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RANKING DECISION MAKING UNITS BASED ON THE COST EFFICIENCY MEASURE

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Abstract: Data envelopment analysis (DEA) divides decision making units (DMUs) in two groups; efficient DMUs and inefficient DMUs. One problem that has been discussed frequently in DEA literature is lack of discrimination of efficient DMUs. This means that the conventional DEA does not provide more information about the efficient DMUs. This short paper proposes a new ranking system of decision making units (DMUs) based on the cost efficiency of these DMUs, which their input prices are available, and the effect of each DMU on the cost efficiency of other DMUs.

Key Words: data envelopment analysis, cost efficiency, ranking

1. Introduction

Data envelopment analysis (DEA) introduced by Charnes et al. [3] is a nonparametric method and employed to evaluate the efficiency performance of a decision making unit (DMU) that transform multiple-inputs to multiple-outputs.

One of the interesting research subjects in DEA is to discriminate among

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efficient DMUs. Many researchers have focused to propose methods for ranking the best performers among others [2, 7, 8, 9, 12, 13, 14, 15]. Also some researchers have found some characterizations of the production possibility set which is used to rank DMUs [10, 11]. For a review of ranking methods, readers are refered to Adler et al. [1]. Cost Efficiency (CE) evaluates the producing ability of the current output of a DMU at minimal cost, given its input prices. In the other words, CE is interpreted as a measure of potential cost reduction given the outputs produced and current input prices at each DMU. The trace of CE concept is found in [6], in which Farrell originated many of the ideas underlying efficiency assessment. Fare et al. [5] developed the Farrell concept of CE and formulated a Linear Programming (LP) model for CE assessment. This LP model requires input and output quantity data as well as input prices at each DMU. The main methodological difference between our model and the others is that the other approaches find the technical efficiency measures of DMUs using only the input and output quantities of DMUs and so DMUs are ranked based on their technical efficiencies and their efforts to construct the production possibility set while by contrast in our methodology the input prices have the main role to rank DMUs.

2. Preliminaries

In this section, we present some basic definitions, models and concepts that will be used in the remaining sections. For further details in DEA solving procedures interested readers are referred to [4].

Suppose that we have n DMUs where each DMU_j, $j = 1, \ldots, n$, produces the same s outputs in (possibly) different amounts, y_{rj} ($r = 1, \ldots s$), using the same m inputs, x_{ij} ($i = 1, \ldots m$), also in (possibly) different amounts.

Arranging the data sets in matrices $X = (x_{ij})_{m\times n}$ and $Y = (y_{rj})_{s\times n}$, $i = 1, ..., m,\ r = 1, ..., s,\ j = 1, \ldots, n$, the production possibility set (PPS), T, can be defined by $T_c = \{(x, y) :\ x \geq X\lambda,\ y \leq Y\lambda,\ \lambda \geq 0\}$, where $\lambda \in \mathbf{R}^n_+$. For evaluating the efficiency of DMU_o ($o \in \{1, \ldots, n\}$), the DEA input-oriented model is as follows:

$$
\begin{array}{ll}
\min & \mu = \theta - \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\
s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \quad i = 1, \ldots, m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1, \ldots, s, \\
& \lambda_j \geq 0, \quad j = 1, \ldots, n, \\
& s_i^- \geq 0, \quad i = 1, \ldots, m, \\
& s_r^+ \geq 0, \quad r = 1, \ldots, s,
\end{array}
\tag{1}
$$

where θ is the technical efficiency (TE) measure and ε is a non-Archimedean small and positive number such that the objective function of (1) is bounded. We know that DMU_o is CCR-efficient if in Model (1) $\theta^* = 1$, $S^{-^*} = 0$ and $S^{+^*} = 0$ (i.e., $\mu^* = 1$), otherwise DMU_o is CCR-inefficient. The set of CCR-efficient DMUs is devided to two distinct sets consisting of extreme efficient DMUs and non-extreme efficient DMUs. In order to determine the CCR-efficient DMUs, the DEA computer code can use a two-phase LP problem, which may be formalized as follows:

Phase 1) solve $\theta^* = \min \theta$ subject to the constraints of Problem (1).

Phase 2) incorporate value θ^* instead of θ in (1) and replace its objective function by $\max(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+)$.

It is important to note that DMU_o is extreme efficient if and only if Model (1) has a unique optimal solution: $(\lambda_o^* = 1,\ \lambda_j^* = 0,\ j = 1, \ldots, o-1, o+1, \ldots, n,\ S^+ = 0,\ S^- = 0)$.

Farrell (1957) [6]proposed a measure of CE, which assumes that all input prices of all DMUs are fixed and known, although they may possibly be different among DMUs.

