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SIGNED EDGE TOTAL DOMINATION NUMBERS OF TWO CLASSES OF GRAPHS

Hong Xia[1], Feng Wei[2], Xu Chunlei[3], Jirimutu[4 §]
[1,2,3,4]College of Mathematics
Inner Mongolia University for Nationalities
Tongliao, 028043, P.R. CHINA

Abstract: Let $\gamma_{st}^{'}(G)$ be the signed edge total domination number of a graph G. A fan graph $F_{1\cdot n}$, of order $n+1$, $n>0$, is obtain from a path with n vertices, denoted P_n, and a single vertex which is adjacent to every vertices of P_n. Let C_m be an $m-$cycle. The graph $n-C_m$, $n \geq 2$ and $m \geq 3$, has $n(m-1)+1$ edges and it consists of the union of n copies of C_m with precisely one common edge. In addition, the $m-$cycle have exactly two vertices in common. In this paper, we calculate the $\gamma_{st}^{'}(F_{1\cdot n})$, $n \geq 1$ and $\gamma_{st}^{'}(n-C_m)$, $m \geq 3$, $n \geq 2$.

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Key Words: signed edge total domination function, signed edge total domination number, fan graphs $F_{1\cdot n}$, $n-C_m$

1. Introduction

In this paper, the graphs are undirectional simple graphs and for other terminologies we follow[1].

Let $G=(V,E)$ be a graph with vertex set $V=V(G)$ and edge set $E=E(G)$. For any vertex $v \in V$, $N_G(v)$ denotes the open neighborhood of v in G and $N_G[v]=N_G(v)\bigcup\{v\}$ the closed one. $d_G(v)=\mid N_G(v)\mid$ is called the degree of v in G, δ and $\triangle$ denote the minimum degree and maximum degree of G, respectively. Two edges e_1, e_2 of G are called *adjacent* if they are distinct and have a common end-vertex. The *open neighborhood* $N_G(e)$ of an edge $e \in E(G)$

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§Correspondence author

is the set of all edges adjacent to e. if $e = uv \in E$, $N_G[e] = N_G(e) \bigcup \{e\}$ the closed one. $d_G(e)$ is called the degree of e in G. If the graph is clear from the context, $N_G(v)$, $N_G[v]$, $d_G(v)$ and $N_G(e)$, $N_G[e]$, $d_G(e)$ can simply be denoted by $N(v)$, $N[v]d(v)$ and $N(e)$, $N[e]d(e)$. If $d(v)$ is odd (even), then v is called an odd (even) vertex of G. Similarly, if $d(e)$ is even (odd), then e is called an even (odd) edge and $d(e) = d(u) + d(v) - 2$. In this paper, we define $E_o = \{e \in E \mid d(e) \text{ is odd}\}$ and $E_e = \{e \in E \mid d(e) \text{ is even}\}$.

For a function $f : E \to \{+1, -1\}$ and a subset S of $E(G)$, define $f(S) = \sum_{e \in S} f(e)$. A function $f : E \to \{+1, -1\}$ is called a *signed edge total dominating function* of G, if $f(N_G(e)) \geq 1$ for each edge $e \in E(G)$. The minimum of the values of $\omega(f) = f(E(G))$, taken over all signed edge dominating function f of G, is called the *signed edge total domination number* of G and denoted $\gamma'_{st}(G)$.

And define $\gamma'_{st}(K_1) = 0$, $\gamma'_{st}(K_2) = 1$.

Here are some known results on $\gamma'_{st}(G)$.

Lemma 1. (see [3]) *Let C_m be a cycle of length $m \geq 3$, then $\gamma'_{st}(C_m) = m$.*

Lemma 2. (see [6]) *For any two disjoint graphs G_1 and G_2, $\gamma'_{st}(G_1 \bigcup G_2) = \gamma'_{st}(G_1) + \gamma'_{st}(G_2)$.*

Lemma 3. (see [8]) *Let f be a signed edge total dominating function of G and $e \in E$. if $e \in E_o$, then $\sum_{e' \in N(e)} f(e') \geq 1$, if $e \in E_e$, then $\sum_{e' \in N(e)} f(e') \geq 2$.*

