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ON FUZZY ALMOST CONTRA γ-CONTINUOUS FUNCTIONS

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Abstract: Joseph and Kwack (see [11]) introduced the notion of (θ, s)-continuous functions in order to investigate S-closed spaces. The aim of this paper is to introduce fuzzy almost contra γ-continuous functions related to S-closed spaces and to investigate some properties of such fuzzy functions.

AMS Subject Classification: 57A40, 57C08
Key Words: fuzzy γ-open set, fuzzy γ-closed set, fuzzy γ-continuity, fuzzy almost contra γ-continuity, fuzzy weakly almost contra γ-continuity, fuzzy strong normal space

1. Introduction

Joseph and Kwack (see [11]) introduced (θ, s)-continuous functions in order to investigate S-closed due to Thompson [23]. A function f is called (θ, s)-continuous if the inverse image of each regular open set is closed. Moreover,

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Chang in [3] introduced fuzzy S-closed spaces in 1968. Fuzzy continuity is one of the main topics in fuzzy topology. Various authors introduce various types of fuzzy continuity. One of them is fuzzy γ-continuity. In 1999, Hanafy in [9] introduced the concept of fuzzy γ-continuity.

The purpose of this paper is to introduce fuzzy almost α-continuous function and to investigate some of its properties. Using these properties of fuzzy almost contra continuous functions, properties of fuzzy almost contra α-continuous functions, fuzzy almost contra precontinuous functions, fuzzy almost contra β-continuous and fuzzy almost contra semicontinuous functions are obtained.

2. Preliminaries

In the present paper, X and Y are always fuzzy topological spaces. The class of fuzzy sets on a universal set X will be denoted by I^X and fuzzy sets on X will be denoted by Greek letters as μ, ρ, η, etc. A family τ of fuzzy sets in X is called a fuzzy topology for X if

(1) $0, 1 \in \tau$,

(2) $\mu \wedge \rho \in \tau$, whenever $\mu, \rho \in \tau$ and

(3) $\vee\{\mu_\alpha : \alpha \in I\} \in \tau$, whenever each $\mu_\alpha \in \tau(\alpha \in I)$.

Moreover, the pair (X, τ) is called a fuzzy topological space. Every member of τ is called a fuzzy open set. The complement of a fuzzy open set is fuzzy closed.

Let μ be a fuzzy set in X. We denote the complement, the interior and the closure of μ by $1 - \mu$ or $\mu^1, int(\mu)$ and $\mathrm{cl}(\mu)$, respectively. A fuzzy set in X is called a fuzzy point if and only if it takes the value 0 for all $y \in X$ except one, say, $x \in X$. If its value at x is $\alpha(0 < \alpha \leq 1)$ we denote this fuzzy point by x_α where the point x is called its support, see [16]. For any fuzzy point $x_\in$ and any fuzzy set μ, we write $x_\epsilon \in \mu$ if and only if $\epsilon \leq \mu(x)$. Two fuzzy sets λ and β are said to be quasi-coincident (q-coincident, shortly), denoted by $\lambda q \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ (see [16]) and by $\overline{q}$ we denote "is not q-coincident". It is known (see [16]) that $\lambda \leq \beta$ if and only if $\lambda \overline{q}(1 - \beta)$.

Definition 2.1. A fuzzy set μ in a space X is called

(1) fuzzy β-open [12] if $\mu \leq cl(int(cl(\mu)))$;

(2) fuzzy semi-open [1] if $\mu \leq cl(int(\mu))$;

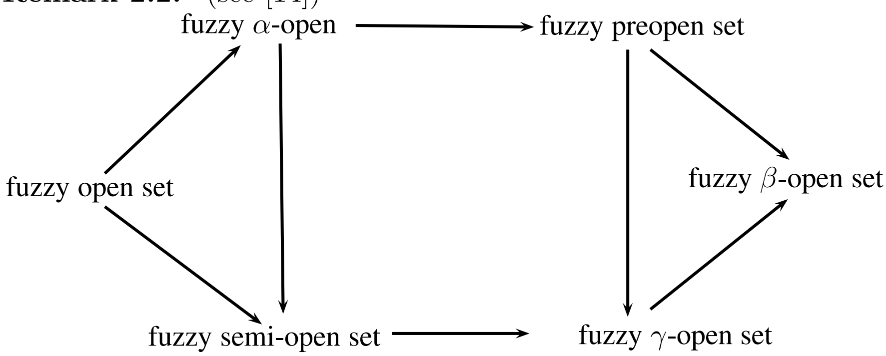
(3) fuzzy α-open [2] if $\mu \leq int(cl(int(\mu)))$;

(4) fuzzy preopen [1] if $\mu \leq int(cl(\mu))$;

(5) fuzzy γ-open [9] if $\mu \leq int(cl(\mu)) \vee cl(int(\mu))$.

The complements of the above mentioned open sets are called their respective closed sets.

Remark 2.2. (see [14])



None of the above implifications is reversible.

Definition 2.3. [8] A space X is said to be fuzzy extremly disconnected if the closure of every fuzzy open set of X is fuzzy open in X.

Definition 2.4. [1] Let (X, τ) be a fuzzy topological space. A fuzzy set μ of X is called

(1) fuzzy regular open if $\mu = int(cl(\mu))$;

(2) fuzzy regular closed if $\mu = cl(int(\mu))$.

