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RELIABILITY MODELING OF A COMPUTER SYSTEM WITH PRIORITY TO S/W REPLACEMENT OVER H/W REPLACEMENT SUBJECT TO MOT AND MRT

Ashish Kumar[1], S.C. Malik[2 §]

[1]Department of Statistics
Maharshi Dayanand University
Rohtak, 124001, INDIA

[2]Department of Statistics
Maharshi Dayanand University
Rohtak, 124001, INDIA

Abstract: The present study deals with the reliability modeling of a computer system of two identical units-one is operative and other is kept as spare in cold standby. In each unit h/w and s/w components work together and fail independently. There is a single server who visits the system immediately to carry out repair activities of the units. The unit under goes for preventive maintenance after a maximum operation time directly from normal mode. The h/w components under go for repair at their failure and are replaced by new one in case these are not repaired up to a maximum repair time. However, only replacement facility is available for s/w components. Priority to s/w replacement in the unit is given over h/w replacement. The failure time distribution of the components follow negative exponential whereas the distributions of preventive maintenance, repair and replacement times are taken as arbitrary with different probability density functions. Several reliability indices have been obtained using semi-Markov and regenerative point technique. The graphs are drawn to highlight the behaviour of the results with respect to preventive maintenance

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§Correspondence author

rate.

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1. Introduction

With the growth of computer systems, their reliability is rapidly becoming a critical business concern. Reliability in computer systems is important so as to maintain reliable operation. Computer failures cause organizations several hours or days of downtime and serious breaches in data confidentiality and integrity. A major challenge to the industrialists now a day is to provide reliable h/w and s/w components for the computer systems. For this purpose, most of the scientists and academicians are also trying to explore new techniques for reliability improvement of the computer systems. In spite of these efforts, a little work has been dedicated to the reliability modeling of computer systems. And, most of the research work carried out so far in the subject of s/w and h/w reliability has been limited to the consideration of either h/w subsystem alone or s/w subsystem alone. But there are many complex systems in which h/w and s/w components work together to provide computer functionality. Friedman and Tran (1992) and Welke et al. (1995) developed a combined reliability model for the whole system in which hardware and software components work together. Recently, Malik and Anand (2010) and Malik et al. (2011) formulated reliability models of a computer system with independent failure of h/w and s/w components.

Furthermore, the continued operation and ageing of these systems gradually reduce their performance and reliability. It is, therefore, of great importance to operate such systems with high reliability. It is proved that preventive maintenance can slow the deterioration process of a repairable system and restore the system in a younger age or state. Thus, the method of preventive maintenance can be used to improve reliability and profit of the systems. Malik et al. (2010) suggested a reliability model for complex systems introducing the concept of preventive maintenance of the unit after a maximum operation time. Also, the reliability of a system can be increased by making replacement of the components by new one in case repair time is too long i.e., if it extends to a pre-specific time. Singh and Agrafiotis (1995) studied stochastically a two-unit cold standby system subject to maximum operation and repair time. Kumar and Malik (2012) discussed stochastically a two-unit cold standby system with the concept of priority subject to maximum operation and maximum repair times.

In view of the above facts and to fill up the gap, the present paper is devoted to evaluate some reliability indices of a computer system in which h/w and s/w components fails independently. A reliability model of two identical units-one is operative and other is kept as spare in cold standby. In each unit h/w and s/w components work together and fail independently. There is a single server who visits the system immediately to perform preventive maintenance, h/w repair, replacement and s/w replacement. The unit under goes for preventive maintenance after a maximum operation time directly from normal mode. The h/w components under go for repair at their failure and are replaced by new one in case these are not repaired up to a maximum repair time. Further, only replacement facility is available for s/w components. Priority to s/w replacement of the unit is given over h/w replacement. The failure time distribution of the components follow negative exponential whereas the distributions of preventive maintenance, repair and replacement time are taken as arbitrary with different probability density functions. Several reliability indices such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server due to PM, busy period of the server due to h/w repair, busy period of the server due to h/w replacement, busy period of the server due to s/w replacement, expected number of h/w replacements, expected number of s/w replacements, expected number of visits by the server and profit function are obtained using semi-Markov and regenerative point technique. The graphical behaviour of some important reliability and economic measures has also been shown for a particular case to make the study more concrete.

1.1. Notations

E	The set of regenerative states
NO	The unit is operative and in normal mode
Cs	The unit is in cold standby
a/b	Probability that the system has hardware/ software failure
λ_1/λ_2	Constant hardware/software failure rate
α_0	Maximum constant rate of Operation Time
β_0	Maximum constant rate of Repair Time