As to the signed edge total domination number of graphs we know the following results: In [3], Bohdan Zelinka introduced the concept of signed edge total domination number $\gamma'_{st}(G)$ of a graph G. In [4], Baogen Xu obtained a lower bound of $\gamma'_{st}(G)$ for all $k-$ regular graphs of order n, and determine the exact value of $\gamma'_{st}(W_{n+1})$ of for all wheels W_{n+1}, and present a conjecture as follows:

Conjecture. *For any graph G of order n, $\gamma'_{st}(G) \leq \frac{4(n-1)}{3}$.*

In [5], H. Karami presented some lower bounds for $\gamma'_{st}(G)$, and proved that $\gamma'_{st}(G) \geq 2 - m/3$ for every tree T of size $m \geq 2$, and classied all trees T which satisfy the equality. In [6], Baogen Xu determined the smallest signed edge total domination number for all tree of order n, and characterized all connected graphs with $\gamma'_{st}(G) = \mid E(G) \mid$. In [7], Xiuhua Yuan determined the signed edge total domination number of complete graph K_n. In [8], Jinfeng Zhao obtained some new lower bounds of $\gamma'_{st}(G)$. In[9], Weiguo Wu present some lower bounds on the signed total edge domination number of a graph G and find some exact values on $\gamma_{st}^{'}(G)$ when G is a complete graph, a complete bipartite graph, the grid $P_2 \times P_k$ or the grid $P_2 \times C_k$. In this paper, we obtain the exact value of signed edge total domination number of fan graphs $F_{1\cdot n}$, $n \geq 1$ and $n - C_m$, $m \geq 3$, $n \geq 2$.

By Lemma 2, we consider connected graphs only.

2. Main Results

Theorem 1. *For any positive integers n, let $F_{1\cdot n} = P_n \vee K_1$ is fan graphs of order $n+1$, then:*

$$\gamma_{st}^{'}(F_{1\cdot n}) = \begin{cases} 1, & \text{when } n = 1 \\ 3, & \text{when } 2 \leq n \leq 6 \\ 5, & \text{when } 7 \leq n \leq 9 \\ 2\lfloor \frac{n}{2} \rfloor - 3, & \text{when } n \geq 10 \end{cases}$$

Proof. Let $G = F_{1\cdot n}$, f be a signed edge total domination function of G such that $\gamma_{st}^{'}(G) = f(E) = \sum_{e \in E} f(e)$. Assume v_0 is the center vertex of fan $F_{1\cdot n}$, and other vertices are $v_1, v_2, \cdots, v_n$ in proper order, namely, $V(K_1) = v_0$, all vertices of P_n are $v_1, v_2, \cdots, v_n$ in proper order. Clearly, $E(G) = \{v_0 v_i \mid 1 \leq i \leq n\} \bigcup \{v_j v_{j+1} \mid 1 \leq j \leq n-1\}$, $\mid E(G) \mid = 2n - 1$. Write $A = \{v_0 v_i \mid 1 \leq i \leq n\}$, $B = \{v_j v_{j+1} \mid 1 \leq j \leq n-1\}$, hence, $E(G) = A \bigcup B$. For convenience, an edge $e \in E(G)$ is said to be a $+1$ edge of G if $f(e) = +1$, analogously, an edge $e \in E(G)$ is said to be a -1 edge of G if $f(e) = -1$. Let s(or t) be the number of $+1$ edges(or -1 edges)of G. Thus, $2n - 1 = s + t$, $\gamma_{st}^{'}(G) = s - t$.

We discuss the signed edge total domination number of G in four cases.

Case 1. $n = 1$. By the definition of the signed edge total domination number, $\gamma_{st}^{'}(G) = 1$ is true.

Case 2. $2 \leq n \leq 6$. When $n = 2$, $F_{1\cdot 2} = C_3$, by **lemma 1**, $\gamma_{st}^{'}(F_{1\cdot 2}) = 3$. When $n = 3$, define an $SETDF$ f_3 of $F_{1\cdot 3}$ as follows:

$$f_3(e) = \begin{cases} -1, & \text{when } e = v_1 v_2; \\ +1, & \text{otherwise.} \end{cases}$$

When $n = 4$, define an $SETDF$ f_4 of $F_{1\cdot 4}$ as follows:

$$f_4(e) = \begin{cases} -1, & \text{when } e = v_0 v_1, \\ & \text{or } e = v_1 v_2; \\ +1, & \text{otherwise .} \end{cases}$$

