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# REFINEMENTS OF HADAMARD TYPE INEQUALITIES FOR $(\alpha, m)$-CONVEX FUNCTIONS

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**Abstract:** The aim of this work is to establish several new results related to the left hand side of the Hermite-Hadamard inequality for functions whose first derivative in absolute value aroused to the $q$th $(q \geq 1)$ power are $(\alpha, m)$-convex.Some applications to special means of positive real numbers are considered.

**AMS Subject Classification:** 26D10, 39B62
**Key Words:** Hermite-Hadamard inequality, $(\alpha, m)$-convex function, convexity, Hölder inequality

## 1. Introduction

Let $f : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ be a convex function on the interval $I$ of real numbers and $a, b \in I$ with $a < b$. Then we have the following double inequality, which is well known as the Hermite-Hadamard inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2}.$$

In 1993 [3], V. Mihesan introduced the class of $(\alpha, m)$-convex functions as the following:

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**Definition 1.** The function $f:[0,b]\longrightarrow\mathbb{R}$ is said to be $(\alpha,m)$-convex, where $(\alpha,m)\in[0,1]^2$, if for every $x,y\in[0,b]$ and $t\in[0,1]$ we have

$$f(tx+m(1-t)y)\le t^\alpha f(x)+m(1-t^\alpha)f(y).$$

For recent results and generalizations concerning $(\alpha,m)$-convex functions, see [1],[7].

In 2011 [4], M. Özdemir et al. obtined some bounds for the right hand side of Hadamard type inequality for $(\alpha,m)$-convex functions.

The aim of this work is to establish some new bounds to the left hand side of Hermite-Hadamard type inequality for $(\alpha,m)$-convex functions.

## 2. Hermite-Hadamard type inequalities

In order to prove main results, we need the following lemma.

**Lemma 2.** *Assume $a,b\in\mathbb{R}$ with $a<b$ and $f:[a,b]\longrightarrow\mathbb{R}$ is a differentiable function on $(a,b)$. If $f'\in L^1[a,b]$ then the following equality holds:*

$$f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x=\frac{b-a}{4}$$
$$\int_0^1(1-t)\left[f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)-f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right]\mathrm{d}t.$$

*Proof.* We get a simple proof by performing an integration by parts in the integrals on the right side and changing the variable. □

**Theorem 3.** *Let $f:I\subseteq[0,b^*]\longrightarrow\mathbb{R}$ be a differentiable mapping on $I^\circ$ such that $f'\in L^1[a,b]$, where $a,b\in I$ with $a<b$ and $b^*>0$. If $|f'|$ is $(\alpha,m)$-convex on $[a,b]$ for $(\alpha,m)\in[0,1]^2$, then the following inequalities hold:*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right|\le(b-a)$$
$$\times\left\{A(|f'(a)|+|f'(b)|)+m\left(\frac{1}{8}-A\right)\left[\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right]\right\},$$

*where* $A=\dfrac{1-\left(\frac{1}{2}\right)^{\alpha+2}(\alpha+3)}{(\alpha+1)(\alpha+2)}$ *and*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right|$$

$$\leq \frac{(b-a)}{4(\alpha+1)(\alpha+2)}\left[|f'(a)|+|f'(b)|+m\alpha(\alpha+3)\left|f'\left(\frac{a+b}{2m}\right)\right|\right].$$

*Proof.* From Lemma 2, we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| = \left(\frac{b-a}{4}\right)$$
$$\times\int_0^1 (1-t)\left[\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|+\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|\right]\mathrm{d}t \qquad (1)$$

Since $|f'|$ is $(\alpha,m)$-convex on $[a,b]$, so for any $t\in[0,1]$:

$$\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right| \leq \left(\frac{1+t}{2}\right)^{\alpha}|f'(a)|+m\left(1-\left(\frac{1+t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{b}{m}\right)\right| \qquad (2)$$

$$\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right| \leq \left(\frac{1+t}{2}\right)^{\alpha}|f'(b)|+m\left(1-\left(\frac{1+t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{a}{m}\right)\right|. \qquad (3)$$

Therefore, as the following way (1), (2) and (3) imply the first ineuality:

