterized as stationary fluctuations around a deterministic trend or as non-stationary processes that have no tendency to return to a deterministic path. They showed that most of macroeconomic time series data have unit root. Also, Hamilton [2] gave us an idea about the need to use the unit root test to find the correct model for the series of the nominal interest rate of the United States from 1947-1989 and the real GNP for the United States from 1947-1989, see Figures 17.2-17.3 of Hamilton [2]. He also suggested that there is no guarantee in economic theory telling that the nominal interest rate series should be a deterministic time trend model, although Figure 17.2 shows an upward trend over the sample data. The fit model for these data might be a random walk without trend or a stationary process model with a constant term. Therefore the statistical predictive inference, in particular estimation of predictor and prediction interval, of macroeconomic time series data set is doubtful to apply to this data set. Diebold and Kilian [3] showed the usefulness of the unit root tests as diagnostic tools for selecting forecasting model. They constructed a data generating process consisting of the canonical AR(1) process with a linear trend. They compared a predictor using prediction mean square error (PMSE) based on the ordinary least square estimator(OLS) of the autoregressive parameter with a predictor based on the OLS estimator after Dickey-Fuller unit root test proposed by Dickey and Fuller [4]. When time series data have an autoregressive parameter near one, Diebold and Kilian [3] found that the predictor of this series should be computed from the random walk model otherwise the stationary model. This method will help to reduce the mean squares error compared to the usual method. In addition, these authors suggested that more powerful unit root test than those of Dickey-Fuller unit root tests might help to improve the forecast from a trending AR(1) model. The relative efficiency of predictors for an unknown mean AR(1) process after Dickey-Fuller unit root tests using the scaled prediction mean square error was studied by Niwitpong [5] [6]. He showed that the preliminary weighted symmetric unit roots, which are more powerful tests than Dickey-Fuller unit root tests, performs better than other preliminary unit root tests for improving the forecast from an AR(1) model. The analysis in this chapter differs from the work of Diebold and Kilian [3] in using other more powerful unit root tests as preliminary unit root tests and adding a trend component in AR(1) which also differs from the work of Niwitpong [6]. We extend the work of Chiangpradit and Niwitpong [7] to AR(p) model. The purpose of this chapter is therefore 1) to develop the scaled PMSE after preliminary unit root tests and compared to the standard PMSE for an AR(p) model 2) to compare the one-step-ahead predictor of an AR(p) process with a linear trend, using the scaled PMSE after preliminary unit root tests, with the one-step-ahead predictor, using the scaled PMSE that does not use a preliminary unit root test. This chapter is organized as follows: Section 2 presents an AR(p) process with a linear trend and unit root tests. The scaled prediction mean square error after unit root tests is discussed in Section 3. Monte Carlo simulation estimation scaled PMSE is presented in Section 4. The empirical application is showed in Section 5. Finally section is conclusion.

2. An $AR(p)$ Process and Unit Root Tests

Suppose an autoregressive process of order p, AR(p), with a linear trend $\{Y_t\}$ satisfies

$$Y_t = \alpha + \delta t + \rho_1 Y_{t-1} + \rho_2 Y_{t-2} + \ldots + \rho_p Y_{t-p} + e_t \tag{1}$$

where $\rho_1, \rho_2, ..., \rho_p$ are autoregressive parameters and the $e_t's$ are independent and identically $N(0, \sigma^2)$ distributed, t = 1, 2, 3,..., T. To resolve the uncertainty we use a preliminary unit root test to choose between a trend stationary process and a random walk process. The unit root tests used in this paper are the Augmented Dickey-Fuller (ADF) test proposed by Said and Dickey [11], Phillips-Perron (PP) test proposed by Perron [8], Kwiatkowski-Phillips- Schmidt-Shin (KPSS) test proposed by Kwiatkowski, et al. [9] and Elliott-Rothenberg-Stock (DF-GLS) test proposed by Elliott et al. [10].