Pm/PM	The unit is under preventive Maintenance/under preventive maintenance continuously from previous state
WPm/WPM	The unit is waiting for PM/waiting for preventive maintenance continuously from previous state
HFur/HFUR	The unit is failed due to hardware and is under repair/under repair continuously from previous state
HFurp/HFURP	The unit is failed due to h/w and is under replacement/under replacement continuously from previous state
HFwr/HFWR	The unit is failed due to h/w and is waiting for repair/waiting for repair continuously from previous state
SFurp/SFURP	The unit is failed due to the s/w and is under replacement/under replacement continuously from previous state
SFwrp/SFWRP	The unit is failed due to the software and is waiting for replacement/waiting for replacement continuously from previous state
$h(t)/H(t)$	pdf/cdf of replacement time of unit due to software
$g(t)/G(t)$	pdf/cdf of repair time of the hardware
$m(t)/M(t)$	pdf/cdf of replacement time of the hardware
$f(t)/F(t)$	pdf/cdf of the time for PM of the unit
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0, t]$
pdf/cdf	Probability density function/Cumulative density function
$q_{ij \cdot kr}(t)/Q_{ij \cdot kr}(t)$	pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k, r once in $(0, t]$
$\mu_i(t)$	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	Probability that the server is busy in the state S_i upto time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

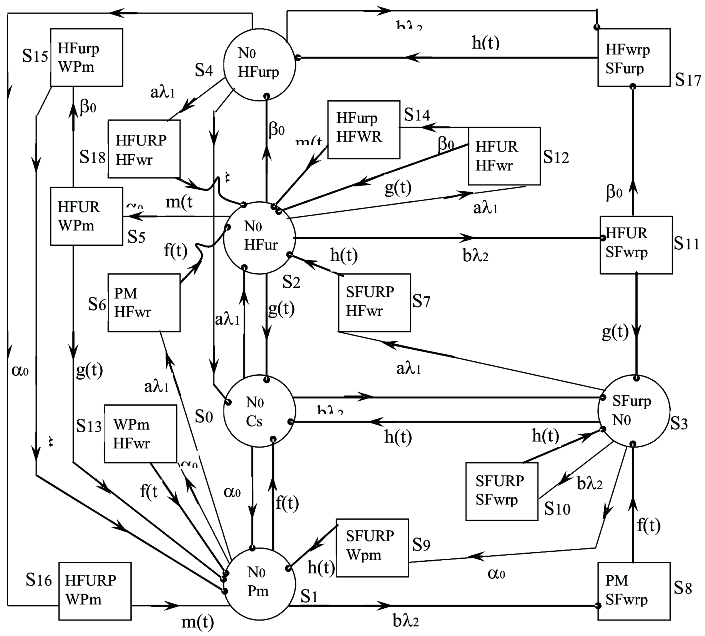

Figure 1

m_{ij} Contribution to mean sojourn time (μ_i) in state S_i when system transit directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*'}(0)$

Ⓢ/Ⓒ Symbol for Laplace-Stieltjes convolution/Laplace convolution

$\sim$ /$*$ Symbol for Laplace Steiltjes Transform (LST)/Laplace Transform (LT)

$'$ (desh) Used to represent alternative result

2. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \tag{1}$$

as

$p_{01} = \frac{\alpha_0}{A}$, $p_{02} = \frac{a\lambda_1}{A}$, $p_{03} = \frac{b\lambda_2}{A}$, $p_{10} = f^*(A)$, $p_{16} = \frac{a\lambda_1}{A}[1 - f^*(A)] = p_{12.6}$, $p_{18} = \frac{b\lambda_2}{A}[1 - f^*(A)] = p_{13.8}$, $p_{1.13} = \frac{\alpha_0}{A}[1 - f^*(A)] = p_{11.13}$, $p_{20} = g^*(B)$, $p_{24} = \frac{\beta_0}{B}[1 - g^*(B)]$, $p_{25} = \frac{\alpha_0}{B}[1 - g^*(B)]$, $p_{2.11} = \frac{b\lambda_2}{B}[1 - g^*(B)]$, $p_{2.12} = \frac{a\lambda_1}{B}[1 - g^*(B)]$, $p_{30} = h^*(A)$, $p_{37} = \frac{a\lambda_1}{A}[1 - h^*(A)] = p_{32.7}$, $p_{39} = \frac{\alpha_0}{A}[1 - h^*(A)] = p_{3,1.9}$, $p_{40} = m^*(A)$, $p_{3,10} = \frac{\lambda_2}{A}[1 - h^*(A)] = p_{33.10}$, $p_{51} = g^*(\beta_0)$, $p_{5,16} = 1 - g^*(\beta_0)$, $p_{4.16} = \frac{\alpha_0}{A}[1 - m^*(A)] = p_{4,1.16}$, $p_{62} = f^*(0)$, $p_{72} = h^*(0)$, $p_{83} = f^*(0)$, $p_{91} = h^*(0)$, $p_{10.3} = h^*(0)$, $p_{11.3} = g^*(\beta_0)$, $p_{11.17} = 1 - g^*(\beta_0)$, $p_{4,17} = \frac{b\lambda_2}{A}[1 - m^*(A)]$, $p_{12.2} = g^*(\beta_0)$, $p_{12.15} = 1 - g^*(\beta_0)$, $p_{13.1} = f^*(0)$, $p_{17.4} = h^*(0)$, $p_{4.18} = \frac{a\lambda_1}{A}[1 - m^*(A)] = p_{42.18}$, $p_{15.2} = m^*(0)$, $p_{16.1} = m^*(0)$, $p_{18.2} = m^*(0)$, $p_{21.5} = \frac{\alpha_0}{B}[1 - g^*(B)]g^*(\beta_0)$, $p_{21,5.15} = \frac{\alpha_0}{B}[1 - g^*(B)][1 - g^*(\beta_0)]$, $p_{23.11} = \frac{b\lambda_2}{B}[1 - g^*(B)][g^*(\beta_0)]$, $p_{2,17.11} = \frac{b\lambda_2}{B}[1 - g^*(B)][1 - g^*(\beta_0)]$, $p_{22.12} = \frac{a\lambda_1}{B}[1 - g^*(B)]g^*(\beta_0)$, $p_{22.12,14} = \frac{a\lambda_1}{B}[1 - g^*(B)][1 - g^*(\beta_0)]$, $A = a\lambda_1 + b\lambda_2 + \alpha_0$ and $B = a\lambda_1 + b\lambda_2 + \alpha_0 + \beta_0 + \theta$ (2)