When $n = 5$, define an $SETDF$ f_5 of $F_{1\cdot 5}$ as follows:

$$f_5(e) = \begin{cases} -1, & \text{when } e = v_0v_1, \\ & \text{or } e = v_1v_2, \\ & \text{or } e = v_{n-1}v_n; \\ +1, & \text{otherwise.} \end{cases}$$

When $n = 6$, define an $SETDF$ f_6 of $F_{1\cdot 6}$ as follows:

$$f_6(e) = \begin{cases} -1, & \text{when } e = v_0v_1, \\ & \text{or } e = v_1v_2, \\ & \text{or } e = v_{n-1}v_n, \\ & \text{or } e = v_0v_n; \\ +1, & \text{otherwise.} \end{cases}$$

Clearly, f_i be a signed edge total domination function of $F_{1\cdot i}$ such that $\gamma_{st}^{'}(F_{1\cdot i}) = \sum_{e\in E} f_i(e)$, we have $\gamma_{st}^{'}(F_{1\cdot i}) = 3$, where $i = 3, 4, 5, 6$.

Case 3. $7 \leq n \leq 9$.

When $n = 7$, define an $SETDF$ f_7 of $F_{1\cdot 7}$ as follows:

$$f_7(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } e = v_0v_1, \\ & \text{or } e = v_0v_4, \\ & \text{or } e = v_{n-1}v_n; \\ +1, & \text{otherwise.} \end{cases}$$

When $n = 8$, define an $SETDF$ f_8 of $F_{1\cdot 8}$ as follows:

$$f_8(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } e = v_0v_1, \\ & \text{or } e = v_0v_4, \\ & \text{or } e = v_{n-1}v_n, \\ & \text{or } e = v_0v_n; \\ +1, & \text{otherwise.} \end{cases}$$

When $n = 9$, define an $SETDF$ f_9 of $F_{1\cdot 9}$ as follows:

$$f_9(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } e = v_0v_1, \\ & \text{or } e = v_4v_5, \\ & \text{or } e = v_5v_6, \\ & \text{or } e = v_{n-1}v_n, \\ & \text{or } e = v_0v_n; \\ +1, & \text{otherwise.} \end{cases}$$

Clearly, f_i be a signed edge total domination function of $F_{1\cdot i}$ such that $\gamma_{st}^{'}(F_{1\cdot i}) = \sum_{e\in E} f_i(e)$, we have $\gamma_{st}^{'}(F_{1\cdot i}) = 5$, where $i = 7, 8, 9$.

Case 4. $n \geq 10$. We firstly prove

$\gamma_{st}^{'}(G) \geq 2\lfloor \frac{n}{2} \rfloor - 3$ (1)

Suppose that (1) does not hold. Then $\gamma_{st}^{'}(G) \leq 2\lfloor \frac{n}{2} \rfloor - 4$, hence $t \geq n - \lfloor \frac{n}{2} \rfloor + \frac{3}{2}$.

Case 4.1. All of -1 edges are in A, by the pigeonhole principle, there must exists a triangle of G containing two -1 edges, for the third edge e of the triangle, which is on P_n, clearly, $\sum_{e'\in N(e)} f(e^{'}) \leq 0$, which is a contradiction.

Case 4.2. All of -1 edges are in B, namely, all of -1 edges are on P_n, there must exists a path P_4 of length 3 such that its edges which are at two ends are -1 edges, we assume the path $P_4 = (v_1v_2v_3v_4)$, namely, v_1v_2 and v_3v_4 are -1 edges, write $e = v_2v_3$, clearly, $\sum_{e'\in N(e)} f(e^{'}) \leq 0$, which is a contradiction.

Case 4.3. Let the number of -1 edges in B is $r (1 \leq r \leq \frac{n-1}{2})$, then the number of -1 edges in A are at most $\frac{n-2r}{2}$. Otherwise, there must be a triangle of G containing two -1 edges, which arises a contradiction the same as *Case 4.1.* Thus, $t \leq r + \frac{n-2r}{2} = \frac{n}{2}$, which contradicts $t \geq n - \lfloor \frac{n}{2} \rfloor + \frac{3}{2}$.

So $\gamma_{st}^{'}(G) \geq 2\lfloor \frac{n}{2} \rfloor - 3$ is true.