The complement of fuzzy regular open set is fuzzy regular closed.
The collection of all fuzzy regular closed sets of X is denoted by FRC(X).

Definition 2.5. [17] A subset ρ in a space X is said to be a fuzzy locally closed (briefly, a fuzzy LC) set if $\rho = \alpha \wedge \beta$, where α is a fuzzy open set and β is a fuzzy closed set.

Theorem 2.6. *[20] Let X be a fuzzy extremely disconnected space and $\mu \leq X$, the following properties are equivalent.*

(1) μ is a fuzzy open set.

(2) μ is fuzzy α-open and a fuzzy LC set,

(3) μ *is fuzzy preopen and a fuzzy LC set.*

(4) μ *is fuzzy semi-open and a fuzzy LC set.*

(5) μ *is fuzzy* γ*-open and a fuzzy LC set.*

Let μ *be a fuzzy set in a fuzzy topological space* X*. The fuzzy* γ*-closure and fuzzy* γ*-interior of* μ *are defined as* $\wedge\{\rho : \mu \leq \rho, \rho$ *is fuzzy* γ*-closed*$\}$, $\vee\{\rho : \mu \geq \rho, \rho$ *is fuzzy* γ*-open*$\}$ *and denoted by* γ*-cl*(μ) *and* γ*-int*(μ)*, respectively.*

A fuzzy set μ *is quasi-coincident with a fuzzy set* ν*, denoted by* $\mu q \nu$*, if there exists* $x \in X$ *such that* $\mu(x) + \nu(x) > 1$*. If* μ *is not quasi-coincident with* ν*, then we write* $\mu \overline{q} \nu$*. It is known that* $\mu \leq \nu$ *iff* $\mu \overline{q} 1 - \nu$*.*

Lemma 2.7. *[10] Let* A *and* B *be fuzzy sets in a fuzzy topological space* (X, τ)*. Then*

(1) if $A \cap B = 0_X$*, then* $A \overline{q} B$*,*

(2) $A \leq B$ *iff* $x_r q B$ *for each* $x_r q A$*,*

(3) $A \overline{q} B$ *iff* $A \leq B^1$*.*

(4) $x_r(\cup A_\alpha)(\alpha \in \Lambda)$ *iff there is* $\alpha_0 \in \Lambda$ *such that* $x_r q A_{\alpha_0}$*.*

Definition 2.8. [3] Let $f : (X, \tau) \to (Y, \rho)$ be a function. Let A be a fuzzy subset in X and B be a fuzzy subset in Y. Then the Zadeh's functions $f(A)$ and $f^{-1}(B)$ are defined by

(1) $f(A)$ is a fuzzy subset of Y where

$$f(A) = \begin{cases} \sup\limits_{z \in f^{(-1)}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

for each $y \in Y$.

(2) $f^{-1}(B)$ is a fuzzy subset of X where $f^{-1}(B)(x) = B(f(x))$, for each $x \in X$.

Lemma 2.9. *[3] Let* $f : (X, \tau) \to (Y, \rho)$ *be a function. For fuzzy sets* A *and* B *of* X *and* Y *respectively, the following statements hold:*

(1) $ff^{-1}(B) \leq B$*;*

(2) $f^{-1}f(A) \geq A$*;*

(3) $f(A^1) \geq (f(A))^1$*;*

(4) $f^{-1}(B^1) = (f^{-1}(B))^1$*;*

(5) if f is injective, then $f^{-1}(f(A)) = A$;

(6) if f is surjective, then $ff^{-1}(B) = B$;

(7) if f is bijective, then $f(A^1) = (f(A))^1$.

Definition 2.10. [3] Let $f : (X, \tau) \to (Y, \rho)$ be a function. Then f is said to be

(1) fuzzy open if the image of every fuzzy open set of X is fuzzy open in Y.

(2) fuzzy closed if the image of every fuzzy closed set ofX is fuzzy closed in Y.

(3) fuzzy continuous if the inverse image of every fuzzy open set of Y is fuzzy open in X.

Let $f : X \to Y$ be a fuzzy function from a fuzzy topological space X to a fuzzy topological space Y. Then the function $g : X \to X \times Y$ defined by $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ is called the fuzzy graph function of f, see [1].

Recall that for a fuzzy function $f : X \to Y$, the subset $\{(x_\epsilon, f(x_\epsilon)) : x_\epsilon \in X\} \leq X \times Y$ is called the fuzzy graph of f and is denoted by $G(f)$.

3. Fuzzy Almost Contra γ-Continuous Functions

In this section, the notion of fuzzy almost contra γ-continuous functions is introduced.

Definition 3.1. Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be fuzzy almost contra γ-continuous if inverse image of each fuzzy regular open set in Y is fuzzy γ-closed in X.