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right|$$
$$\leq \left(\frac{b-a}{4}\right)\int_0^1\left[(1-t)\left(\frac{1+t}{2}\right)^{\alpha}(|f'(a)|+|f'(b)|)\right.$$
$$\left.+m\left(1-\left(\frac{1+t}{2}\right)^{\alpha}\right)(1-t)\left(\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right)\right]\mathrm{d}t$$
$$=(b-a)\left\{\frac{1-\left(\frac{1}{2}\right)^{\alpha+2}(\alpha+3)}{(\alpha+1)(\alpha+2)}(|f'(a)|+|f'(b)|)\right.$$
$$\left.+m\left(\frac{1}{8}-\frac{1-\left(\frac{1}{2}\right)^{\alpha+2}(\alpha+3)}{(\alpha+1)(\alpha+2)}\right)\left[\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right]\right\}.$$

Since $|f'|$ is $(\alpha,m)$-convex on $[a,b]$ and $\frac{1+t}{2}a+\frac{1-t}{2}b = ta+(1-t)\frac{a+b}{2}$ then for any $t\in[0,1]$, we have the following inequalities:

$$\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right| \leq t^{\alpha}|f'(a)|+m(1-t^{\alpha})\left|f'\left(\frac{a+b}{2m}\right)\right|, \qquad (4)$$

$$\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right| \le t^{\alpha}|f'(b)|+m(1-t^{\alpha})\left|f'\left(\frac{a+b}{2m}\right)\right|. \tag{5}$$

So the inequalities (1), (4) and (5) imply the second inequality:

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{4}\right)$$

$$\int_0^1\left[(1-t)t^{\alpha}(|f'(a)|+|f'(b)|)+2m(1-t)(1-t^{\alpha})\left|f'\left(\frac{a+b}{2m}\right)\right|\right]\mathrm{d}t$$

$$=\left(\frac{b-a}{4(\alpha+1)(\alpha+2)}\right)\left[|f'(a)|+|f'(b)|+m\alpha(\alpha+3)\left|f'\left(\frac{a+b}{2m}\right)\right|\right]. \qquad \square$$

**Theorem 4.** *Let $f: I \subseteq [0,b^*] \longrightarrow \mathbb{R}$ be a differentiable mapping on $I^{\circ}$ such that $f' \in L^1[a,b]$, where $a,b \in I$ with $a<b$ and $b^*>0$. If $|f'|^q$ is $(\alpha,m)$-convex on $[a,b]$ for $(\alpha,m)\in[0,1]^2$ and $q>1$, then the following inequalities hold:*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{4}\right)\left(\frac{1}{2}\right)^{\frac{q-1}{q}} \times$$

$$\left\{\left[A|f'(a)|^q+m\left(\frac{1}{8}-A\right)\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}\right.$$

$$\left.+\left[A|f'(b)|^q+m\left(\frac{1}{8}-A\right)\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\}, \tag{6}$$

*where* $A=\dfrac{1-\left(\frac{1}{2}\right)^{\alpha+2}(\alpha+3)}{(\alpha+1)(\alpha+2)}$ *and*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{q}}$$

$$\times\left[\left(|f'(a)|^q+m\alpha\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right)^{\frac{1}{q}}+\left(|f'(b)|^q+m\alpha\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right)^{\frac{1}{q}}\right]$$

$$\le\left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{q}}\left[|f'(a)|+|f'(b)|+2(m\alpha)^{\frac{1}{q}}\left|f'\left(\frac{a+b}{2m}\right)\right|\right]. \tag{7}$$

*Proof.* Since $|f'|^q$ is $(\alpha,m)$-convex on $[a,b]$, we have for any $t\in[0,1]$:

$$\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q \le \left(\frac{1+t}{2}\right)^\alpha |f'(a)|^q + m\left(1-\left(\frac{1+t}{2}\right)^\alpha\right)\left|f'\left(\frac{b}{m}\right)\right|^q \tag{8}$$

Using inequality (8) and Hölder's inequality for $q>1$ and $p=\dfrac{q}{q-1}$, we get (6)

$$\begin{aligned}
&\int_0^1 (1-t)\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\\
&=\int_0^1 (1-t)^{1-\frac{1}{q}}(1-t)^{\frac{1}{q}}\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|\mathrm{d}t\\
&\le\left(\int_0^1(1-t)\mathrm{d}t\right)^{\frac{1}{p}}\left(\int_0^1(1-t)\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{\frac{1}{q}} \qquad (9)\\
&\le\left(\frac{1}{2}\right)^{\frac{1}{p}}\left\{\int_0^1\left[(1-t)\left(\frac{1+t}{2}\right)^\alpha|f'(a)|^q\right.\right.\\
&\quad\left.\left.+m(1-t)\left(1-\left(\frac{1+t}{2}\right)^\alpha\right)\left|f'\left(\frac{b}{m}\right)\right|^q\right]\mathrm{d}t\right\}^{\frac{1}{q}}\\
&=\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left[A|f'(a)|^q+m\left(\frac{1}{8}-A\right)\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}.
\end{aligned}$$