Firstly, The Augmented Dickey Fuller test are proposed by Said and Dickey [11]. An autoregressive process of order p shown in (1) can be written as

$$\Delta Y_t = \alpha^* + \delta^* t + \rho_1^* Y_{t-1} + \rho_2^* \Delta Y_{t-1} + ... + \rho_p^* \Delta Y_{t-p+1} + e_t, \tag{2}$$

where $\Delta Y_t = Y_t - Y_{t-1}$. In this formulation, we consider AR(p) model as the baseline model. The hypothesis testing case 4 of [2], pp 529 are considered. The null hypothesis H_0 and the alternative hypothesis H_a are as follows,

$$H_0 : \rho_1^* = 0$$

$$H_1 : \rho_1^* < 0.$$

Said and Dickey [11] proposed unit root tests based on the ordinary least squares estimator for the null hypothesis H_0. The test statistic is

$$\hat{\tau} = (\hat{\rho_1^*})/SE(\hat{\rho_1^*}),$$

where SE $(\hat{\rho_1^*})$ is the estimated standard error of $\hat{\rho_1^*}$. The quantiles of test statistic $\hat{\tau}$, estimated by simulation, for $H_0 : \rho_1^* = 0$ are provided in Table B.7 of [2].

Secondly,Phillips and Perron [8] proposed the nonparametric test statistics for the unit root null by using consistent estimate of variances as follows

$$PP = T(\hat{\rho_1^*}) - \frac{T^6}{24D_x}(s^2 - s_e^2)$$

where $D_x = det(X^{'}X)$ and the regressors are

$$X = (1, t, Y_{t-1}, \Delta Y_{t-1}, \ldots, \Delta Y_{t-p+1}).$$

The quantiles of PP test statistic, estimated by simulation, for $H_0 : \rho_1^* = 0$ are provided in Table B.5 of [2].

Thirdly, Kwiatkowski, et al.[9] proposed an test for testing trend (the KPSS test). That is, now the null hypothesis is a stationary process and the alternative hypothesis is a unit root. Hence, if we then reject the null hypothesis, we can believe that the series has a unit root. The test statistic is

$$KPSS = \frac{\sum_{t=1}^{T} S_t^2}{\hat{\sigma}^2}$$

where S_t is the partial sum of e_t defined by

$$S_t = \sum_{t=1}^{T} e_t^2, \text{ t = 1, 2, \ldots, T.}$$

The quantiles of the KPSS test statistic, estimated by simulation, for H_0 : *stationary* are provided in Table 4.4 of [12].

Finally, Elliott, et al.[10] proposed DF–GLS test, which applies a generalized least squares(GLS) estimator. The test statistic for $H_0 : \rho_1^* = 0$ is

$$DF - GLS = \frac{S(a=\bar{a}) - \bar{a}S(a=1)}{\hat{w}^2}.$$

Where $S(a = \bar{a})$ and $S(a = 1)$ are the sums of squared error from a least square regression on Y_a and Z_a with

$$\begin{aligned} Y_a &= (Y_1, Y_2 - aY_1, \ldots, Y_T - aY_{T-1}), \\ Z_a &= (z_1, z_2 - az_1, \ldots, z_T - az_{T-1}). \end{aligned}$$

Y_a is a T dimensional column vector and $z_t = (1, t)'$ the estimator for the variance of the error process e_t is

$$\hat{w}^2 = \frac{\hat{\sigma}_e^2}{(1 - \sum \hat{\alpha}_i)^2}.$$

Where $\hat{\sigma}_e^2$ and $\hat{\alpha}_i$ for i = 1, ..., p are taken form the auxiliary ordinary least square(OLS) regression

$$\Delta Y_t = \alpha_0 + \alpha_1 \Delta Y_{t-1} + \ldots + \alpha_p \Delta Y_{t-p} + e_t.$$

And the scalar $\bar{a}$ is set to $\bar{a} = 1 + \bar{c}/T$, where $\bar{c}$ denotes a constant. [10] suggested that $\bar{c}$ should be set to -13.5 in the case of a linear trend. The quantiles of the DF-GLS test statistic, estimated by simulation, for $H_0 : \rho_1^* = 0$ are provided in Table I of [10].

