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A SIMPLE HEURISTIC FOR SOLVING GENERALIZED FUZZY TRANSPORTATION PROBLEMS

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Abstract: In this paper, a new method is proposed for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. In the proposed method transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed method a numerical example is solved and the obtained result is compared with the results of existing approaches.

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1. Introduction

In today's highly competitive market, the pressure on organizations is to find

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better ways to create and deliver value added service to the customers in order to become stronger. How and when to send the products to the customers in quantities in a cost-effective manner becomes more challenging. Transportation models provide a powerful framework to meet the challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

The basic transportation problem was originally developed by Hitchcock [12]. The transportation problems can be modeled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information (Variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [3] developed a stepping stone method which provides an alternative way of determining the simplex method information. Dantzig and Thapa[7] used simplex method to the transportation problem as the primal simplex transportation method. An Initial Basic Feasible Solution (IBFS) for the transportation problem can be obtained by using the North-West Corner rule, Row Minima, Column Minima, Matrix Minima, or Vogel's Approximation Method. The Modified Distribution Method is useful for finding the optimal solution of the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution.

Zimmermann [16] showed that solutions obtained by fuzzy linear programming method and are always efficient. Subsequently, Zimmermann's fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas and Kuchta [2] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution.

In this paper, a new method is proposed for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation cost only but there is no uncertainty about the supply and demand of the product. In the proposed algorithm transportation costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed algorithm a numerical example is solved and the obtained result is compared with the results of existing approaches. The proposed method is easy to understand and to apply in real life transportation problems for the decision makers.

2. Preliminaries

In this section, some basic definitions, arithmetic operations and an existing method for comparing generalized fuzzy numbers are presented.

Definition 2.1. A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \le x < b \\ 1 \quad , & b \le x \le c \\ \frac{(x-d)}{(c-d)}, & c < x \le d \\ 0 & otherwise. \end{cases}$$

Definition 2.2. A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\Re$, is said to be generalized fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}} : \Re \to [0, \omega]$ is continuous.

(ii)$\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii)$\mu_{\tilde{A}}(x)$ Strictly increasing on [a , b] and strictly decreasing on [c , d].

(iv)$\mu_{\tilde{A}}(x) = \omega$, for all $x \in [b, c]$, where $0 < \omega \le 1$.

Definition 2.3. A fuzzy number $\tilde{A} = (a, b, c, d; \omega)$ is said to be generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega\frac{(x-a)}{(b-a)}, & a \le x < b \\ \omega, & b \le x \le c \\ \omega\frac{(x-d)}{(c-d)}, & c < x \le d \\ 0 & otherwise. \end{cases}$$

Definition 2.4. In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers $\Re$, are presented [10].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2)$ be two generalized trapezoidal fuzzy number then:

(i) $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min(\omega_1, \omega_2))$.

(ii) $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \min \omega_1, \omega_2))$.

(iii) $\tilde{A}_1 \otimes \tilde{A}_2 \cong (a, b, c, d; \min(\omega_1, \omega_2))$, where:

$$a = \min(\mathrm{a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2}), \quad b = \min(b_1 \mathrm{b_2, b_1 c_2, c_1 b_2, c_1 c_2},$$

$$c = \max(b_1 b_2 b_1 c_2, c_1 b_2, c_1 c_2), \quad d = \max(a_1 a_2, a_1 d_2, a_2 d_1, d_1 d_2).$$

(iv) $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega_1) & \lambda < 0. \end{cases}$

Ranking Function. An efficient approach for comparing the fuzzy numbers is by the use of ranking function,$\Re : F(\Re) \to \Re$, where $F(\Re)$ is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.:

(i)$\tilde{A} >_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) > \Re(\tilde{B})$;

(ii)$\tilde{A} <_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) < \Re(\tilde{B})$;

(iii)$\tilde{A} =_{\Re} \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$.

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; \omega_2)$ be two generalized trapezoidal fuzzy numbers and $\omega = \min(\omega_1, \omega_2)$ then $\Re(\tilde{A}) = \frac{\omega(a_1+b_1+c_1+d_1)}{4}$ and $\Re(\tilde{B}) = \frac{\omega(a_2+b_2+c_2+d_2)}{4}$.

3. Fuzzy Transportation Problem

The fuzzy transportation problems, in which a decision maker is uncertain about the precise values of transportation cost from ith source to the jth destination, but sure about the supply and demand of the product, can be formulated as follows:

Minimize $\tilde{x}_0$
Subject to
$\sum_{j=1}^{n} x_{ij} \leq a_i, \qquad i = 1, 2, 3.......m$
$\sum_{i=1}^{m} x_{ij} \geq b_j, \qquad j = 1, 2, 3......n$
$x_{ij} \geq 0, \qquad \forall\, i, j$

If $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ then the FTP is said to be balanced FTP, otherwise it is called an unbalanced FTP.

4. Proposed Method

In this section, a new method is proposed for finding a fuzzy optimal solution using ranking function, in which transportation costs are represented as generalized trapezoidal fuzzy numbers instead of normal fuzzy trapezoidal numbers.

Step 1. Initialization
Construct the fuzzy transportation table for the given fuzzy transportation problem and then, convert it into a balanced one, if it is not.

Step 2. Develop the cost table
(a) Subtract each row entries of the fuzzy transportation table from each row minimum.
(b) Subtract each column entries of the resulting fuzzy transportation table after using the step 2(a). From the column minimum.

