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The article about a choice of an optimum import tariff, taking into account interests of the importer and the consuming state was published in this journal in 2013 [1]. Unfortunately, it is impossible to recognize this work as original and moreover, it is direct copying. The history of emergence of this work dates back 1997 when I for the first time became interested in optimization of the import duties. I prepared and published result of research in the "Economics and Mathematical Methods" journal in Russian [2]. In 2002-2003 having provided the main result with examples from economy of Russia, I translated the work into English to publish it in an electronic journal. At this time, having got a grant according to Fulbright's program, I appeared in Merrick School of Business of University of Baltimore where I met Professor H. Arsham. Naturally, we discussed the most of my works in English. Further, the text of a draft copy in English which was available to Professor H. Arsham is provided in small print.

Introduction

One of the brightest works on a history of protectionism belongs to Russian great compatriot Dmitri Ivanovitch Mendeleyev " The explanatory tariff or research on development of industry in Russia in connection with its general customs tariff of 1891 " [1]. It gives the analysis of reasons, why it is impossible to follow blindly the theorists of free trade. Though the ideas of D.Mendeleyev are poorly grounded theoretically and are poorly formalized, he formulated the tasks of protectionism, i.e. he determined, who should receive and how much and where these funds should be directed. From the same positions the analysis of protectionism in the present note is conducted. But here the formalized quantitative expressions are given as well. As a result both the economic criteria of imposing the import tariffs can be obtained and the optimal value of a tariff can be chosen. The attempt is undertaken to consider the dynamic model of protectionism taking into account the time-dependent factors in international trade. Inclusion in the analysis of such parameter as smuggling is caused by a simple observation, that if the barrier is fixed too high, the importers will find a pass through the border for smaller money. This idea was also spoken out by Mendeleyev.

Mathematical formalization

Let's consider the interrelations between the size of import of the given goods, the volume of domestic production, the difference between the customs duty and VAT for the domestic manufacturer, the volume of smuggling of the given goods and efficiency of the customs control. The mathematical model is constructed on the elementary assumptions which admit a clear economic interpretation.

Let's assume, that some commodity (or the service) on a domestic market is sold for the price p, and all volume of sales consists of three parts: x – home-made, y – import and z – smuggling. The values are measured in terms of a unit of the commodity; the price p and a price q of unit of the product in the country, where the import is produced and the smuggling exists is measured in conditional units (for example, dollar). We will designate the import tariff as τ, and the VAT as – s.

According to the introduced designations, the following amount of money flows to the budget of the country from the customs and the tax committees

$$S = \tau q y + s x p + \alpha z p, \tag{1}$$

where the latter term reflects the realization of αz units of smuggling at the price p inside the country (α – a coefficient of the customs house effectiveness).

The basic assumption. Let's consider, that the domestic consumers pay the constant sum M for the given commodity, and it is not substituted. Thus, the curve of demand has the form:

$$p(x + y + z) = M. \tag{2}$$

Let's consider also that the import is determined by the benefit of the importer who is capable to change the value y depending on conditions of the domestic market. The profit of the importer will be calculated by the formula:

$$D = y(p - q - \tau q). \tag{3}$$

The profit is a driving force of smugglers. So

$$G = z[(1 - \alpha)p - q]. \tag{4}$$

The motives of changes of volume of the domestic manufacture can be various: the creation of jobs, the formation of the export potential of the country, strengthening of manufacture connected with national security, etc. In each case the law of change of the value x depending on conditions of the market will act. Now we will consider elementary problems with the help of which it is possible to do a small analytical research.

Problem 1. Let's assume that $x = \text{const}$, $\quad z = 0$, i.e. there is no smuggling, and domestic manufacture is a constant value. Then, at fixed $\tau \geq 0$ the following relations will exist:

$$p_\tau(x + y_\tau) = M, \quad D_\tau = y_\tau \left[\frac{M}{x + y_\tau} - q(1 + \tau) \right], \quad S_\tau = \tau q y_\tau + s x p_\tau.$$

Let's assume that the importer implements the best strategy for him, i.e. $y_\tau = \arg\max_{y_\tau} D_\tau(y_\tau)$. We will obtain the optimum strategy from a condition: $\frac{d}{dy_\tau} D_\tau = 0$. This equation has the solution:

$$y_\tau = \sqrt{\frac{Mx}{q(1 + \tau)}} - x.$$

It is obvious, that $y_\tau > 0$, while $\tau < \frac{M}{qx} - 1$. The price on the home market under this condition is calculated by the formula:

$$p_\tau = \sqrt{\frac{Mq(1 + \tau)}{x}}.$$

Let's substitute the optimum y_τ, p_τ in the formula for calculating the budget profits:

$$S_\tau = sx\sqrt{\frac{Mq(1 + \tau)}{x}} + \tau q \left(\sqrt{\frac{Mx}{q(1 + \tau)}} - x \right).$$