It can be easily verified that

$$\begin{aligned}
p_{01} + p_{02} + p_{03} &= p_{10} + p_{16} + p_{18} + p_{1.13} \\
&= p_{20} + p_{24} + p_{25} + p_{2,11} + p_{2.12} \\
&= p_{30} + p_{37} + p_{39} + p_{3,10} \\
&= p_{40} + p_{4.16} + p_{4.17} + p_{4.18} \\
&= p_{5.1} + p_{5.16} = p_{62} = p_{72} = p_{83} = p_{91} = p_{10.3} \\
&= p_{11.3} + p_{11.14} = p_{12.2} + p_{12.15} = p_{13.1} = p_{14.1} \\
&= p_{15.2} = p_{16.4} = p_{17.4} = p_{18.3} = p_{19.2} \\
&= p_{10} + p_{12.6} + p_{11.13} + p_{13.8} \\
&= p_{20} + p_{24} + p_{21.5} + p_{21,5.15} + p_{23,11} + p_{2,17.11} \\
&\quad + p_{22,12} + p_{22.12,14} \\
&= p_{30} + p_{31.9} + p_{32.7} + p_{33.10} \\
&= p_{40} + p_{4,1.16} + p_{42.18} + p_{4.17} = 1
\end{aligned} \tag{3}$$

The mean sojourn times (μ_i) is the state S_i are

$$\mu_0 = \frac{1}{A}, \quad \mu_1 = \frac{1}{A+\alpha}, \quad \mu_2 = \frac{1}{\theta + B}, \quad \mu_3 = \frac{1}{A+\beta},$$

$$\mu_4 = \frac{1}{A+\gamma}, \quad \mu'_1 = \frac{1}{\alpha}\mu'_3 = \frac{1}{\beta}, \quad \mu'_4 = \frac{\beta^2 + A(a\lambda_1 + \alpha_0) + b\beta\lambda_2}{\beta(\beta+A)^2},$$

$$\mu'_2 = \frac{(\beta_0+\theta)}{(B+\theta)^2} + \frac{\begin{pmatrix} (A) - \theta^2\gamma(\theta+\beta_0)^2 + \gamma\theta(B) \\ +\beta_0(\beta_0+\theta)(\theta+B)(B) \\ -\beta_0\theta_\gamma(\theta+\beta_0) \\ +(B+\beta_0)\gamma(B)(\theta+\beta_0) \end{pmatrix}}{\gamma(\theta+B)^2(\theta+\beta_0)^2(B)}, \quad \mu_{17} = \frac{1}{\beta}, \tag{4}$$

Also

$$\begin{aligned} &m_{01} + m_{02} + m_{03} = \mu_0, \quad m_{10} + m_{16} + m_{18} + m_{1.13} = \mu_1 \\ &m_{20} + m_{24} + m_{25} + m_{2.11} + m_{2.12} = \mu_2 \\ &m_{40} + m_{4.17} + m_{4.18} + m_{4.16} = \mu_4 \\ &m_{51} + m_{5.16} = \mu_5 m_{11.17} + m_{11.3} = \mu_{11}, \quad m_{12.14} + m_{12.2} = \mu_{12} \\ &m_{62} = \mu_6, \quad m_{72} = \mu_7, \quad m_{83} = \mu_8, \end{aligned}$$

$$\begin{aligned} &m_{91} = \mu_9, m_{10.3} = \mu_{10}, \quad m_{10} + m_{12.6} + m_{13.8} + m_{11.13} = \mu'_1 \\ &m_{20} + m_{24} + m_{21.5} + m_{21.5,15} + m_{22.12} + m_{22.12,14} + m_{23.11} + m_{2,17.11} = \mu'_2 \\ &m_{30} + m_{39} + m_{32.7} + m_{33.10} = \mu'_3 \quad , \\ &m_{40} + m_{42.18} + m_{4.17} + m_{41.16} = \mu'_4 \end{aligned} \tag{5}$$

3. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the c.d.f of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\phi_i(t) = \sum_j Q_{i,j}(t) \circledR \phi_j(t) + \sum_k Q_{i,k}(t) \tag{6}$$

where j is an un-failed regenerative state to which the given regenerative state i can transit and k is a failed state to which the state i can transit directly.