Theorem 3.2. *For a fuzzy function $f : X \to Y$, the following statements are equivalent:*

(1) f is fuzzy almost contra γ-continuous,

(2) for every fuzzy regular closed set μ in $Y, f^{-1}(\mu)$ is fuzzy γ-open,

(3) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\epsilon X$ if $f(x_\epsilon)q\mu$, then $x_\epsilon q\gamma\text{-}int(f^{-1}(\mu))$,

(4) for any fuzzy regular closed set $\mu \leq Y$ and for any $x_\epsilon \in X$ if $f(x_\epsilon)q\mu$, then there exists a fuzzy γ-open set η such that $x_\epsilon q\eta$ and $f(\eta) \leq \mu$,

(5) $f^{-1}(int(cl(\mu)))$ *is fuzzy* γ*-closed for every fuzzy open set* μ*,*

(6) $f^{-1}(cl(int(\rho)))$ *is fuzzy* γ*-open for every fuzzy closed subset* ρ*,*

(7) for each fuzzy singleton $x_\epsilon \in X$ *and each fuzzy regular closed set* η *in* Y *containing* $f(x_\epsilon)$*, there exists a fuzzy* γ*-open set* μ *in* X *containing* x_ϵ *such that* $f(\mu) \leq \eta$*.*

Proof. $(1) \Leftrightarrow (2)$: Let ρ be a fuzzy regular open set in Y. Then, ρ^1 is fuzzy regular closed. By $(2), f^{-1}(\rho^1) = (f^{-1}(\rho))^1$ is fuzzy γ-open. Thus, $f^{-1}(\rho)$ is fuzzy γ-closed.
Converse is similar.
$(2) \Leftrightarrow (3)$: Let $\mu \leq Y$ be a fuzzy regular closed set and $f(x\epsilon)q\mu$. Then $x_\epsilon q f^{-1}(\mu)$ and from $(2), f^{-1}(\mu) = \gamma\text{-int}(f^{-1}(\mu))$. Hence $x_\epsilon q\gamma\text{-int}(f^{-1}(\mu))$. Thus, (3) holds. The reverse is obvious.
$(3) \Rightarrow (4)$: Let $\mu \leq Y$ be any fuzzy regular closed set and for $x_\epsilon \in X$ let $f(x_\epsilon)q\mu$. Then $x_\epsilon q\gamma\text{-int}(f^{-1}(\mu))$. Take $\eta = \gamma\text{-int}(f^{-1}(\mu))$, then $f(\eta) = f(\gamma\text{-int}(f^{-1}(\mu))) \leq f(f^{-1}(\mu)) \leq \mu$, where η is fuzzy regular open in X and $x_\epsilon q\eta$.
$(4) \Rightarrow (3)$: Let $\mu \leq Y$ be any fuzzy regular closed set and let $f(x_\epsilon)q\mu$. From (4), there exists fuzzy γ-open set η such that $x_\epsilon q\eta$ and $f(\eta) \leq \mu$. Hence $\eta \leq f^{-1}(\mu)$ and then $x_\epsilon q\gamma\text{-int}(f^{-1}(\mu))$.
$(1) \Leftrightarrow (5)$: Let μ be a fuzzy open set. Since $int(cl(\mu))$ is fuzzy regular open, by (1), it follows that $f^{-1}(int(cl(\mu)))$ is fuzzy γ-closed.
The converse can be shown easily.
$(2) \Leftrightarrow (6)$: It can be obtained similar as $(1) \Leftrightarrow (5)$.
$(2) \Leftrightarrow (7)$: Obvious.

Theorem 3.3. *Let* $f : X \to Y$ *be a fuzzy function and let* $g : X \to X \times Y$ *be the fuzzy graph function of* f*, defined by* $g(x_\epsilon) = (x_\epsilon, f(x_\epsilon))$ *for every* $x_\epsilon \in X$*. If* g *is fuzzy almost contra* γ*-continuous, then* f *is fuzzy almost contra* γ*-continuous.*

Proof. Let η be a fuzzy regular closed set in Y, then $X \times \eta$ is a fuzzy regular closed set in $X \times Y$. Since g is fuzzy almost contra γ-continuous, then $f^{-1}(\eta) = g^{-1}(X \times \eta)$ is fuzzy γ-open in X. Thus, f is fuzzy almost contra γ-continuous.

Definition 3.4. [20] A fuzzy filter base Λ is said to be fuzzy γ-convergent to a fuzzy singleton x_ϵ in X if for any fuzzy γ-open set η in X containing x_ϵ, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.

Definition 3.5. [5] A fuzzy filter base Λ is said to be fuzzy rc-convergent to a fuzzy singleton x_ϵ in X if for any fuzzy regular closed set η in X containing

x_ϵ, there exists a fuzzy set $\mu \in \Lambda$ such that $\mu \leq \eta$.

Theorem 3.6. *If a fuzzy function $f : X \to Y$ is fuzzy almost contra γ-continuous, then for each fuzzy singleton $x_\epsilon \in X$ and each fuzzy filter base Λ in $X\gamma$-converging to x_ϵ, the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\epsilon)$.*

Proof. Let $x_\epsilon \in X$ and Λ be any fuzzy filter base in $X\gamma$-converging to x_ϵ. To prove that the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\epsilon)$, let λ be a fuzzy regular closed set in Y containing $f(x_\epsilon)$. Since f is almost fuzzy contra γ-continuous, there exists a fuzzy γ-open set μ in X containing x_ϵ such that $f(\mu) \leq \lambda$. Since Λ is fuzzy γ-converging to x_ϵ, there exists a $\xi \in \Lambda$ such that $\xi \leq \mu$. This means that $f(\xi) \leq \lambda$ and therefore the fuzzy filter base $f(\Lambda)$ is fuzzy rc-convergent to $f(x_\epsilon)$.