Let's assume that the state establishes a level of the customs duty basing on the condition of the maximal deductions to the budget from the tax and the customs service, i.e. from achievement of the highest value of the function S_τ. Let's simplify the latter function with replacement by $\mu = \sqrt{1 + \tau}$:

$$S_\mu = q\left(\mu^2 - 1\right) \left(\sqrt{\frac{xM}{q}} \frac{1}{\mu} - x \right) + s\sqrt{xMq}\mu \tag{5}$$

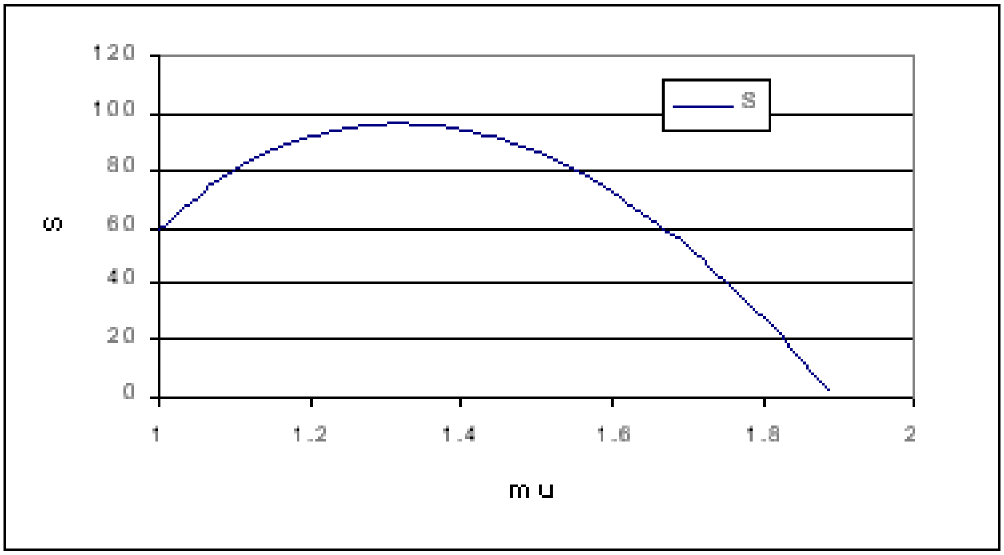

Figure 1: A sample kind of dependence of budget profits on the customs tariff parameter.

The diagram of dynamics of budget profits S_μ has approximately the following form:

It is obvious, that it is interesting, taking into account the restrictions on τ to obtain the highest value of S_μ at $1 < \mu < \sqrt{\frac{M}{qx}}$. Let $m = \sqrt{\frac{M}{qx}}$ then the highest value $S_\mu$is determined by the root of the equation:

$$\mu^3 - \frac{m(1+s)}{2}\mu^2 - \frac{m}{2} = 0.$$

In the given limits the root exists and it is the unique root of the equation. It is possible to be convinced graphically. The root can be calculated by the well-known formula of Cortano:

$$\mu = \frac{m(s+1)}{6} + \sqrt[3]{\frac{m^3(1+s)^3}{216} + \frac{m}{4} + \frac{m}{4}\sqrt{1 + \frac{m^2(1+s)^3}{27}}} +$$

$$\sqrt[3]{\frac{m^3(1+s)^3}{216} + \frac{m}{4} - \frac{m}{4}\sqrt{1 + \frac{m^2(1+s)^3}{27}}}.$$

Problem 2. Let's assume that the industry of the country responds to growth of the price by expansion of production. And the expansion of manufacture, i.e. increase of x , can be carried out only at the expense of the funds received from the sale of an additionally manufactured product at the prices, which increased because of the customs duty. The equation of change will have the form:

$$\dot{x}(t) = cx(t)[p_\tau(t) - p^0], \tag{6}$$

where p_τ - the price which was established after introduction of the duty, p^0 - the price before introduction of the duty (it reflects the cost price because if x units of product were on sale at that price, so P^0 would be higher than the cost price by the value of trade, transport, tax and other costs). The expansion of manufacture (increase of x) will be possible, if $p_\tau > p^0$ according to the equation (6). Here c means a share of the income going to expansion of manufacture. From (6) it is seen, that its growth will be stopped, when $p_\tau = p^0$, i.e. with increase of x the price determined by the function of demand (2), will fall down to p^0, if the importer does not change his behaviour. Let's assume that the importer continues to import y units of a product, and the manufacturer, having received the credit, managed to increase the manufacture up to $\hat{x}$, so that:

$$p^0(y_\tau + \hat{x}) = M. \tag{7}$$

A more rigorous approach requires to take into account the time of repayment of credit and interest, or it is necessary to assume, that the necessary sum is received as the state grant. The specified assumptions allow to determine the profit of the state from the following expression:

$$S = \tau q y_\tau + sp^0 \left(\frac{M}{p^0} - y_\tau \right).$$

Let's show that the profit of the state is higher, if $y_\tau < y$, i.e. $\hat{x} > x$, so:

$$\Delta S = (\tau q y_\tau + sxp_\tau) - \left(\tau q y_\tau + sp^0 \left(\frac{M}{p^0} - y_\tau \right) \right) = s \left(\frac{xM}{x + y_\tau} - M + p^0 y_\tau \right) =$$

$$= \frac{s y_\tau}{x + y_\tau} (y_\tau - y) < 0.$$

Problem 3. Let's assume that there is a non zero smuggling. It, obviously, can be only when there exists a customs barrier. Otherwise there is no sense to risk. For the sake of simplification we will assume, that $x = \text{const}$. If α is a parameter of efficiency of the customs house, the customs will confiscated αz units of z ones of the given product intended for smuggling, and $(1 - \alpha)z$ find their way to the home market. Let's consider that the relationships are valid:

$$p^0(y^0 + x^0) = M, \quad p_\tau(y_\tau + z_\tau + x^0) = M,$$

where y_τ - the optimal strategy of the importer. Then the profit of the importer is equal to:

$$D = y_\tau \left[\frac{M}{y_\tau + z_\tau + x^0} - q(1 + \tau) \right],$$

and that of a smuggler:

$$G = z_\tau \left[(1 - \alpha) \frac{M}{y_\tau + z_\tau + x^0} - q \right].$$

From the above mentioned three conclusions follow:
1) if $(1 - \alpha)p_\tau < q$, then $z_\tau = 0$, i.e. the smuggling is absolutely unprofitable irrespective of existence of import;
2) If $\alpha p_\tau > q\tau$, then $z_\tau = 0$, i.e. the smuggling is unprofitable in comparison to legal import;
3) If $\alpha p_\tau \leq q\tau$, the smuggling will be significant (though, apparently, there will be also honest importers).

The last formulae allow to solve the problems of optimization of a level of the customs duties in different formulations: with one, two or even with three goal functions. For example, the optimal value of import $y_\tau = \sqrt{\frac{M(x^0 + z_\tau)}{q(1+\tau)}} - x^0 - z_\tau$, as before, is obtained from the condition $\frac{d}{dy} D = 0$. Then the state will receive:

$$S = \tau q \left[\sqrt{\frac{M(x^0 + z_\tau)}{q(1 + \tau)}} - x^0 - z_\tau \right] + \sqrt{\frac{Mq(1 + \tau)}{x^0 + z_\tau}} \left(sx^0 + \alpha z_\tau \right).$$

It is possible to take advantage of this formula for calculation z_τ, α, by the known S. If z_τ, α are given, it is possible to calculate the optimal τ, s.

Two examples

Let's consider a rather simple example - the manufacture and the trade with TV sets in Russia, which, however, can turn to bulky research at the detailed analysis. All assumptions given below are simplified. Now about 40 million families live in Russia, of which almost half is underpaid, i.e. they don't dream of purchasing a new TV set at all. The remaining potential buyers could buy 20 millions TV sets. Let's assume that for various reasons the TV set is bought once in 10 years. Then we will obtain a rough annual volume of purchase. Certainly, there are many various models which have different characteristics, but, for the sake of simplicity, we will assume that in Russia half of sales are domestic models (including the Byelorussian), and half are import ones. Before imposing the import duties the price of one averaged TV set is \$220. The estimation of a wholesale price of imported TV sets in the countries of the manufacturer is complicated. Therefore we will consider the price of \$200 as an almost real (transportation included). Then $M = 440$ [million of pieces x dollars]. Let's calculate the amount of the tariff using the algorithm mentioned above. It is determined by the extremum of the function:

$$S_\mu = 200\left(1 - \mu^2 - \frac{\sqrt{2.2}}{\mu} - 1.2\mu\sqrt{2.2}\right),$$

which is reached at $\mu \approx 1.32$, which corresponds to the tariff $\tau \approx 0.73$. And the retail price, for the particular model, will rise up to \$390. The import will fall down to 0.128 [millions pieces]. Thus the tax revenue of the state will increase from 44 up to 96.7 million dollars. The Russian manufacturers of TV sets will receive 171 million dollars more as a net profit per one year. This money can be invested in construction of new factories or production of the up-to-date models of TV sets. The consumer will suffer huge losses: almost 874 thousand of Russian families will refuse to buy a new TV set within one year. But these are negative consequences of any form of protectionism.