3. The Scaled Prediction Mean Square Error after Unit Root Tests

In this section, we review the scaled PMSE and the scaled PMSE after unit root tests. Our aim for model (2), whether the hypothesis H_1 is satisfied or not, is to predict Y_{T+1}, based on data $Y_1, Y_2, \ldots, Y_T$. If H_0 is satisfied then $\Delta Y_t = \alpha^* + \delta^* t + \rho_2^* \Delta Y_{t-1} + \ldots + \rho_p^* \Delta Y_{t-p+1} + e_t$ and the predictor of Y_{T+1} , based on data $Y_1, Y_2, \ldots, Y_T$ is $\hat{\alpha}^* + \hat{\delta}^*(T+1) + Y_T + \hat{\rho}_2^* \Delta Y_{T-1} + \ldots + \hat{\rho}_p^* \Delta Y_{T-p+1}$. If H_a is satisfied then $\Delta Y_t = \alpha^* + \delta^* t + \rho_1^* Y_{t-1} + \rho_2^* \Delta Y_{t-2} + \ldots + \rho_p^* \Delta Y_{t-p+1} + e_t$ and the predictor of Y_{T+1}, based on data $Y_1, Y_2, \ldots, Y_T$ is $\hat{\alpha}^* + \hat{\delta}^*(T+1) + (1 + \hat{\rho}_1^* Y_T + \hat{\rho}_2^* \Delta Y_{T-1} + \ldots + \hat{\rho}_p^* \Delta Y_{T-p+1}$. For the predictor $\hat{\alpha}^* + \hat{\delta}^*(T+1) + \hat{\rho}_1^* Y_T + \hat{\rho}_2^* \Delta Y_{T-1} + \ldots + \hat{\rho}_p^* \Delta Y_{T-p+1}$ of Y_{T+1}, the error is

$$\begin{aligned}\gamma_0(Y_1, Y_2, \ldots, Y_T, Y_{T+1}) &= e_{T+1} + (\alpha^* - \hat{\alpha}^*) + (\delta^* - \hat{\delta}^*)(T+1) \\ &+\rho_1^* Y_T + (\rho_2^* - \hat{\rho}_2^*)\Delta Y_{T-1} + \ldots + (\rho_p^* - \hat{\rho}_p^*)\Delta Y_{T-p+1}.\end{aligned}$$

For the predictor $\hat{\alpha}^* + \hat{\delta}^*(T+1) + \hat{\rho}_1^* Y_T + \hat{\rho}_2^* \Delta Y_{T-1} + \ldots + \hat{\rho}_p^* \Delta Y_{T-p+1}$, the error is

$$\begin{aligned}\gamma_1(Y_1, Y_2, \ldots, Y_T, Y_{T+1}) &= e_{T+1} + (\alpha^* - \hat{\alpha}^*) + (\delta^* - \hat{\delta}^*)(T+1) \\ &+(\rho_1^* - \hat{\rho}_1^*)Y_T + (\rho_2^* - \hat{\rho}_2^*)\Delta Y_{T-1} + \ldots + (\rho_p^* - \hat{\rho}_p^*)\Delta Y_{T-p+1}.\end{aligned}$$

Now we consider a unit root test based on the statistics $\hat{\tau}$, PP, KPSS, DF-GLS. We may write the sample space $\Omega = A_1 \cup A_2$ when A_1 and A_2 are the events that the hypothesis H_0 is accepted and rejected respectively.

Let

$$\begin{aligned}\phi(Y_1, Y_2, \ldots, Y_T, Y_{T+1}) &= \gamma_0(Y_1, Y_2, \ldots, Y_T, Y_{T+1}); if \omega \epsilon A_1, \\ \phi(Y_1, Y_2, \ldots, Y_T, Y_{T+1}) &= \gamma_1(Y_1, Y_2, \ldots, Y_T, Y_{T+1}); if \omega \epsilon A_2.\end{aligned}$$

Let $PMSE_p$ denote the PMSE after unit root test. In other words,

$$PMSE_p = E(\phi(Y_1, Y_2, \ldots, Y_T, Y_{T+1}))^2.$$

The scaled $PMSE_p$ is defined to be $PMSE_p/\sigma^2$.

In other words, the scaled $PMSE_p$ is

$$\begin{aligned}\frac{PMSE_p}{\sigma^2} &= E(\frac{1}{\sigma^2}\phi(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2) \\ &= E(\tfrac{1}{\sigma^2}\phi(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2)(I(A_1, \omega) + I(A_2, \omega)) \\ &= E(\tfrac{1}{\sigma^2}\gamma_0(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2)(I(A_1, \omega)) \\ &+ E(\tfrac{1}{\sigma^2}\gamma_1(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2)(I(A_2, \omega)),\end{aligned}$$

where $I(A_1, \omega) = 1 : if \Lambda_i < c_i$ and 0 otherwise and $I(A_2, \omega) = 1 : if \Lambda_i \geq c_i$ and 0 otherwise where $\Lambda_i, i = 1, 2, 3, 4$ are respectively, the statistics $\hat{\tau}$, PP, KPSS, DF-GLS and $c_i, i = 1, 2, 3, 4$ are the corresponding critical values in [2]. We may show that