Definition 3.7. [20] A space X is called fuzzy γ-connected if X cannot be expressed as $X = \mu_1 \vee \mu_2$ where

(1) μ_1, μ_2 are fuzzy γ-open sets.

(2) $\mu_1, \mu_2 \neq 0_X$.

(3) $\mu_1 \overline{q} \mu_2$.

Definition 3.8. [18] A space X is called fuzzy connected if X cannot be expressed as $X = \mu_1 \vee \mu_2$ where

(1) μ_1, μ_2 are fuzzy open sets.

(2) $\mu_1, \mu_2 \neq 0_X$.

(3) $\mu_1 \overline{q} \mu_2$.

Theorem 3.9. *If $f : X \to Y$ is a fuzzy almost contra γ-continuous surjection and X is fuzzy γ-connected, then Y is fuzzy connected.*

Proof. Suppose that Y is not a fuzzy connected. Then there is a proper fuzzy clopen subset η in Y. Therefore η is fuzzy regular clopen in Y. Since f is fuzzy almost contra γ-continuous surjection, $f^{-1}(\eta)$ is a proper fuzzy γ-clopen set in X. Thus X is not fuzzy γ-connected and this is a contradiction. Hence Y is fuzzy connected.

Definition 3.10. A fuzzy space X is said to be fuzzy γ-normal if every pair of fuzzy closed sets μ and η with $\mu \neq 0_X, \eta \neq 0_X$ and $\mu \overline{q} \eta$, can be separated by non-quasicoincident fuzzy γ-open sets.

Definition 3.11. [5] A fuzzy space X is said to be fuzzy strongly normal if for every pair of fuzzy closed sets μ and η with $\mu\overline{q}\eta$, there exist fuzzy open sets ρ and ξ such that $\mu \leq \rho, \eta \leq \xi$ and $cl(\rho)\overline{q}cl(\xi)$.

Theorem 3.12. *If Y is fuzzy strongly normal and $f : X \to Y$ is fuzzy almost contra γ-continuous closed injection, then X is fuzzy γ-normal.*

Proof. Let η and ρ be fuzzy closed sets of X with $\eta, \rho \neq 0_X$ and $\eta\overline{q}\rho$. Since f is injective and fuzzy closed, $f(\eta)$ and $f(\rho)$ are fuzzy closed sets with $f(\eta)\overline{q}f(\rho)$. Since Y is fuzzy strongly normal, there exist fuzzy open sets μ and ξ such that $f(\eta) \leq \mu$ and $f(\rho) \leq \xi$ and $cl(\mu)\overline{q}cl(\xi)$. Since μ and ξ are fuzzy open in $X, cl(\mu)$ and $cl(\xi)$ are fuzzy regular closed in $X.f$ is fuzzy almost contra γ-continuous implies $f^{-1}(cl(\mu))$ and $f^{-1}(cl(\xi))$ are fuzzy γ-open sets in X, with $f^{-1}(cl(\mu))\overline{q}f^{-1}(cl(\xi))$. Also $\eta \leq f^{-1}(cl(\mu)), \rho \leq f^{-1}(cl(\xi))$. Thus, X is fuzzy γ-normal.

Definition 3.13. [5] A space X is said to be fuzzy weakly T_2 if each element of X is an intersection of fuzzy regular closed sets.

Definition 3.14. [20] A space X is said to be fuzzy γ-T_2 if for each pair of distinct points x_ϵ and y_ν in X, there exist fuzzy γ-open sets μ and η containing x_ϵ and y_ν, respectively such that $\mu\overline{q}\eta$.

Definition 3.15. [20] A space X is said to be fuzzy γ-T_1 if for each pair of distinct fuzzy singletons x_ϵ and y_ν in X, there exist fuzzy γ-open sets μ and η containing x_ϵ and y_ν, respectively, such that $y_\nu \notin \mu$ and $x_\epsilon \notin \eta$.

Theorem 3.16. *If $f : X \to Y$ is a fuzzy almost contra γ-continuous injection and Y is fuzzy Urysohn, then X is fuzzy γ-T_2.*

Proof. Let x_ϵ and t_γ be any two distinct fuzzy singletons in X. Since f is injective, $f(x_\epsilon) \neq f(t_\gamma)$ in Y. By assumption Y is fuzzy Urysohn and therefore there exist fuzzy open sets η and ρ in Y such that $f(x_\epsilon) \in \eta$ and $f(t_\gamma) \in \rho$ and $cl(\eta)\overline{q}cl(\rho)$. Since η and ρ is fuzzy open, $cl(\eta)$ and $cl(\rho)$ are fuzzy regular closed in $Y.f$ is fuzzy almost contra γ-continuous implies that there exists fuzzy γ-open sets μ and ξ in X containing x_ϵ and t_γ respectively, such that $f(\mu) \leq cl(\eta)$ and $f(\xi) \leq cl(\rho)$. Since $cl(\eta)\overline{q}cl(\rho)$, we have $f(\mu)\overline{q}f(\xi)$ and hence $\mu\overline{q}\xi$. This shows that X is fuzzy γ-T_2.