We will give one more example from the tourist business. The goods is a tourist package called conditionally as "Rest at the sea for two weeks". Let's consider those resorts, the rest on which is considered as the same. For example, the Russian resorts and the resorts of Bulgaria. According to the basic assumption, the population pays for the given product the sum M, which, in this case, is calculated by the formula: $M = px + qy$, where:

q - the price of the package in Bulgaria;

y - an annual number of the tourists in Bulgaria;

p - the price of the package for the Russian resorts;

x - an annual number of the tourists on the Russian resorts.

The partial inspection of travel companies of St.-Petersburg in 1997-1998 years showed, that $q = \$450,\ y = 3370000,\ p = \$710,\ x = 678000$. Thus $M = 1997880000$.

We will calculate the tariff by the algorithm mentioned above. Namely, it gives an extremum of the function $S_\mu = q\left(\mu^2 - 1\right)\left(\sqrt{\frac{xM}{q}}\frac{1}{\mu} - x\right) + s\sqrt{xMq}\mu$. And $s = 0$, as the tourist packages are not levied with VAT.

$$S_\mu = 450\left(\mu^2 - 1\right)\left(\sqrt{\frac{678000 \cdot 1997880000}{450}}\frac{1}{\mu} - 678000\right) =$$

$$= 45000\left(\sqrt{301013920}\mu - 6780\mu^2 - \frac{\sqrt{301013920}}{\mu} + 6780\right).$$

The highest value S_μ is defined by the root of equation:

$$\mu^3 - \frac{m}{2}\mu - \frac{m}{2} = 0,$$

where $m = \sqrt{\frac{M}{xq}} = \sqrt{\frac{1997880000}{450 \cdot 678000}} \approx 2.56$. Thus it is necessary to solve the equation:

$$\mu^3 - 1.28\mu^2 - 1.28 = 0.$$

We will calculate the root of this equation by the Cortano formula mentioned above:

$$\mu = \frac{m}{6} + \sqrt[3]{\frac{m^3}{216} + \frac{m}{4} + \frac{m}{4}\sqrt{1 + \frac{m^2}{27}}} + \sqrt[3]{\frac{m^3}{216} + \frac{m}{4} - \frac{m}{4}\sqrt{1 + \frac{m^2}{27}}} \approx 1.7;$$

Since $\mu = \sqrt{1+\tau}$ the optimal tariff is equal to 1.89.

The calculations showed that for the state it is necessary to introduce the customs tariff from only the fiscal reasons for the package tours to Bulgaria fixing a price of \$2000. Obviously, for this money the Russian citizens won't go neither to Bulgaria, nor to the resorts but will prefer Turkey, Spain, Egypt and other resort zones of the world. I.e. at increase of the customs tariff there will be substitutions of the goods by other ones which are more expensive before the introduction of the duty. From this negative experience of using the algorithm it is obvious, that it is necessary to supplement it by assumptions about substitutions of the goods by others.

Conclusions

The models mentioned above show us how the authorities could calculate the optimal import tariff if some protection policy is accepted. The models allow to identify a customs quality or smuggling in the country.

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Full similarity of the main content of work [1] with the given above draft copy is obvious. The introduction, the conclusion and references are made by H. Arsham independently. Unfortunately Professor H. Arsham made some mistakes in this publication: 1) on the page 392 the letter "e" missed in 7th line from bottom; 2) the figure 1 "floated away"; 3) on the page 393 the big formula is missed before "2.3 Case II"; 4) in formula (6) one x(t) is extra. And so on. I would like to note that this result was published in the manual [3] in Russian in 2000 and in the book [4] in 2008. In 2011 I returned to this task again. The creation of optimum set according to Pareto for negotiating on optimization of the import duty [5] was a research objective now. This new result was reported at the international conference [6] and was published with some changes in [7] in Russian. It should be noted that changes which more correspond to the legislation on foreign trade have been made for the model since 2011.

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