Taking LT of above relation (6) and solving for $\widetilde{\phi}_0(s)$, we have

$$R^*(s) = \frac{1 - \widetilde{\phi_0}(s)}{s} \tag{7}$$

The reliability of the system model can be obtained by taking Laplace inverse transform of (7).

The mean time to system failure (MTSF) is given by

$$\mathrm{MTSF} = \lim_{s\to 0} \frac{1-\widetilde{\phi_0}(s)}{s} = \frac{N_1}{D_1} \tag{8}$$

where $N_1 = \mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{03}\mu_3 + p_{24}p_{02}\mu_4$ and $D_1 = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} - p_{02}p_{24}p_{40}$.

4. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \tag{9}$$

where j is any successive regenerative state to which the regenerative state i can transit through $n \geq 1$(natural number) transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ is up at time t without visiting to any other regenerative state, we have

$$\begin{aligned} &M_0(t) = e^{-(a\alpha_1+b\alpha_2+\alpha_0)t}, \qquad M_1(t) = e^{-(a\alpha_1+b\alpha_2+\alpha_0)t}\overline{F(t)},\\ &M_2(t) = e^{-(a\alpha_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G(t)},\ M_3(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H(t)},\\ &M_4(t) = e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M(t)} \end{aligned} \tag{10}$$

Taking LT of above relations (9) and solving for $A_0^*(s)$, the steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \tag{11}$$

where

$$\begin{aligned} N_2 = (&-p_{24} - p_{2,17.11}p_{17.4})\{\mu_0[(1-p_{11.13})(1-p_{33.10})p_{42.18}\\ &+ (1-p_{33.10})p_{12.6}p_{41.16} - p_{13.8}(p_{31.9}p_{42.18} - p_{41.16}p_{32.7})]\\ &+ \mu_1[p_{01}p_{42.18}(1-p_{33.10}) - p_{02}(1-p_{33.10})p_{41.16}\\ &+ p_{03}(p_{31.9}p_{42.18} - p_{32.7}p_{41.16})] + \mu_3[p_{01}p_{42.18}p_{13.8} \end{aligned}$$

$$
\begin{aligned}
&- p_{02}p_{13.8}p_{41.16} + p_{03}((1-p_{11.13})p_{42.18} + p_{12.6}p_{41.16})] \\
&- \mu_4[p_{01}(p_{32.7}p_{13.8} + p_{12.6}(1-p_{33.10})) \\
&+ p_{02}(-p_{31.9}p_{13.8} + (1-p_{33.10})(1-p_{11.13})) \\
&+ p_{03}((1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6})]\} \\
&+ (1-p_{17.4}p_{4.17})\{\mu_0[(1-p_{11.13})\{(1-p_{22.12 -} p_{22.12,14}) \\
&\times (1-p_{33.10}) - p_{32.7}p_{23.11}\} - p_{12.6}\{(p_{21.+p_{21.5,15})5} \\
&\times (1-p_{33.10}) + p_{31.9}p_{23.11}\} - p_{13.8}\{(p_{21.5} + p_{21.5,15})p_{32.7} \\
&+ p_{31.9}(1-p_{22.12 -} p_{22.12,14})\}] + \mu_1\{p_{01}[(1-p_{22.12 -} p_{22.12,14}) \\
&\times (1-p_{33.10}) - p_{32.7}p_{23.11}] + p_{02}[(p_{21.5} + p_{21.5,15}) \\
&\times (1-p_{33.10}) + p_{31.9}p_{23.11}] + p_{03}[p_{32.7}(p_{21.5} + p_{21.5,15}) \\
&+ p_{31.9}(1-p_{22.12 -} p_{22.12,14})]\} + \mu_2\{p_{01}[p_{12.6}(1-p_{33.10}) \\
&- p_{32.7}p_{13.8}] + p_{02}[(1-p_{11.13})(1-p_{33.10}) - p_{31.9}p_{13.8}] \\
&+ p_{03}[(1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6}]\} + \mu_3\{p_{01}[p_{23.11}p_{12.6} \\
&+ (1-p_{22.12 -} p_{22.12,14})p_{13.8}] + p_{02}[(1-p_{11.13})p_{23.11} \\
&+ p_{13.8}(p_{21.5} + p_{21.5,15})] + p_{03}[-p_{12.6}(p_{21.5} + p_{21.5,15}) \\
&+ (1-p_{11.13})(1-p_{22.12 -} p_{22.12,14})]\}\}
\end{aligned}
$$