$$\begin{aligned}&\frac{1}{\sigma^2}\gamma_0(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2 \\ &\quad = (\frac{e_{T+1}}{\sigma} + \frac{(\alpha^* - \hat{\alpha}^*)}{\sigma} + \frac{(\delta^* - \hat{\delta}^*)(T+1)}{\sigma} + \frac{\rho_1^* Y_T}{\sigma} + \frac{(\rho_2^* - \hat{\rho}_2^*)\Delta Y_{T-1}}{\sigma} \\ &\qquad + \ldots + \frac{(\rho_p^* - \hat{\rho}_p^*)\Delta Y_{T-p+1}}{\sigma})^2 \\ &= (\eta_{T+1} + \lambda + \nu_{T+1} + \rho_1^* X_T + (\rho_2^* - \hat{\rho}_2^*)\Delta X_{T-1} + \ldots + (\rho_p^* - \hat{\rho}_p^*)\Delta X_{T-p+1})^2,\end{aligned}$$

and

$$\begin{aligned}&\frac{1}{\sigma^2}\gamma_1(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2 \\ &\quad = (\frac{e_{T+1}}{\sigma} + \frac{(\alpha^* - \hat{\alpha^*})}{\sigma} + \frac{(\delta^* - \hat{\delta}^*)(T+1)}{\sigma} + \frac{(\rho_1^* - \hat{\rho_1^*})Y_T}{\sigma} + \frac{(\rho_2^* - \hat{\rho}_2^*)\Delta Y_{T-1}}{\sigma} \\ &\qquad + \ldots + \frac{(\rho_p^* - \hat{\rho}_p^*)\Delta Y_{T-p+1}}{\sigma})^2 \\ &= (\eta_{T+1} + \lambda + \nu_{T+1} + (\rho_1^* - \hat{\rho_1}^*) X_T + (\rho_2^* - \hat{\rho}_2^*)\Delta X_{T-1} + \ldots + (\rho_p^* - \hat{\rho}_p^*)\Delta X_{T-p+1})^2,\end{aligned}$$

which are functions of $(X_1, X_2, \ldots, X_T, \eta_{T+1}, \nu_{T+1}$ and $\rho_1^*, \rho_2^*, \ldots, \rho_p^*)$ when we defined $X_T = Y_T/\sigma$, $\lambda = \frac{(\alpha^* - \hat{\alpha}^*)}{\sigma}$, $\eta_T = \frac{e_T}{\sigma}$ and $\nu_T = \frac{\delta^* - \hat{\delta}^*}{\sigma}T$.

Therefore

$$\frac{1}{\sigma^2}\gamma_0(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2(I(A_1, \omega))$$

and

$$\frac{1}{\sigma^2}\gamma_1(Y_1, Y_2, \ldots, Y_T, Y_{T+1})^2)(I(A_2, \omega)$$

are functions of

$$(X_1, X_2, \ldots, X_T, \eta_{T+1}, \nu_{T+1})$$

and

$$\rho_1^*, \rho_2^*, \ldots, \rho_p^*.$$

4. Monte Carlo Simulation Estimation Scaled PMSE

In this section, The scaled PMSE of a one-step-ahead predictor based on OLS estimation and the scaled $PMSE_p$ based on after preliminary unit roots test are computed using Monte Carlo simulation. Suppose that each Monte Carlo simulation consists of M independent runs. Let the observed value of X_t,λ,ν_t and $\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_p$ be denoted by $X_T^{(k)}$,$\lambda^{(k)}$,$\nu_{T+1}^{(k)}$ and $\hat{\rho}_1^{*(k)}, \hat{\rho}_2^{*(k)}, \ldots, \hat{\rho}_p^{*(k)}$. We estimate the scaled PMSE by

$$1 + \frac{1}{M}\sum_{k=1}^{M}\sum_{i=1}^{p}(\lambda^{(k)} + \nu_{T+1}^{(k)} + (\rho_i^{*(k)} - \hat{\rho}_i^{*(k)})X_{T-i+1}^{(k)})^2$$