Since $\frac{1+t}{2}a+\frac{1-t}{2}b=ta+(1-t)\frac{a+b}{2}$ and $|f'|^q$ is $(\alpha,m)$-convex on $[a,b]$, then the inequalities (4), (5) for $|f'|^q$ and (1) and Hölder's inequality for $q>1$ and $p=\dfrac{q}{q-1}$ imply

$$\begin{aligned}
&\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right|\le\left(\frac{b-a}{4}\right)\left(\int_0^1(1-t)^p\mathrm{d}t\right)^{\frac{1}{p}}\\
&\left\{\left(\int_0^1\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q\mathrm{d}t\right)^{\frac{1}{q}}+\left(\int_0^1\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|^q\mathrm{d}t\right)^{\frac{1}{q}}\right\} \qquad (10)\\
&=\left(\frac{b-a}{4}\right)\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left\{\left[\int_0^1\left(t^\alpha|f'(a)|^q+m(1-t^\alpha)\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right)\mathrm{d}t\right]^{\frac{1}{q}}\right.
\end{aligned}$$

$$+\left[\int_0^1 \left(t^\alpha |f'(b)|^q + m(1-t^\alpha)\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right) \mathrm{d}t\right]^{\frac{1}{q}}\Bigg\}$$

$$= \left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{q}}$$

$$\left[\left(|f'(a)|^q + m\alpha\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right)^{\frac{1}{q}} + \left(|f'(b)|^q + m\alpha\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right)^{\frac{1}{q}}\right].$$

Using the fact that

$$\sum_{i=1}^n (a_i + b_i)^r \le \sum_{i=1}^n a_i^r + \sum_{i=1}^n b_i^r \tag{11}$$

for $0 < r < 1$, $a_1, a_2, \ldots, a_n \ge 0$ and $b_1, b_2, \ldots, b_n \ge 0$, we obtain (7). □

If in Theorem 4, we use Lemma 2 and inequalities (2), (3) for $|f'|^q$ and (10), (11), then we get the following theorem.

**Theorem 5.** *With the assumption of Theorem 4, we have inequalities:*

$$\left|f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}$$

$$\times\left\{\left[\frac{2-2^{-\alpha}}{\alpha+1}|f'(a)|^q + m\left(1-\frac{2-2^{-\alpha}}{\alpha+1}\right)\left|f'\left(\frac{b}{m}\right)\right|^q\right]^{\frac{1}{q}}\right.$$

$$\left.+\left[\frac{2-2^{-\alpha}}{\alpha+1}|f'(b)|^q + m\left(1-\frac{2-2^{-\alpha}}{\alpha+1}\right)\left|f'\left(\frac{a}{m}\right)\right|^q\right]^{\frac{1}{q}}\right\}$$

$$\le \left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\left(\frac{2-2^{-\alpha}}{\alpha+1}\right)^{\frac{1}{q}}(|f'(a)| + |f'(b)|)\right.$$

$$\left.+m^{\frac{1}{q}}\left(1-\frac{2-2^{-\alpha}}{\alpha+1}\right)^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)\right].$$

If in Theorem 4, we use Lemma 2 and inequalities (4), (5) for $|f'|^q$ and (9), (11), then we get the following theorem.

**Theorem 6.** *With the assumption of Theorem 4, we have the inequalities:*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \left(\frac{b-a}{4}\right)\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left(\frac{1}{(\alpha+1)(\alpha+2)}\right)^{\frac{1}{q}}$$
$$\times\left\{\left[|f'(a)|^q+\frac{m\alpha(\alpha+3)}{2}\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[|f'(b)|^q+\frac{m\alpha(\alpha+3)}{2}\left|f'\left(\frac{a+b}{2m}\right)\right|^q\right]^{\frac{1}{q}}\right\}$$
$$\le\left(\frac{b-a}{4}\right)\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left(\frac{1}{(\alpha+1)(\alpha+2)}\right)^{\frac{1}{q}}$$
$$\times\left[|f'(a)|+|f'(b)|+2\left(\frac{m\alpha(\alpha+3)}{2}\right)^{\frac{1}{q}}\left|f'\left(\frac{a+b}{2m}\right)\right|\right].$$