and

$$
\begin{aligned}
D_2 = {} & (-p_{24} - p_{2,17.11}p_{17.4})\{\mu_0[(1-p_{11.13})(1-p_{33.10})p_{42.18} \\
&+ (1-p_{33.10})p_{12.6}p_{41.16} - p_{13.8}(p_{31.9}p_{42.18} - p_{41.16}p_{32.7})] \\
&+ \mu_1'[p_{01}p_{42.18}(1-p_{33.10}) - p_{02}(1-p_{33.10})p_{41.16} \\
&+ p_{03}(p_{31.9}p_{42.18} - p_{32.7}p_{41.16})] + \mu_3'[p_{01}p_{42.18}p_{13.8} \\
&- p_{02}p_{13.8}p_{41.16} + p_{03}((1-p_{11.13})p_{42.18} + p_{12.6}p_{41.16})] \\
&- (\mu_4' + p_{4.17\mu_{17}'})[p_{01}(p_{32.7}p_{13.8} + p_{12.6}(1-p_{33.10})) \\
&+ p_{02}(-p_{31.9}p_{13.8} + (1-p_{33.10})(1-p_{11.13})) \\
&+ p_{03}((1-p_{11.13})p_{32.7} + p_{31.9}p_{12.6})]\} + (1-p_{17.4}p_{4.17}) \\
&\times \{\mu_0[(1-p_{11.13})\{(1-p_{22.12 -} p_{22.12,14})(1-p_{33.10}) \\
&- p_{32.7}p_{23.11}\} - p_{12.6}\{(p_{21.5} + p_{21.5,15})(1-p_{33.10}) \\
&+ p_{31.9}p_{23.11}\} - p_{13.8}\{(p_{21.5} + p_{21.5,15})p_{32.7} \\
&+ p_{31.9}(1-p_{22.12 -} p_{22.12,14})\}] + \mu_1'\{p_{01}[(1-p_{22.12 -} p_{22.12,14}) \\
&\times (1-p_{33.10}) - p_{32.7}p_{23.11}] + p_{02}[(p_{21.5} + p_{21.5,15}) \\
&\times (1-p_{33.10}) + p_{31.9}p_{23.11}] + p_{03}[p_{32.7}(p_{21.5} + p_{21.5,15})
\end{aligned}
$$

$$
\begin{aligned}
&+ p_{31.9}(1 - p_{22.12 -} p_{22.12,14})]\} + (\mu_2' + p_{2.17,11}\mu_{17}') \\
&\times \{p_{01}[p_{12.6}(1 - p_{33.10}) - p_{32.7}p_{13.8}] + p_{02}[(1 - p_{11.13}) \\
&\times (1 - p_{33.10}) - p_{31.9}p_{13.8}] + p_{03}[(1 - p_{11.13})p_{32.7} + p_{31.9}p_{12.6}]\} \\
&+ \mu_3'\{p_{01}[p_{23.11}p_{12.6} + (1 - p_{22.12 -} p_{22.12,14})p_{13.8}] \\
&+ p_{02}[(1 - p_{11.13})p_{23.11} + p_{13.8}(p_{21.5} + p_{21.5,15})] \\
&+ p_{03}[-p_{12.6}(p_{21.5} + p_{21.5,15}) + (1 - p_{11.13})(1 - p_{22.12 -} p_{22.12,14})]\}\}
\end{aligned}
$$

5. Busy Period Analysis for Server

Let $B_i^p(t)B_i^R(t)B_i^S(t)$ and $B_i^{HRp(t)}$ be the probabilities that the server is busy in Preventive maintenance of the system, repairing the unit due to hardware failure, replacement of the software and hardware components at an instant 't' given that the system entered state i at $t = 0$. The recursive relations for $B_i^P(T)B_i^R(t)B_i^S(t)$ and $B_i^{HRp}(t)$ are as follows:

$$
\begin{aligned}
B_i^p(t) &= W_i(t) + \sum_j q_{i,j}^{(n)}(t)©B_j^p(t), \\
B_i^R(t) &= W_i(t) + \sum_j q_{i,j}^{(n)}(t)©B_j^R(t), \\
B_i^S(t) &= W_i(t) + \sum_j q_{i,j}^{(n)}(t)©B_j^S(t), \\
B_i^{HRp}(t) &= W_i(t) + \sum_j q_{i,j}^{(n)}(t)©B_j^{HRp}(t)
\end{aligned} \tag{12}
$$

where j is any successive regenerative state to which the regenerative state i can transit through $n \geq 1$ (natural number) transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance, hardware and software failure up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$
\begin{aligned}
W_1 &= e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{F}(t) + (\alpha_0 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{F}(t) \\
&\quad + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{F}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{F}(t), \\
W_2 &= e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}\overline{G}(t) + (\alpha_0 e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1)\overline{G}(t) \\
&\quad + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1)\overline{G}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0+\beta_0)t}©1)\overline{G}(t),
\end{aligned}
$$

$$
\begin{aligned}
W_3 &= e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{H}(t) + (\alpha_0 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t})\overline{H}(t) \\
&\quad + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{H}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t})\overline{H}(t), \\
W_4 &= e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}\overline{M}(t) + (\alpha_0 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{M}(t) \\
&\quad + (a\lambda_1 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{M}(t) + (b\lambda_2 e^{-(a\lambda_1+b\lambda_2+\alpha_0)t}©1)\overline{M}(t), \\
W_{17} &= \overline{H}(t)
\end{aligned}
$$