From the previous section, the scaled $PMSE_p$ is estimated by

$$1 + \sum_{k\epsilon M_0}\sum_{i=2}^{p}(\lambda^{(k)} + \nu_{T+1}^{(k)} + \rho_1^{*(k)} + X_T^{(k)} + (\rho_i^{*(k)} - \hat{\rho}_i^{*(k)})\Delta X_{T-i+1}^{(k)})\frac{1}{M}$$
$$+ \sum_{k\epsilon M_1}\sum_{i=1}^{p}(\lambda^{(k)} + \nu_{T+1}^{(k)} + (\rho_i^{*(k)} - \hat{\rho}_i^{*(k)})\Delta X_{T-i+1}^{(k)})\frac{1}{M}$$

where M_0 is the set of simulation runs for which H_0 fails to reject and M_1 is the set of simulation runs for which H_0 is rejected. The relative efficiency of the predictor based on the estimators using the scaled $PMSE$ compared to the predictor based on the estimators using the scaled $PMSE_p$ is defined to be

$$\frac{scaledPMSE}{scaledPMSE_p}$$

We used R program ([14],[15]) to generate the data from an autoregressive process of order 2, AR(2) with a linear trend in model (1). We set the autoregressive parameters by illustrating two examples. First, consider the AR(2) model $(Y_t - \mu) = 1.2(Y_{t-1} - \mu) - 0.2(Y_{t-2} - \mu) + e_t$. In this case the ρ_1 is 1.2, not 1 and yet the mean cancels out on both sides which is characteristic of unit root process is one of the reasons for interest in unit roots. The term "unit root" refers to the roots of the backshift operator, not the coefficients. That is, we express the model as$(1-1.2B+0.2B^2)(Y_t-\mu)=e_t$ where $B(Y_t)$ is Y_{t-1}, and we then treat the polynomial $(1-1.2B+0.2B^2)=(1-0.2B)(1-B)$, as an algebraic polynomial and find its roots which are B=5 and B=1. The B=1 is a unit root. Note also that $(1-0.2B)(1-B)Y_t$ is $(1-0.2B)(Y_t-Y_{t-1})$ which is why differences are taken when unit roots are encountered. Second, consider $(Y_t-\mu)=1(Y_{t-1}-\mu)-0.16(Y_{t-2}-\mu)+e_t$. In this case the is ρ_1 is 1, but this is not a unit root process. Therefore we set the values of the AR(2) process parameters are ρ_1 = -1.109, -1.127, -1.145, -1.200, -1.320, -1.490 and ρ_2 = 0.01. When ρ_1 = -1.109 and ρ_2 = 0.01 we find its roots which are B=10 and B=0.99. When ρ_1 = -1.127 and ρ_2 = 0.01 we find its roots which are B=10 and B=0.97. When ρ_1 = -1.145 and ρ_2 = 0.01 we find its roots which are B=10 and B=0.95. When ρ_1 = -1.200 and ρ_2 = 0.01 we find its roots which are B=11 and B=0.90. When ρ_1 = -1.320 and ρ_2 = 0.01 we find its roots which are B=12 and B=0.80. When ρ_1 = -1.490 and ρ_2 = 0.01 we find its roots which are B=14 and B=0.70. The parameters of a linear trend are α = 10 and δ = 0.1. The random variables e_t are generated from normal distribution with mean zero and variance one. The sample sizes in these simulations are equal to 25, 50, 100 and 250 respectively. The number of simulation M = 1000 and a significance level of 0.05. The relative efficiency of the predictor based on the estimators using the scaled PMSE compared to the predictor based on the estimators using the scaled PMSEp are reported in Table 1. From this table, we can see that the efficiency of the estimators using the scaled PMSEp when the root near unit root process. and all T investigated for all preliminary unit root tests considered here. Table 1 also shows that all preliminary unit root tests considered here perform well to improve the predictors from a trending AR(2) process when the root near 1. In addition, the preliminary unit root tests of KPSS and DF-GLS are slightly superior to other unit root tests.

5. An Empirical Application

To illustrate the application of the estimations that have been presented in this paper, we have used real economic time series data, the 54 advertising expenditures of the Lydia Pinkham data. The unit root tests compared in this example are Augmented Dickey-Fuller, Phillips Perron, KPSS and DF-GLS unit root test. All unit root tests show that the autoregressive parameter should be one. We calculate a one-step-ahead predictor of AR(1) process with a linear