Theorem 3.17. *If $f : X \to Y$ is a fuzzy almost contra γ-continuous injection and Y is fuzzy weakly T_2, then X is fuzzy γ-T_1.*

Proof. Let x_ϵ and t_γ be any two distinct fuzzy points in X. Since f is injective, $f(x_\epsilon)$ and $f(t_\gamma)$ are distinct fuzzy points in $Y.Y$ is fuzzy weakly T_2 implies that there exist fuzzy regular closed sets η and ρ in Y such that

$f(x_\epsilon) \in \eta, f(t_\gamma) \notin \eta, f(x_\epsilon) \notin \rho$ and $f(t_\gamma) \in \rho$. Since f is fuzzy almost contra γ-continuous, by Theorem 3.2, $f^{-1}(\eta)$ and $f^{-1}(\rho)$ are fuzzy γ-open sets in X such that $x_\epsilon \in f^{-1}(\eta), t_\gamma \notin f^{-1}(\eta), x_\epsilon \notin f^{-1}(\rho)$ and $t_\gamma \in f^{-1}(\rho)$. This shows that X is fuzzy γ-T_1.

Theorem 3.18. *Let (X_i, τ_i) be fuzzy topological spaces $\forall i \in I$ and I be finite. Let $f : (X, \tau) \to (\prod_{i\in I} X_i, \sigma)$ be a fuzzy function where $(\prod_{i\in I} X_i, \sigma)$ is the product space. If f is fuzzy almost contra γ-continuous, then pr_i of is fuzzy almost contra γ-continuous where pr_i is projection function for each $i \in I$.*

Proof. Let ρ_i be a fuzzy regular closed set in (X_i, τ_i). Since pr_i is fuzzy continuous and open function, $pr_i^{-1}(\rho_i) = X_1 \times X_2 \times \cdots X_{i-1} \times \rho_i \times X_{i+1} \times X_{i+2} \times \cdots \times X_n$ where $I = \{1, 2, \cdots, n\}$ is finite, is a fuzzy regular closed set in $(\prod_{i\in I} X_i, \sigma)$. By assumption f is fuzzy almost contra γ-continuous

$$f^{-1}((pr_i)^{-1}(\rho_i)) = (pr_i \circ f)^{-1}(\rho_i)$$

is fuzzy γ-open in X. Hence $pr_i \circ f$ is fuzzy almost contra γ-continuous for each $i \in I$.

Definition 3.19. The fuzzy graph $G(f)$ of a fuzzy function $f : X \to Y$ is said to be fuzzy strongly contra γ-closed if for each $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy γ-open set μ in X containing x_ϵ and a fuzzy regular closed set η in Y containing y_ν such that $(\mu \times \eta)\overline{q}G(f)$.

Lemma 3.20. *The following properties are equivalent for the fuzzy graph $G(f)$ of a fuzzy function f :*

(1) $G(f)$ is fuzzy strongly contra γ-closed;

(2) for each $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$, there exist a fuzzy γ-open set μ in X containing x_ϵ and a fuzzy regular closed set η containing y_ν such that $f(\mu)\overline{q}\eta$.

Theorem 3.21. *If $f : X \to Y$ is fuzzy almost contra γ-continuous and Y is fuzzy Urysohn, $G(f)$ is fuzzy strongly contra γ-closed in $X \times Y$.*

Proof. Suppose that Y is fuzzy Urysohn. Let $(x_\epsilon, y_\nu) \in (X \times Y)\backslash G(f)$. It follows that $f(x_\epsilon) \neq y_\nu$. Since Y is fuzzy Urysohn, there exist fuzzy open sets η and ρ such that $f(x_\epsilon) \in \eta, y_\nu \in \rho$ and $cl(\eta)\overline{q}cl(\rho)$.$\eta$ and ρ are fuzzy open in Y implies $cl(\eta)$ and $cl(\rho)$ are fuzzy regular closed in Y. Since f is fuzzy almost contra γ-continuous, there exists a fuzzy γ-open set μ in X containing x_ϵ such that $f(\mu) \leq cl(\eta)$. Therefore, $f(\mu)\overline{q}cl(\rho)$ and thus $G(f)$ is fuzzy strongly contra γ-closed in $X \times Y$.

Theorem 3.22. *Let $f : X \to Y$ have a fuzzy strongly contra γ-closed graph. If f is injective and fuzzy almost contra γ-continuous, then X is fuzzy γ-T_1.*

Proof. Let x_ϵ and y_ν be any two distinct points of X. Since f is injective $f(x_\epsilon) \neq f(y_\nu)$. Then, we have $(x_\epsilon, f(y_\vee)) \in (X \times Y)\backslash G(f)$. By Lemma 3.20, there exist a fuzzy γ-open set μ in X containing x_ϵ and a fuzzy regular closed set ρ in Y containing $f(y_\nu)$ such that $f(\mu)\overline{q}\rho$. Since f is fuzzy almost contra γ-continuous, $f^{-1}(\rho)$ is fuzzy γ-open in X such that $y_\gamma \in f^{-1}(\rho)$. As $f(\mu)\overline{q}\rho$, we have $\mu\overline{q}f^{-1}(\rho)$. Taking $\eta. = f^{-1}(\rho)$, we have fuzzy γ-open sets μ and η in X such that $x_\epsilon \in \mu, y_\nu \in \eta$ respectively, whereas $x_\epsilon \notin \eta y_\nu \notin \mu$. This proves that X is γ-T_1.