We prove $\gamma_{st}^{'}(G) \leq 2\lfloor \frac{n}{2} \rfloor - 3$ by the following definition of the signed edge total domination function of G.

When $n \equiv 0 (mod\ 4), n \equiv 2 (mod\ 4)$, let

$$f(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } v_6v_7, \\ & \text{or } v_7v_8; \\ -1, & \text{when } e = v_0v_1, \\ & \text{or } v_0v_4; \\ (-1)^{i+1}, & \text{when } e = v_0v_i, 10 \leq i \leq n; \\ +1, & \text{otherwise.} \end{cases}$$

It is easy to verify that $\sum_{e'\in N(e)} f(e^{'}) \geq 1$ for any $e \in E(G)$, we have $\sum_{e\in E(G)} f(e) = 2\lfloor \frac{n}{2} \rfloor - 3$.

When $n \equiv 1(mod\ 4), n \neq 13$, let

$$f(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } v_6v_7, \\ & \text{or } v_7v_8, \\ & \text{or } v_{n-5}v_{n-4}, \\ & \text{or } v_{n-4}v_{n-3}, \\ & \text{or } v_{n-1}v_n; \\ -1, & \text{when } e = v_0v_1, \\ & \text{or } v_0v_4, \\ & \text{or } v_0v_n; \\ (-1)^{i+1}, & \text{when } e = v_0v_i,\ 10 \leq i \leq n-7; \\ +1, & \text{otherwise.} \end{cases}$$

It is easy to verify that $\sum_{e' \in N(e)} f(e') \geq 1$ for any $e \in E(G)$, we have $\sum_{e \in E(G)} f(e) = 2\lfloor \frac{n}{2} \rfloor - 3$.

When $n = 13$, let

$$f(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } v_6v_7, \\ & \text{or } v_7v_8, \\ & \text{or } v_{n-1}v_n; \\ -1, & \text{when } e = v_0v_1, \\ & \text{or } v_0v_4, \\ & \text{or } v_0v_n, \\ & \text{or } v_0v_{10}; \\ +1, & \text{otherwise.} \end{cases}$$

Clearly, f be a signed edge total domination function of $F_{1\cdot 13}$ such that $\gamma'_{st}(F_{1\cdot 13}) = \sum_{e \in E} f(e)$, we have $\gamma'_{st}(F_{1\cdot 13}) = 9$.

When $n \equiv 3(mod\ 4)$, let

$$f(e) = \begin{cases} -1, & \text{when } e = v_1v_2, \\ & \text{or } v_6v_7, \\ & \text{or } v_7v_8, \\ & \text{or } v_{n-1}v_n; \\ -1, & \text{when } e = v_0v_1, \\ & \text{or } v_0v_4, \\ & \text{or } v_0v_n; \\ (-1)^{i+1}, & \text{when } e = v_0v_i,\ 10 \leq i \leq n-3; \\ +1, & \text{otherwise.} \end{cases}$$

It is easy to verify that $\sum_{e' \in N(e)} f(e') \geq 1$ for any $e \in E(G)$, we have $\sum_{e \in E(G)} f(e) = 2\lfloor \frac{n}{2} \rfloor - 3$.

Combining the above four cases, we have completed the proof of **Theorem 1**.

Theorem 2. *For any positive integers $m \geq 3$ and $n \geq 2$, then*

$$\gamma'_{st}(n - C_m) = \begin{cases} 3, & \text{when } m = 3,\ n \geq 2 \\ n + 1, & \text{when } m = 4, n = 2 \text{ or } n = 3 \\ n - 1, & \text{when } m = 4,\ n \geq 4 \\ mn - n - 1, & \text{when } m \geq 5,\ n \geq 2 \end{cases}$$