T	ρ_1	ρ_2	ADF	PP	KPSS	DF-GLS
25	-1.109	0.01	1.1061	1.0983	1.1089	1.1098
25	-1.127	0.01	1.0873	1.0679	1.0892	1.0913
25	-1.145	0.01	1.0792	1.0689	1.0807	1.0891
25	-1.200	0.01	1.0219	1.0248	1.0389	1.0471
25	-1.320	0.01	0.9895	0.9984	0.9951	0.9991
25	-1.490	0.01	0.9241	0.9835	0.9857	0.9724
50	-1.109	0.01	1.0744	1.0702	1.0786	1.0793
50	-1.127	0.01	1.0417	1.0424	1.0506	1.0564
50	-1.145	0.01	1.0256	1.0324	1.0368	1.0436
50	-1.200	0.01	0.9361	0.9353	0.9424	0.9457
50	-1.320	0.01	0.9325	0.9278	0.9465	0.9490
50	-1.490	0.01	0.9713	0.9724	0.9799	0.9805
100	-1.109	0.01	1.0458	1.0453	1.0502	1.0555
100	-1.127	0.01	1.0275	1.0220	1.0255	1.0259
100	-1.145	0.01	0.9910	0.9904	0.9909	0.9955
100	-1.200	0.01	0.9246	0.9218	0.9345	0.9398
100	-1.320	0.01	0.9022	0.9017	0.9113	0.9237
100	-1.490	0.01	0.9903	0.9901	0.9912	0.9930
250	-1.109	0.01	1.0141	1.0099	1.0277	1.0322
250	-1.127	0.01	0.9946	0.9914	0.9944	0.9965
250	-1.145	0.01	0.9982	0.9955	1.0000	1.0000
250	-1.200	0.01	1.0000	1.0000	1.0000	1.0000
250	-1.320	0.01	1.0000	1.0000	1.0000	1.0000
250	-1.490	0.01	1.0000	1.0000	1.0000	1.0000

Table 1: The relative efficiency of the predictor based on the estimators using the scaled $PMSE$ compared to the predictor based on the estimators using the scaled $PMSE_p$.

trend, using the scaled PMSE after preliminary unit root tests, equal to 0.064 and with a one-step-ahead predictor, using the scaled PMSE that does not use a preliminary unit root test, equal to 0.093. This result shows that the pretesting predictor is better than the usual predictor.

6. Conclusions and Discussion

In the time series data, the hypothesis of a unit root test has attracted in recent work on economic [1]. We have proposed one-step-ahead predictor of AR(p) model with a linear trend after preliminary unit root tests. The results show that the pretesting is favorable in small sizes. All preliminary unit root tests considered here perform well to improve the predictors from a trending AR(p) process when the root near unit root process. In addition, the preliminary unit root test of KPSS and DF-GLS are slightly preferable to other unit root tests.

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References

[1] C.R Nelson, C.I. Plosser, Trends and random walks in macroeconomic time series, *Journal of Monetary Economics*, **10** (1982), 139-162.

[2] J.D. Hamilton, *Time Series Analysis*, Princeton University Press, New Jersey (1994).

[3] F.X. Diebold, L. Kilian, Unit root tests are useful for selecting forcasting models, *Journal of Business & Economic Statistics*, **18** (2000), 265-273.

[4] D.A. Dickey, W.A. Fuller, Distribution of estimators for autoregressive time series with a unit root, *Journal of the American Statistical Association*, **74** (1979), 427-431.

[5] S. Niwitpong, *Predictive Inference for Times Series*, PhD Thesis, La Trobe University, Australia (2007).

[6] S. Niwitpong, Effect of preliminary unit root tests on predictors for an unknown mean Guassian AR(1) process, *Thailand Statistician*, **7**, No. 1 (2009), 71-79.

[7] M. Chiangpradit, S. Niwitpong, Predictor of AR(1) model with a linear trend after preliminary unit root tests, *Applied Mathematical Sciences*, **5**, No. 67 (2011), 3347-3356.

[8] P.C.B. Phillips, P. Perron, Testing for a unit root in time series regression, *Biometrika*, **75** (1988), 335-346.

[9] Kwiatkowski, et al., The KPSS stationarity test as a unit root test, *Economics Letters*, **38**(4) (1992), 387-392.

[10] G. Elliott, T.J. Rothenberg, J.H. Stock, Efficient tests for an autoregressive unit root, *Econometrica*, **64**(4) (1996), 813-83.

[11] S.E. Said, D. Dickey, Testing for unit root in autoregressive moving average models with unknown order, *Biometrika*, **71** (1984), 599-607.

[12] G.S. Maddala, I.M. Kim, *Unit Roots, Cointegration and Structural Change*, Oxford University Press, New York (1998).

[13] B. Plaff, *Analysis of Integrated and Cointegration Time Series with R*, Springer, New York (2008).

[14] The R Development Core Team, *An Introduction to R*, Vienna, R Foundation for Statistical Computing, http://www.R-project.org (2010a).

[15] The R Development Core Team, *A Language and Environment for Statistical Computing*, Vienna, R Foundation for Statistical Computing, http://www.R-project.org (2010b).