Taking LT of above relations (12) and with $B_i(t)$ solving for $B_i^p(t)$, $B_i^R(t)$, $B_i^s(t)$ and $B_i^{HRp}(t)$ the time for which server is busy due to PM, h/w repair and h/w and s/w replacements respectively is given by

$$
B_0^H = \lim_{s\to 0} sB_0^{*H}(s) = \frac{N_3^H}{D_2}, \qquad B_0^S = \lim_{s\to 0} sB_0^{*S}(s) = \frac{N_3^S}{D_2},
$$

$$
B_0^R = \lim_{s\to 0} sB_0^{*R}(S) = \frac{N_S^R}{D_2}
$$

and

$$
B_0^{HRp} - \lim_{s\to 0} sB_0^{*HRp}(S) = \frac{N_s^{HRp}}{D_2} \tag{13}
$$

where

$$
\begin{aligned}
N_3^P(t) &= W_1^*(0)[(p_{24} + p_{2,17.11}p_{17.4})[-p_{01}p_{42.18}(1 - p_{33.10}) \\
&\quad + (1 - p_{33.10})p_{02}p_{41.16} - p_{03}(p_{31.9}p_{42.18} - p_{32.7}p_{41.16})] \\
&\quad + (1 - p_{17.4}p_{4.17})[p_{01}(1 - p_{22.12} - p_{22.12,14})(1 - p_{33.10}) \\
&\quad - p_{02}(p_{21.5} + p_{21.5,15})(1 - p_{33.10}) + p_{03}\{p_{32.7}(p_{21.5} + p_{21.5,15}) \\
&\quad + p_{31.9}(1 - p_{22.12} - p_{22.12,14})\}]] \\
N_3^R(t) &= (1 - p_{4.17}p_{17.4})W_2^*(0)[p_{01}\{p_{32.7}p_{13.8} \\
&\quad + p_{12.6}(1 - p_{33.10})\} + p_{02}\{-p_{31.9}p_{13.8} \\
&\quad + (1 - p_{33.10})(1 - p_{11.13})\} + p_{03}\{(1 - p_{11.13})p_{32.7} \\
&\quad + p_{31.9}p_{12.6}\}] \\
N_3^S(t) &= (1 - p_{17.4}p_{4.17})p_{2.17,11}W_{17}^*\{p_{01}\{p_{12.6}(1 - p_{33.10}) \\
&\quad + p_{32.7}p_{13.8}\} + p_{02}[(1 - p_{11.13})(1 - p_{33.10}) - p_{31.9}p_{13.8}] \\
&\quad + p_{03}[p_{32.7}(1 - p_{11.13}) + p_{31.9}p_{12.6}]\} - W_3^*\{-p_{01} \\
&\quad \times [p_{12.6}p_{23.11} + (1 - p_{22.12} - p_{22.12,14})p_{13.8}] \\
&\quad - p_{02}[(1 - p_{11.13})p_{23.11} + (p_{21.5} + p_{21.5,15})p_{13.8}]
\end{aligned}
$$

$$
\begin{aligned}
&- p_{03}[(1-p_{11.13})(1-p_{22.12}-p_{22.12,14})\\
&- (p_{21.5}+p_{21.5,15})p_{12.6}]\} - (p_{24}+p_{2,17.11}p_{17.4})\\
&\times \{W_3^*[p_{01}p_{42.18}p_{13.8}+p_{03}\{(1-p_{11.13})p_{42.18}\\
&+ p_{41.16}p_{12.6}\}] - [p_{01}\{p_{32.7}p_{13.8}+p_{12.6}(1-p_{33.10})\}\\
&+ p_{02}\{-p_{31.9}p_{13.8}+(1-p_{33.10})(1-p_{11.13})\}\\
&+ p_{03}\{(1-p_{11.13})p_{32.7}+p_{31.9}p_{12.6}\}]W_{17}^*p_{4.17}\}\\
N_3^{HRp}(t) = {}&(p_{24}+p_{17.4}p_{2.17,11})W_4^*(0)[p_{01}\{p_{32.7}p_{13.8}+p_{12.6}(1-p_{33.10})\}\\
&+ p_{02}\{-p_{31.9}p_{13.8}+(1-p_{33.10})(1-p_{11.13})\}\\
&+ p_{03}\{(1-p_{11.13})p_{32.7}+p_{31.9}p_{12.6}\}] \qquad (14)
\end{aligned}
$$

6. Expected Number of Replacements of the Units

Let $R_i^H(t)$ and $R_i^S(t)$ the expected number of replacements of the failed hardware and software components by the server in (0,t] given that the system entered the regenerative state i at $t=0$.