Proof. Let $C_m^1, C_m^2, \cdots, C_m^n$ denote n cycle of graph $n - C_m$. Two vertices of common edge of C_m^i are denoted by v_0 and v_{m-1}, respectively. Other $m-2$ vertices of C_m^i are denoted by v_j^i for $j = 1, 2, \cdots, m-2$ and $i = 1, 2, \cdots, n$. For convenience, we put $v_0^1 = v_0^2 = \cdots = v_0^n = v_0$, $v_{m-1}^1 = v_{m-1}^2 = \cdots = v_{m-1}^n = v_{m-1}$, and take subscript j's modulo m. Obviously, $\mid E(n - C_m) \mid = (m-1)n + 1$, $E(n - C_m) = \{v_0 v_{m-1}\} \bigcup \{v_0 v_1^i \mid i = 1, 2, \cdots, n\} \bigcup \{v_{m-2}^i v_{m-1} \mid i = 1, 2, \cdots, n\} \bigcup \{v_j^i v_{j+1}^i \mid j = 1, 2, \cdots, m-3; i = 1, 2, \cdots, n\}$. Write $A = \{v_0 v_{m-1}\}$, $B = \{v_0 v_1^i \mid i = 1, 2, \cdots, n\}$, $C = \{v_{m-2}^i v_{m-1} \mid i = 1, 2, \cdots, n\}$, $D = \{v_j^i v_{j+1}^i \mid j = 1, 2, \cdots, m-3; i = 1, 2, \cdots, n\}$. Then $E(n - C_m) = A \bigcup B \bigcup C \bigcup D$. For convenience, an edge $e \in E(n - C_m)$ is said to be a $+1$ edge of $n - C_m$ if $f(e) = +1$, analogously, an edge $e \in E(n - C_m)$ is said to be a -1 edge of $n - C_m$ if $f(e) = -1$. Let s(or t) be the number of $+1$ edges(or -1 edges) of $n - C_m$. Thus, $(m-1)n + 1 = s + t$, $\gamma'_{st}(n - C_m) = s - t$.

Case 1. $m = 3$, $n \geq 2$. We firstly prove

$$\gamma'_{st}(n - C_3) \geq 3 \tag{1}$$

Suppose that (1) does not hold. Then $\gamma'_{st}(n - C_3) \leq 2$, thus, $t \geq n - \frac{1}{2}$. Notice that t is an integer, so $t \geq n$.

Case 1.1. If $f(v_0 v_{m-1}) = +1$, namely, all of -1 edges are in $B \bigcup C$. Notice that $\mid E(n - C_3) \mid = 2n + 1$, $\mid N(v_0 v_{m-1}) \mid = 2n$, $t \geq n$. Obviously, $\sum_{e' \in N(v_0 v_{m-1})} f(e') \leq 0$, which is a contradiction.

Case 1.2. If $f(v_0 v_{m-1}) = -1$, namely, the number of -1 edges in $B \bigcup C$ is at least $n - 1$.

Case 1.2.1. When the number of -1 edge in $B \bigcup C$ is at least n, which arises a contradiction the same as *Case 1.1.*

Case 1.2.2. When the number of -1 edges in $B\bigcup C$ is exactly $n-1$. Notice that $\mid B\mid=\mid C\mid=n$, by pigeonhole principle, the number of -1 edges either in B or in C is at least $\lceil\frac{n-1}{2}\rceil$. Suppose the number of -1 edges in B is at least $\lceil\frac{n-1}{2}\rceil$. Obviously, $\sum_{e'\in N(e)}f(e')\leq 0$ for any $+1$ edge e in B, which is a contradiction.

So we have $\gamma'_{st}(n-C_3)\geq 3$.

We prove $\gamma'_{st}(n-C_3)\leq 3$ by the following definition of the signed edge total domination function of $n-C_3$.

When $n\equiv 1(mod\ 2)$, let

$$f(e_i)=\begin{cases}+1, & \text{when } e_i=v_0v_{m-1} \text{ or } v_0v_1^1 \text{ or } v_{m-2}^1v_{m-1};\\(-1)^i, & \text{when } e_i=v_0v_1^i \text{ or } v_{m-2}^iv_{m-1},(i=2,3,\cdots,n).\end{cases}$$

When $n\equiv 0(mod\ 2)$, let

$$f(e_i)=\begin{cases}+1, & e_i=v_0v_1^1 \text{ or } v_{m-2}^1v_{m-1};\\-1, & e_i=v_0v_{m-1};\\(-1)^i, & e_i=v_0v_1^i \text{ or } v_{m-2}^iv_{m-1},(i=2,3,\cdots,n).\end{cases}$$

It is easy to verify that $\sum_{e'\in N(e)}f(e')\geq 1$ for any $e\in E(G)$, we have $\sum_{e\in E(G)}f(e)=3$, so $\gamma'_{st}(n-C_3)\leq 3$. Combining with (1), we have $\gamma'_{st}(n-C_3)=3$.