Step 3. Find the opportunity cost table
Now there will be at least one fuzzy zero in each row and in each column in the reduced fuzzy cost matrix. Select the first fuzzy zero cell (row-wise) occurring in the fuzzy cost matrix. Suppose (i,j) th fuzzy zero cell is selected, count the total number of fuzzy zeros (excluding the selected one) in the i th row and j th column. Now select the next fuzzy zero cell and count the total number of fuzzy zeros in the corresponding row and column in the same manner. Continue it for all fuzzy zero cell in the fuzzy cost matrix.

Step 4. Optimality criterion
Now choose a fuzzy zero cell for which the number of fuzzy zeros counted in step 3 is minimum and supply maximum possible amount to that cell. If tie occurs for some fuzzy cost zeros in step 3 then choose a (k,l)th fuzzy cost zero breaking tie such that the fuzzy total sum of all the elements in the kth row and lth column is maximum. Allocate maximum possible amount to that cell.

Step 5. Revise the opportunity cost table
After performing step 4, delete the row or column for further calculation where the supply from a given source is depleted or the demand for a given destination is satisfied.

Step 6. Develop the new revised opportunity cost table
Check whether the resultant matrix possesses at least one fuzzy cost zero cell

	D1	D2	D3	Availability(ai)
S1	(1,4,9,19;.5)	(1,2,5,9;.4)	(2,5,8,18;.5)	10
S2	(8,9,12,26;.5)	(3,5,8,12;.2)	(7,9,13,28;.4)	14
S3	(11,12,20,27;.5)	(0,5,10,15;.8)	(4,5,8,11;.6)	15
Demand(bj)	15	14	10	

Table 1

in each row and in each column. If not, repeat step 2, otherwise go to step 7.

Step 7. Determination of cell for allocation
Repeat step 3 to step 6 until and unless all the demands are satisfied and all the supplies are Exhausted.

5. Numerical Example

To illustrate the proposed method, the following Fuzzy Transportation Problem is solved.

Problem 5.1. Table1 gives the availability of the product available at three sources and their demand at three destinations, and the approximate cost for transporting one unit quantity of product from each source to each destination is represented by generalized trapezoidal fuzzy number. Determine the fuzzy optimal transportation of products such that the total fuzzy transportation cost is minimum.

Since $\sum_{i=1}^{3} a_i = \sum_{j=1}^{3} b_j = 39$, so the chosen problem is a balanced FTP.

By applying the above proposed method allocations for the given fuzzy transportation problem are obtained as follows

The minimum fuzzy transportation cost is

$$10(1,4,9,19;.5) + 5(8,9,12,26;.5) + 9(3,5,8,12;.2) + 5(0,5,10,15;.8) + 10(4,5,8,11;.6) = (117,205,352,613;.2).$$

Results with Normalization Process 5.2. If all the values of the parameters used in problem.1 are first normalized and then the problem is

	D1	D2	D3	Availability(ai)
S1	**10** (1,4,9,19;.5)	(1,2,5,9;.4)	(2,5,8,18;.5)	10
S2	**5**(8,9,12,26;.5)	**9**(3,5,8,12;.2)	(7,9,13,28;.4)	14
S3	(11,12,20,27;.5)	**5**(0,5,10,15;.8)	**10**(4,5,8,11;.6)	15
Demand(bj)	15	14	10	

Table 2

solved by using the proposed approach then the fuzzy optimal value is $\tilde{x}_0 =$ (117,205,352,613; 1).

Results without Normalization Process 5.3. If the values of the parameters of the same problem.1 is not normalized and then the problem is solved by using the proposed approach then the fuzzy optimal value is $\tilde{x}_0 =$ (117,205,352,613; .2).

Remark 5.4. Results with normalization process represents that the overall level of satisfaction of decision maker about the statement that minimum transportation cost will lie between 205 and 352 units as 100% while without normalization process the overall level of satisfaction of the decision maker for the same range is 20%. Hence, it is better to use generalized fuzzy numbers instead of normal fuzzy numbers, obtained by using normalization process.

Problem 5.5.

6. Results and discussion

From the investigations and the results given above, it is clear that the proposed method is better than GFNWCM, GFLCM and GFVAM for solving fuzzy transportation problem and also, the solution of the fuzzy transportation problem given by the proposed method is an optimal solution. The advantage of this method is that it requires very simple arithmetical and logical calculation and is very easy to understand. This method is very useful for the decision makers to take a decision during uncertainty.

	D1	D2	D3	D4	Availa-bility(ai)
S1	(1,4,8,11;.6)	(0,1,2,3;.4)	(3,7,10,16;.5)	(1,2,4,5;.5)	70
S2	(4,10,12,18;.3)	(1,3,6,10;.2)	(0,1,3,4;.5)	(3,5,9,15;.2)	55
S3	(4,9,12,15;.2)	(6,10,14,18;.6)	(2,3,5,6;.2)	(4,6,8,10;.2)	90
Demand bj	85	35	50	45	

Table 3

Pr.	GFNWCM	GFLCM	GFVAM	PROPOSED METHOD	Optimal
1	(117,205,352,613;.2)	(197,240,382,643;2)	(147,220,382,603;.2)	(117,205,352,613;.2)	(117,205,352,613;.2)
2	(435,945,1550,2130;.2)	(515,1105,1560,2150;2)	(405,945,1520,1930;.2)	(405,930,1460,1905;.2)	(405,930,1460,1905;.2)

Table 4

7. Conclusion

This new method proposed, provides an optimal fuzzy solution for the fuzzy transportation problems, with an easy to understand step by step technique. The advantage of the proposed method is discussed and a numerical example is solved to illustrate the new methodology. So we can apply this technique for solving the fuzzy transportation problems occurring in real life situations.

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