The recursive relations for $R_i^H(t)$ and $R_i^S(t)$ are given as

$$
\begin{aligned}
R_i^H(t) &= \sum_j q_{i,j}^{(n)}(t)Ⓡ[\delta_j + R_j^H(t)],\\
R_i^S(t) &= \sum_j q_{i,j}^{(n)}(t)Ⓡ[\delta_j + R_j^S(t)] \qquad (15)
\end{aligned}
$$

where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$, if j is the regenerative state where the server does job afresh, otherwise $\delta_j = 0$.

Taking LT of relations and, solving for $\widetilde{R}_0^H(s)$ and $\widetilde{R}_0^S(S)$. The expected numbers of replacements per unit time to the hardware and software failures are respectively of given by

$$
R_0^H(\infty) = \lim_{s\to 0} s\widetilde{R}_0^H(s) = \frac{N_4^H}{D_2} \text{ and } R_0^S(\infty) = \lim_{s\to 0} s\widetilde{R}_0^S(s) = \frac{N_4^S}{D_2} \qquad (16)
$$

where D_2 is already mentioned

$$
\begin{aligned}
N_4^H(t) = {}&(p_{22,12.14}+p_{21.5,15})(1-p_{17.4}p_{4.17})[p_{01}(p_{32.7}p_{13.8}\\
&+ p_{12.6}(1-p_{33.10})) + p_{02}(-p_{31.9}p_{13.8}+(1-p_{33.10})
\end{aligned}
$$

$$
\begin{aligned}
&\times(1-p_{11.13}))+p_{03}((1-p_{11.13})p_{32.7}+p_{31.9}p_{12.6})] \\
&+(p_{24}+p_{17.4}p_{2,17.11})(p_{40}+p_{4,2.18}+p_{41.16}) \\
&\times[p_{01}(p_{32.7}p_{13.8}+p_{12.6}(1-p_{33.10})) \\
&+p_{02}(-p_{31.9}p_{13.8}+(1-p_{33.10})(1-p_{11.13})) \\
&+p_{03}((1-p_{11.13})p_{32.7}+p_{31.9}p_{12.6})], \\
N_4^S(t)=&(1-p_{17.4}p_{4.17})\{(p_{17.4}p_{2.17,11})\{p_{01}\{p_{12.6}(1-p_{33.10}) \\
&+p_{32.7}p_{13.8}\}+p_{02}[(1-p_{11.13})(1-p_{33.10}) \\
&-p_{31.9}p_{13.8}]+p_{03}[p_{32.7}(1-p_{11.13})+p_{31.9}p_{12.6}]\} \\
&+\{p_{01}[p_{12.6}p_{23.11}+(1-p_{22.12-}p_{22.12,14})p_{13.8}] \\
&-p_{02}[(1-p_{11.13})p_{23.11}+(p_{21.5}+p_{21.5,15})p_{13.8}] \\
&+p_{03}[(1-p_{11.13})(1-p_{22.12-}p_{22.12,14}) \\
&-(p_{21.5}+p_{21.5,15})p_{12.6}]\}-(p_{24}+p_{2,17.11}p_{17.4}) \\
&\times\{[p_{01}p_{42.18}p_{13.8}-p_{02}p_{41.16}p_{13.8} \\
&+p_{03}\{(1-p_{11.13})p_{42.18}+p_{41.16}p_{12.6}\}] \\
&-[p_{01}\{p_{32.7}p_{13.8}+p_{12.6}(1-p_{33.10})\} \\
&+p_{02}\{-p_{31.9}p_{13.8}+(1-p_{33.10})(1-p_{11.13})\} \\
&+p_{03}\{(1-p_{11.13})p_{32.7}+p_{31.9}p_{12.6}\}]p_{17.4}p_{4.17}\}
\end{aligned}
$$

7. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server in $(0,t]$ given that the system entered the regenerative state i at $t=0$. The recursive relations for $N_i(t)$ are given as

$$N_i(t)=\sum_j q_{i,j}^{(n)}(t)\circledR[\delta_j+N_j(t)] \tag{17}$$

where j is any regenerative state to which the given regenerative state i transits and $\delta=1$, if j is the regenerative state where the server does job afresh, otherwise $\delta j=0$.

Taking LT of relation (17) and solving for $\widetilde{N}_0(s)$. The expected number of visit per unit time by the server are given by