Case 2. $m=4$.

Case 2.1. $n=2$ either $n=3$. We firstly prove $\gamma'_{st}(n-C_4)\geq n+1$. Suppose that the inequality does not hold, then $\gamma'_{st}(n-C_4)\leq n$, so we have $t\geq n+\frac{1}{2}$. Notice that t is an integer, so $t\geq n+1$. Considering that $\sum_{e'\in N(e)}f(e)=f(v_0v_1^i)+f(v_{m-2}^iv_{m-1})\geq 1$ for any edge $e\in D$, so $f(v_0v_1^i)=f(v_{m-2}^iv_{m-1})=+1(i=1,2,\cdots,n)$. Thus the rest $n+1$ edges of $n-C_4$ must be -1 edges, obviously, $\sum_{e'\in N(v_0v_1^i)}f(e')\leq 0$, which is a contradiction. So we have $\gamma'_{st}(n-C_4)\geq n+1$.

We prove $\gamma'_{st}(G)\leq n+1$ by the following definition of the signed edge total domination function of $n-C_4$.

$$f(e_i)=\begin{cases}-1, & \text{when } e_i\in D;\\+1, & \text{otherwise.}\end{cases}$$

It is easy to verify that $\sum_{e'\in N(e)}f(e')\geq 1$ for any $e\in E(n-C_4)$, we have $\sum_{e\in N(e)}f(e)=n+1$, so $\gamma'_{st}(n-C_4)\leq n+1$, hence $\gamma'_{st}(n-C_4)=n+1$.

Case 2.2. $n \geq 4$. We firstly prove $\gamma'_{st}(n - C_4) \geq n - 1$. Suppose that the inequality does not hold, then $\gamma'_{st}(n - C_4) \leq n - 2$so we have $t \geq n + \frac{3}{2}$. Notice that t is an integer, so $t \geq n + 2$. Considering that $\sum_{e' \in N(e)} f(e') = f(v_0 v_1^i) + f(v_{m-2}^i v_{m-1}) \geq 1$ for any edge $e \in D$, so $f(v_0 v_1^i) = f(v_{m-2}^i v_{m-1}) = +1 (i = 1, 2, \cdots, n)$, thus the rest $n + 1$ edges of $n - C_4$ must be -1 edges, obviously, which contradicts $t \geq n + 2$. So we have $\gamma'_{st}(n - C_4) \geq n - 1$.

We prove $\gamma'_{st}(n - C_4) \leq n - 1$ by the following definition of the signed edge total domination function of $n - C_4$.

$$f(e_i) = \begin{cases} -1, & \text{when } e_i \in A \bigcup D; \\ +1, & \text{otherwise.} \end{cases}$$

It is easy to verify that $\sum_{e' \in N(e)} f(e') \geq 1$ for any $e \in E(n - C_4)$, we have $\sum_{e \in E(e)} f(e) = n - 1$. So $\gamma'_{st}(n - C_4) \leq n - 1$, hence $\gamma'_{st}(n - C_4) = n - 1$.

Case 3. We firstly prove $\gamma'_{st}(n - C_m) \geq mn - n - 1$. Suppose that the inequality does not hold. Then $\gamma'_{st}(n - C_m) \leq mn - n - 2$, so we have $t \geq \frac{3}{2}$. Notice that t is an integer, so $t \geq 2$. Considering that $\sum_{e' \in N(e)} f(e') \geq 1$ for any edge $e \in D$, so $f(e) = +1$ for any edge $e \in E(n - C_m) \setminus A$, then the only edge $e' = v_0 v_{m-1}$ is -1 edge, which contradicts $t \geq 2$, so we have $\gamma'_{st}(n - C_m) \geq mn - n - 1$.

We prove $\gamma'_{st}(G) \leq mn - n - 1$ by the following definition of the signed edge total domination function of $n - C_m$

$$f(e_i) = \begin{cases} -1, & \text{when } e_i \in A; \\ +1, & \text{otherwise .} \end{cases}$$

It is easy to verify that $\sum_{e' \in N(e)} f(e') \geq 1$ for any $e \in E(n - C_m)$, we have $\sum_{e \in E(n - C_m)} f(e) = mn - n - 1$. So $\gamma'_{st}(n - C_m) = mn - n - 1$.

Combining the above three cases, we have completed the proof of **Theorem 2**.

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