To obtain a measure of cost efficiency for DMUs with multiple inputs and outputs, the minimum cost for a DMU's current outputs's production with existing input prices is obtained by solving the following linear problem, which first formulated by Fare et al. (1985) [5]:

$$\begin{array}{ll} \min & \sum_{i=1}^{m} p_{io} x_i^o \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} = x_i^o, \quad i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \\ & \lambda_j \geq 0, \quad j = 1, \ldots, n, \\ & x_i^o \geq 0, \quad i = 1, \ldots, m. \\ & \text{Minimal cost model} \end{array} \tag{2}$$

In the above model, $p_{io} > 0$ is the price of input i of the DMU under assessment (DMU_o). x_i^o is a variable that, at optimality, gives the amount of input i to be employed by DMU_o in order to produce the current outputs at minimal cost, subject to the technological restrictions imposed by the existing PPS. Note that this model assumes that the input prices at each DMU $(p_{io},\ i = 1, 2, \ldots, m)$ are fixed and known, although they can differ between DMUs. Cost efficiency is then obtained as the ratio of minimum cost with current prices (i.e., the optimal value ofmodel (2)) to the current cost at DMU_o , as follows:

$$CE_o = \frac{\sum_{i=1}^{m} p_{io} x_i^{o^*}}{\sum_{i=1}^{m} p_{io} x_{io}}. \tag{3}$$

DMU_o is cost efficient if $CE_o = 1$, otherwise it is cost inefficient.

3. Our Proposed Method

The following relation between cost efficiency and technical efficiency holds:

Theorem 3.1. *In evaluating DMU_o with (1), if $\theta^* < 1$ or $S^{-^*} \neq 0$ then $CE_o < 1$.*

Proof. Suppose that $(\theta^*, \lambda^*, S^{-^*}, S^{+^*})$ is an optimal solution of (1) with $\theta^* < 1$ or $S^{-^*} \neq 0$. Let $(\overline{\lambda}, \overline{x^o}) = (\lambda^*, \theta^* x_o - S^{-^*})$, then $(\overline{\lambda}, \overline{x^o})$ is a feasible solution of (2) therefore $CE_o \leq \frac{P_o \overline{x^o}}{P_o x_o} = \theta^* - \frac{P_o S^{-^*}}{P_o x_o} < 1$. □

Corollary. *If DMU_o is cost efficient, then we have in evaluating DMU_o using Model (1), $\theta^* = 1$ and $S^{-^*} = 0$.*

If a DMU is efficient, it is not necessarily a cost efficient DMU. Also, at least one of the observed DMUs is technically efficient but it is possible for all DMUs to be cost inefficient. For example, consider two DMUs with two inputs and one output as follows:

DMU	I_1	I_2	O	p_1	p_2	μ^*	CE
DMU_1	1	2	1	2	3	1	0.875
DMU_2	2	1	1	3	2	1	0.875

According to the above table, DMUs 1 and 2 are cost inefficient.

For an illustration of our proposed method consider the following example:

DMU	(I_1, I_2, O)	Super efficiency	(p_1, p_2)	CE
DMU_a	(2,2,1)	1.6111	(1,1)	1.0000
DMU_b	(2,2,1)	1.6111	(1,6)	0.7857
DMU_c	(5,1,1)	2.0000	(3,1)	0.5000
DMU_d	(1,6,1)	2.0000	(8,1)	1.0000

The super-efficiency measures of DMU_a is less than the supper efficiency of DMU_c, therefore DMU_c has a higher ranking than DMU_a using the supper efficiency measure. Note that the supper efficiency measures use only the input and output quantities of DMUs. By considering input prices (p_1, p_2), DMU_a is cost efficient ($CE_a = 1$), while DMU_c is not cost efficient. So we expect that DMU_a has a better ranking than DMU_c.

On the other hand, applying all available ranking methods, DMU_a and DMU_b have the same ranking because they have the same input and output quantities, but considering input prices it is not true.

In order to employ our approach, for each DMU_j, $(j = 1, \ldots, n)$, after ranking DMUs with respect to their cost efficiency it is possible that there exist at least two DMUs which have the same cost efficiency. Without loss of generality suppose that $N = \{DMU_j : CE_j = CE_1,\ j = 1, \ldots, n\} = \{DMU_1, \ldots, DMU_k\}$ is a set of such DMUs.

The main idea of our proposed method for ranking such a set of DMUs is as follows:

If excluding DMU_k, $(DMU_k \in N)$, from the observed DMUs cab be caused that the greatest relative of the cost efficiency of all other DMUs are increased, then DMU_k is the best performer among other DMUs in N.