$$N_0(\infty)=\lim_{s\to 0} s\widetilde{N}_0(s)=\frac{N_5}{D_2}, \tag{18}$$

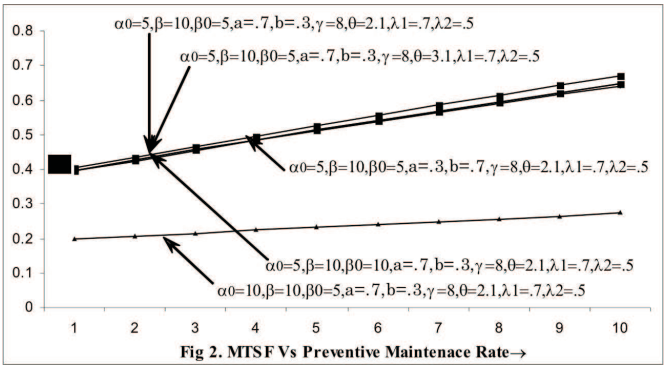

Figure 2

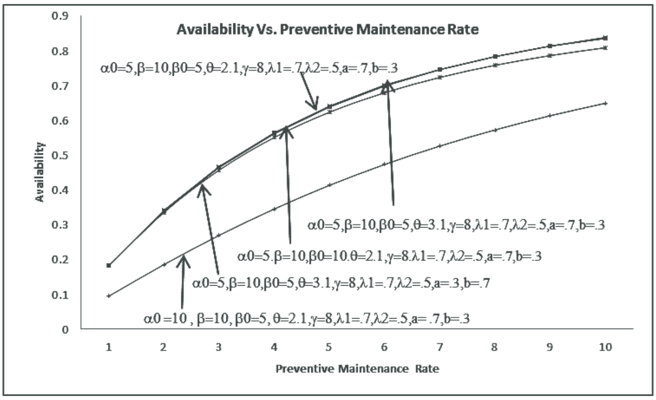

Figure 3

where

$$
\begin{aligned}
N_5 = {} & (-p_{24} - p_{17.4}p_{2,17.11})[(1 - p_{33.10})(1 - p_{11.13})p_{42.18} \\
& + p_{12.6}p_{41.16}(1 - p_{33.10}) - p_{13.8}\{p_{31.9}p_{42.18} - p_{41.16}p_{32.7}\} \\
& + (1 - p_{17.4}p_{4.17})[(1 - p_{11.13})\{(1 - p_{33.10})(1 - p_{22.12} - p_{22.12,14})
\end{aligned}
$$

$$
\begin{aligned}
& - p_{23.11}p_{32.7}\} + p_{12.6}\{(1 - p_{33.10})(-p_{21.5} - p_{21.5,15}) - p_{31.9}p_{23.11}\} \\
& - p_{13.8}\{p_{32.7}(p_{21.5} + p_{21.5,15}) + p_{31.9}(1 - p_{22.12 -}p_{22.12,14})\}]
\end{aligned}
$$

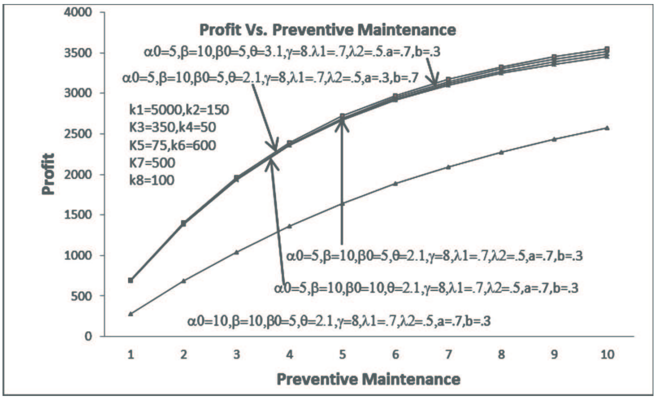

Figure 4

8. Economic Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0 A_0 - K_1 B_0^p - K_2 B_0^R - K_3 B_0^S - K_4 B_0^{HRp} \\ - K_5 R_0^H - K_6 R_0^S - K_7 N_0 \qquad (19)$$

$K_0 =$ Revenue per unit up-time of the system

$K_1 =$ Cost per unit time for which server is busy due preventive maintenance

$K_2 =$ Cost per unit time for which server is busy due to hardware failure

$K_3 =$ Cost per unit replacement of the failed software component

$K_4 =$ Cost per unit replacement of the failed hardware component

$K_5 =$ Cost per unit replacement of the failed hardware

$K_6 =$ Cost per unit replacement of the failed software

$K_7 =$ Cost per unit visit by the server

9. Conclusion

The numerical results considering a particular case $g(t) = \theta e^{-e}$, $h(t) = \beta e^{-\beta t}$, $f(t) = \alpha e^{-\alpha t}$ and $m(t) - \gamma e^{-\gamma t}$ are obtained for some reliability and economic indices of a computer system of two identical units having independent h/w and s/w components. The graphs for mean time to system failure (MTSF), availability and profit are drawn with respect to preventive maintenance rate (α) for fixed values of other parameters as shown respectively in figures 2 to 4. From these figures, it is revealed that MTSF, Availability and profit increase with the increase of PM rate (α) and repair rate (θ) of the hardware components. But the value of these measures decrease with the increase of maximum operation time (α_0). Again, if we increase maximum constant rate of repair time (β_0), then the value of MTSF, availability and profit increase.Thus, on the basis of the results obtained for a particular case, it is suggested that the reliability and profit of a system in which chances of h/w failure are high can be improved by

(i) Reducing the repair time of the h/w components as well as conducting preventive maintenance of the units after a pre-specific period of time.

(ii) Making replacement of the hardware components by new one in case repair time is too long.

(iii) Making replacement of s/w components by new one.

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