4. The Relationships

In this section, the relationships between fuzzy almost contra γ-continuity and other forms of continuity are investigated.

Definition 4.1. A function $f : X \to Y$ is called fuzzy weakly almost contra γ-continuous if for each $x \in X$, and each fuzzy regular closed set η of Y containing $f(x_\epsilon)$, there exists a fuzzy γ-open set μ in X containing x_ϵ such that $int(f(\mu)) \leq \eta$.

Definition 4.2. A function $f : X \to Y$ is called fuzzy (γ, s)-open if the image of each fuzzy γ-open set is fuzzy semi-open.

Theorem 4.3. *If a function $f : X \to Y$ is fuzzy weakly almost contra γ-continuous and fuzzy (γ, s)-open, then f is fuzzy almost contra γ-continuous.*

Proof. Let $x_\epsilon \in X$ and η be a fuzzy regular closed set containing $f(x_\epsilon)$. Since f is fuzzy weakly almost contra γ-continuous, there exists a fuzzy γ-open set μ in X containing x_ϵ such that $int(f(\mu)) \leq \eta$. Since f is fuzzy (γ, s)-open, $f(\mu)$ is a semi-open set in Y and so $f(\mu) \leq cl(int(f(\mu))) \leq \eta$. This shows that f is fuzzy almost contra γ-continuous.

Definition 4.4. Let X and Y be fuzzy topological spaces. A fuzzy function $f : X \to Y$ is said to be

(1) fuzzy almost contra precontinuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy preclosed in X,

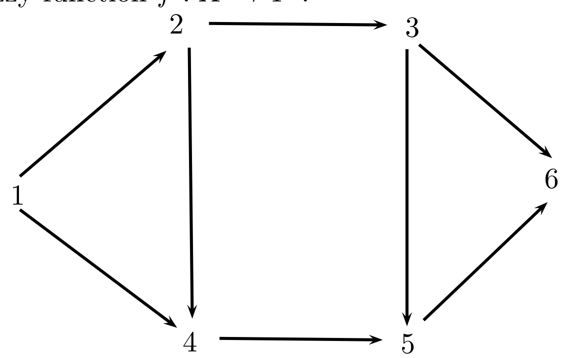
(2) fuzzy almost contra semicontinuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy semi-closed in X,

(3) fuzzy almost contra continuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy closed in X,

(4) fuzzy almost contra α-continuous if the inverse image of each fuzzy regular open set in Y is fuzzy α-closed in X,

(5) fuzzy almost contra β-continuous [5] if the inverse image of each fuzzy regular open set in Y is fuzzy β-closed in X.

Remark 4.5. We have the following diagram describing the properties of a fuzzy function $f : X \to Y$:



where the numbers represent the properties noted against then.

(1) fuzzy almost contra continuous function

(2) fuzzy almost contra α-continuous function

(3) fuzzy almost contra precontinuous function

(4) fuzzy almost contra semi-continuous function

(5) fuzzy almost contra γ-continuous function

(6) fuzzy almost contra β-continuous function

None of the above implications is reversible.

Example 4.6. Fuzzy almost contra α-continuity $\nrightarrow$ fuzzy almost contra continuity.

Let X be a nonempty set and $C_a : X \to [0,1]$ be defined as $C_a(x) = a \forall x \in X$ and $a \in [0,1]$. Then $\tau_1 = \{C_0, C_{6/10}, C_1\}$ and $\tau_2 = \{C_0, C_{3/10}, C_1\}$ are fuzzy topologies and $(X,\tau_1),(X,\tau_2)$ are fuzzy topological spaces. The identity function $f : (X,\tau_1) \to (X,\tau_2)$ is fuzzy almost contra α-continuous but not fuzzy almost contra continuous.

In $(X, \tau_2), C_{7/10}$ is the only fuzzy regular closed set other than C_0 and C_1. Also $f^{-1}(C_{7/10}) = C_{7/10}, f$ being the identity function. In $(X, \tau_1), C_{7/10}$ is fuzzy α-open since $C_{7/10} \leq C_1 = int(cl(int(C_{7/10})))$. Thus f is fuzzy almost contra α-continuous.

Obviously $C_{7/10}$ is not fuzzy open in (X, τ). Hence f is not fuzzy almost contra continuous.

Example 4.7. Fuzzy almost contra semi-continuity $\nrightarrow$ fuzzy almost contra continuity.

In Example 4.6, $C_{7/10} \in FRC(X)$ in (X, τ_2). This is the only fuzzy regular closed set other than C_0 and C_1. And $f^{-1}(C_{7/10}) = C_{7/10}$ is fuzzy α-open and hence fuzzy semi-open in (X, τ_1) but not fuzzy open. This proves that f is fuzzy almost contra semi-continuous but not fuzzy almost contra continuous.

Example 4.8. Fuzzy almost contra semi continuity $\nrightarrow$ fuzzy almost contra α-continuity.