For more explanation, let $CE_{a,b}$ be the cost efficiency of DMU_b after excluding DMU_a from the observed DMUs; i.e.,

$$CE_{a,b} = \frac{C_{a,b}}{\sum_{i=1}^{m} p_{ib} x_{ib}} \tag{4}$$

such that

$$\begin{array}{lll} C_{a,b} = \min & \sum_{i=1}^{m} p_{ib} x_i^b & \\ s.t. & \sum_{j=1\ j\neq a}^{n} \lambda_j x_{ij} = x_i^b, & i = 1, \ldots, m, \\ & \sum_{j=1\ j\neq a}^{n} \lambda_j y_{rj} \geq y_{rb}, & r = 1, \ldots, s, \\ & \lambda_j \geq 0, & j = 1, \ldots, n, j \neq a \\ & x_i^b \geq 0, & i = 1, \ldots, m. \end{array} \tag{5}$$

Note that the above model is feasible. We define Ω_a as follows:

$$\Omega_a = \frac{\sum_{j\neq a} CE_{a,j} - \sum_{j\neq a} CE_j}{n-1}. \tag{6}$$

If $\Omega_a > \Omega_b$ then DMU_a has a higher rank than DMU_b. Ω_a is the arithmetic average of variations of the cost efficiency of DMUs by excluding DMU_a from the observed DMUs and the cost efficiency of all DMUs except DMU_a.

To illustrate see Figure 1. The black and green line segments are parallel and their gradient vectors are the same and equal to (p_{1B}, p_{2B}) . We see that DMU B is cost inefficient and its cost efficiency is $\frac{Ob}{OB}$. If we omit DMU C, the new PPS is identified by A, B and D, that is, segments BC and CD are omitted. By this new PPS DMU B is cost efficient. So by removing DMU C from the observed DMUs then the cost efficiency of DMU B is increased.

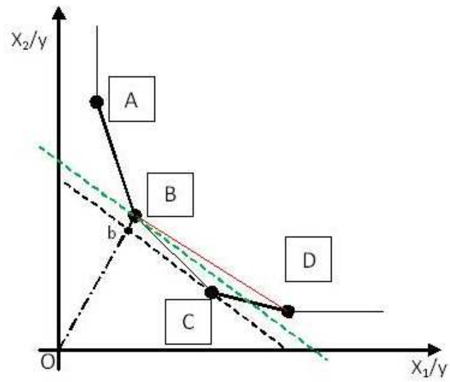

Figure 1: Explanation of the proposed approach

Theorem 3.2. *For each DMU_b and DMU_a we have $CE_{a,b} \geq CE_b$.*

Proof. The optimal value of Model (5) is equal to the optimal value of the following model:

$$\begin{array}{lll} \min & \sum_{i=1}^{m} p_{ib} x_i^b & \\ s.t. & \sum_{j=1}^{n} \lambda_j x_{ij} = x_i^b, & i = 1, \ldots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rb}, & r = 1, \ldots, s, \\ & \lambda_a = 0, \quad \lambda_j \geq 0, & j = 1, \ldots, n, \\ & x_i^o \geq 0, & i = 1, \ldots, m. \end{array} \tag{7}$$

Moreover, the feasible space of (7) is a subset of the feasible space of (2) and therefore the result holds. □

Theorem 3.3. *If DMU_a is not an extreme efficient DMU, then $CE_{a,b} = CE_b$.*

Proof. If DMU_a is not an extreme efficient DMU, by excluding DMU_a from the observed DMUs the new PPS does not change, therefore, the optimal value of (5) is equal to that of (2), i.e., $CE_{a,b} = CE_b$.

Numerical Example. Here we present one example with the data, except for input prices, taken from [1].

DMU	I_1	I_2	O_1	O_2	P_1	P_2
A	150	0.2	14000	3500	30	8
B	400	0.7	14000	21000	20	9
C	320	1.2	42000	10500	15	5
D	520	2.0	28000	42000	25	9
E	350	1.2	19000	25000	22	5
F	320	0.7	14000	15000	20	5

DMU scores for several ranking methods

Our results	CE	CCR		BCC		CEA		CEB		EDM	
C	1.000	A	1.000	A	1.000	A	0.764	A	1.000	A	200.000
D	1.000	B	1.000	B	1.000	B	0.700	D	1.000	B	140.625
E	0.921	C	1.000	C	1.000	D	0.700	E	0.974	C	140.000
A	0.712	D	1.000	D	1.000	E	0.696	B	0.955	D	113.077
B	0.651	E	0.978	E	1.000	C	0.643	C	0.886	E	97.750
F	0.648	F	0.868	F	0.896	F	0.608	F	0.847	F	86.745

CEA stands for the cross-efficiency-aggressive method, CEB for the cross-efficiency-benevolent method and EDM for the extended DEA measure method, see [2].

Note that DMU_C and DMU_D have the same cost efficiency, therefore, for ranking these two DMUs we calculate Ω_C and Ω_D.

$\Omega_C = 0.0719$ and $\Omega_D = 0.0427$, so DMU_C ranks higher than DMU_D.

4. Conclusion

Not only assessing DMUs regardless of the input prices, when the prices of all inputs of DMUs are available, is not a fair job but also ranking DMUs and identifying their productivity. In this paper, we proposed a method to rank DMUs in the presence of all input prices, which is totally dependent on the cost efficiency measure.

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