**Remark 7.** *From Theorem 4-6, we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\mathrm{d}x\right| \le \min\{E_1,E_2,E_3,E_4\},$$

*where*

$$E_1=\left(\frac{b-a}{4}\right)\left(\frac{1}{2}\right)^{\frac{q-1}{q}}$$
$$\times\left\{A^{\frac{1}{q}}(|f'(a)|+|f'(b)|)+m^{\frac{1}{q}}\left(\frac{1}{8}-A\right)^{\frac{1}{q}}\left[\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right]\right\},$$

$$E_2=\left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left(\frac{1}{\alpha+1}\right)^{\frac{1}{q}}$$
$$\times\left[|f'(a)|+|f'(b)|+2(m\alpha)^{\frac{1}{q}}\left|f'\left(\frac{a+b}{2m}\right)\right|\right]$$

$$E_3=\left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\left(\frac{2-2^{-\alpha}}{\alpha+1}\right)^{\frac{1}{q}}(|f'(a)|+|f'(b)|)\right.$$
$$\left.+m^{\frac{1}{q}}\left(1-\frac{2-2^{-\alpha}}{\alpha+1}\right)^{\frac{1}{q}}\left(\left|f'\left(\frac{a}{m}\right)\right|+\left|f'\left(\frac{b}{m}\right)\right|\right)\right]$$

$$E_4=\left(\frac{b-a}{4}\right)\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left(\frac{1}{(\alpha+1)(\alpha+2)}\right)^{\frac{1}{q}}$$

$$\times\left[|f'(a)|+|f'(b)|+2\left(\frac{m\alpha(\alpha+3)}{2}\right)^{\frac{1}{q}}\left|f'\left(\frac{a+b}{2m}\right)\right|\right].$$

## 3. Applications to special means

Now using the results of Section 2, we give some applications to special means of positive real numbers.

(1) The arithmetic mean: $A(a,b)=\frac{a+b}{2}, \quad a,b\in\mathbb{R}, \quad a,b>0.$

(2) The logarithmic mean: $L(a,b)=\frac{b-a}{\ln b-\ln a}, \quad a,b\in\mathbb{R},\ a\neq b, \quad a,b>0.$

(3) The generalized logarithmic mean:

$$L_n(a,b)=\left[\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right]^{\frac{1}{n}},\ n\in\mathbb{R}\setminus\{-1,0\}, a,b\in\mathbb{R}, a\neq b,\ a,b>0.$$

The following propositions hold, by applying Theorems 5, 6 for $f(x)=x^n$ and $f(x)=\dfrac{1}{x}$ and $\alpha=m=1$.

**Proposition 8.** *Let $n\in(-\infty,0)\cup[1,\infty)\setminus\{-1\}$ and $q>1$ and $[a,b]\subset(0,\infty)$. Then we have the following inequalities:*

$$|A^n(a,b)-L_n^n(a,b)|\leq\frac{n(b-a)}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\left(\frac{3}{4}\right)^{\frac{1}{q}}+\left(\frac{1}{4}\right)^{\frac{1}{q}}\right]A(a^{n-1},b^{n-1})$$

$$|A^n(a,b)-L_n^n(a,b)|\leq n\left(\frac{b-a}{4}\right)\left(\frac{1}{3}\right)^{\frac{1}{q}}\left[A(a^{n-1},b^{n-1})+2^{\frac{1}{q}}A^{n-1}(a,b)\right].$$

**Proposition 9.** *Let $[a,b]\subset(0,\infty)$ and $q>1$. Then we have the following inequalities:*

$$\left|A^{-1}(a,b)-L^{-1}(a,b)\right|\leq\frac{b-a}{2}\left(\frac{q-1}{2q-1}\right)^{\frac{q-1}{q}}\left[\left(\frac{3}{4}\right)^{\frac{1}{q}}+\left(\frac{1}{4}\right)^{\frac{1}{q}}\right]A(a^{-2},b^{-2})$$

$$\left|A^{-1}(a,b)-L^{-1}(a,b)\right|\leq\left(\frac{b-a}{4}\right)\left(\frac{1}{3}\right)^{\frac{1}{q}}\left[A(a^{-2},b^{-2})+2^{\frac{1}{q}}A^{-2}(a,b)\right].$$

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