Let X be a nonempty set and $C_a : X \to [0, 1]$ be defined as $C_a(x) = a \forall x \in X$ and $a \in [0, 1]$. Then $\tau_1 = \{C_0, C_{2/10}, C_1\}$ and $\tau_2 = \{C_0, C_{3/10}, C_1\}$ are fuzzy topologies. Then $f : (X, \tau_1) \to (X, \tau_2)$, the identity function is fuzzy almost contra semi-continuous but not fuzzy almost contra α-continuous.

In (X, τ_2), the only fuzzy regular closed set other than C_0 and C_1 is $C_{7/10}$. And $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1), f$ being the identity function. $C_{7/10} \leq C_{8/10} = cl(int(C_{7/10}))$. Thus $C_{7/10}$ is fuzzy semi open in (X, τ_1) proving that f is fuzzy almost contra semicontinuous. But $C_{7/10} \nleq C_{2/10} = int(cl(int(C_{7/10})))$ and so $C_{7/10}$ is not fuzzy α-open in (X, τ_1). This proves that f is not fuzzy almost contra α-continuous.

Example 4.9. Fuzzy almost contra precontinuity $\nrightarrow$ fuzzy almost contra α-continuity.

Let X be a nonempty set and $C_a : X \to [0, 1]$ be defined as $C_a(x) = a \forall x \in X$ and $a \in [0, 1]$. The identity function $f : (X, \tau_1) \to (X, \tau_2)$ where $\tau_1 = \{C_0, C_{5/10}, C_1\}$ and $\tau_2 = \{C_0, C_{3/10}, C_1\}$ is fuzzy almost contra precontinuous but not fuzzy almost contra α-continuous. In $(X, \tau_2), C_{7/10}$ is the only fuzzy regular closed set other than C_0 and C_1. And $f^{-1}(C_{7/10}) = C_{7/10} C_1 \leq C_1 int(cl(C_{7/10}))$. Thus $C_{7/10}$ is fuzzy preopen in (X, τ_1) and this proves that f is fuzzy almost contra precontinuous. But $C_{7/10} \nleq C_{5/10} = int(cl(int(C_{7/10})))$ which proves that $C_{7/10}$ is not fuzzy α-open in (X, τ_1) and hence f is not fuzzy almost contra α-continuous.

Example 4.10. fuzzy almost contra γ-continuity $\nrightarrow$ fuzzy almost contra precontinuity.

In Example 4.8, the only fuzzy regular closed set in (X, τ_2) is $C_{7/10}$, other than C_0 and C_1. Also, f being the identity $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1).C_{7/10}$ is fuzzy semiopen in (X, τ_1) and hence fuzzy γ-open in (X, τ_1). Thus f is fuzzy almost contra γ-continuous. But $C_{7/10} \nleq int(cl(C_{7/10}))$ which means that $C_{7/10}$ is not fuzzy preopen in (X, τ_1). This proves that f is not fuzzy almost contra precontinuous.

Example 4.11. Fuzzy almost contra γ-continuity $\nrightarrow$ fuzzy almost contra semi-continuity.

In Example 4.9, $C_{7/10}$ is the only fuzzy regular closed set in (X, τ_2) other than C_0 and C_1. Also, f being the identity $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1).C_{7/10}$ is fuzzy preopen in (X, τ_1) and hence fuzzy γ-open in (X, τ_1). This proves that f is fuzzy almost contra γ-continuous.

But $C_{7/10} \nleq C_{5/10} = int(cl(C_{7/10}))$ which means that $C_{7/10}$ is not fuzzy semi-open.

Hence f is not fuzzy almost contra semi-continuous.

Example 4.12. In Example 4.8, $C_{7/10}$ is the only fuzzy regular closed set in (X, τ_2). Also f being the identiy function $f^{-1}(C_{7/10}) = C_{7/10}$ in $(X, \tau_1).C_{7/10} \leq C_{8/10} = cl(int(cl(C_{7/10})))$ and hence $C_{7/10}$ is fuzzy β-open in (X, τ_1). This proves that f is fuzzy almost contra β-continuous.

But $C_{7/10} \nleq C_{2/10} = int(cl(C_{7/10}))$ which means that $C_{7/10}$ is not fuzzy preopen. Hence f is not fuzzy almost precontinuous.

Definition 4.13. [5] A fuzzy space is said to be fuzzy P_Σ if for any fuzzy open set μ of X and each $x_\epsilon \in \mu$, there exists fuzzy regular closed set ρ containing x_ϵ such that $x_\epsilon \in \rho \leq \mu$.

Definition 4.14. [9] A fuzzy function $f : X \to Y$ is said to be fuzzy γ-continuous if $f^{-1}(\mu)$ is fuzzy γ-open in X for every fuzzy open set μ in Y.

Theorem 4.15. *Let $f : X \to Y$ be a fuzzy function. Then, if f is fuzzy almost contra γ-continuous and Y is fuzzy P_Σ, then f is fuzzy γ-continuous.*

Proof. Let μ be any fuzzy open set in Y. Since Y is fuzzy P_Σ, there exists a family ψ whose members are fuzzy regular closed sets of Y such that $\mu = \vee\{\rho : \rho \in \psi\}$. Since f is fuzzy almost contra γ-continuous, $f^{-1}(\rho)$ is fuzzy γ-open in X for each $\rho \in \psi$ and $f^{-1}(\mu)$ is fuzzy γ-open in X. Therefore, f is fuzzy almost contra γ-continuous.

Definition 4.16. [5] A space is said to be fuzzy weakly P_Σ if for any fuzzy regular open set μ and each $x_\epsilon \in \mu$, there exists a fuzzy regular closed set ρ containing x_ϵ such that $x_\epsilon \in \rho \leq \mu$.

Definition 4.17. A fuzzy function $f : X \to Y$ is said to be fuzzy almost

γ-continuous at $x_\epsilon \in X$ if for each fuzzy open set η containing $f(x_\epsilon)$, there exists a fuzzy γ-open set μ containing x_ϵ such that $f(\mu) \leq int(cl(\eta))$.

Theorem 4.18. *Let $f : X \to Y$ be a fuzzy almost contra γ-continuous function. If Y is fuzzy weakly P_Σ, then f is fuzzy almost γ-continuous.*

Proof. Let μ be any fuzzy regular open set of Y. Since Y is fuzzy weakly P_Σ, there exists a family ψ whose members are fuzzy regular closed sets of Y such that $\mu \vee \{\rho : \rho \in \psi\}$. Since f is fuzzy almost contra γ-continuous, $f^{-1}(\rho)$ is fuzzy γ-open in X for each $\rho \in \psi$ and $f^{-1}(\mu)$ is fuzzy γ-open in X. Hence, f is fuzzy almost γ-continuous.

Definition 4.19. [9] A fuzzy function $f : X \to Y$ is called fuzzy γ-irresolute if the inverse image of each fuzzy γ-open set is fuzzy γ-open.

Theorem 4.20. *Let X, Y, Z be fuzzy topological spaces and let $f : X \to Y$ and $g : Y \to Z$ be fuzzy functions. If f is fuzzy γ-irresolute and g is fuzzy almost contra γ-continuous, then $g \circ f : X \to Z$ is a fuzzy almost contra γ-continuous function.*

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set. Since g is fuzzy almost contra γ-continuous, $g^{-1}(\mu)$ is fuzzy γ-open in Y. But f is fuzzy γ-irresolute $\Rightarrow f^{-1}(g^{-1}(\mu))$ is fuzzy γ-open in X. Thus $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ is fuzzy γ-open in X and this proves that $g \circ f$ is a fuzzy almost contra γ-continuous function.

Definition 4.21. A fuzzy function $f : X \to Y$ is called always fuzzy γ-open [20] if the image of each fuzzy γ-open set is fuzzy γ-open.

Theorem 4.22. *If $f : X \to Y$ is a surjective always fuzzy γ-open function and $g : Y \to Z$ is a fuzzy function such that $g \circ f : X \to Z$ is fuzzy almost contra γ-continuous, then g is fuzzy almost contra γ-continuous.*

Proof. Let $\mu \leq Z$ be any fuzzy regular closed set. Since $g \circ f$ is fuzzy almost contra γ-continuous, $(g \circ f)^{-1}(\mu)$ is fuzzy γ-open in X. Therefore $f^{-1}(g^{-1}(\mu)) = (g \circ f)^{-1}(\mu)$ is fuzzy γ-open in $X.f$ is always fuzzy γ-open surjection implies $f(f^{-1}(g^{-1}(\mu))) = g^{-1}(\mu)$ is fuzzy γ-open in Y. Thus g is fuzzy almost contra γ-continuous.

Corollary 4.23. *Let $f : X \to Y$ be a surjective fuzzy γ-irresolute and always fuzzy γ-open function and let $g : Y \to Z$ be a fuzzy function. Then, $g \circ f : X \to Z$ is fuzzy almost contra γ-continuous if and only if g is fuzzy almost contra γ-continuous.*

Proof. It can be obtained from Theorem 4.20 and Theorem 4.22.

Definition 4.24. A space X is said to be fuzzy γ-compact [20] (fuzzy S-closed [3]) if every fuzzy γ-open (respectively fuzzy regular closed) cover of X has a finite subcover.

Theorem 4.25. *The fuzzy almost contra γ-continuous image of a fuzzy γ-compact space is fuzzy S-closed.*

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra γ-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular closed cover of Y. Since f is fuzzy almost contra γ-continuous, $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy γ-open cover of X and X being fuzzy γ-compact, there exists a finite subset I_o of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_o\}$. Since f is surjective, we have $Y = \vee\{\eta_i : i \in I_0\}$ and thus Y is fuzzy S-closed.

Definition 4.26. A space X is said to be

(1) fuzzy γ-closed-compact [20] if every fuzzy γ-closed cover of X has a finite subcover,

(2) fuzzy nearly compact [7] if every fuzzy regular open cover of X has a finite subcover.

Theorem 4.27. *The fuzzy almost contra γ-continuous image of a fuzzy γ-closed-compact space is fuzzy nearly compact.*

Proof. Suppose that $f : X \to Y$ is a fuzzy almost contra γ-continuous surjection. Let $\{\eta_i : i \in I\}$ be any fuzzy regular open cover of Y. Since f is fuzzy almost contra γ-continuous, $\{f^{-1}(\eta_i) : i \in I\}$ is a fuzzy γ-closed cover of X. Since X is fuzzy γ-closed-compact, there exists a finite subset I_0 of I such that $X = \vee\{f^{-1}(\eta_i) : i \in I_0\}$. Thus, we have $Y = \vee\{\eta_i : i \in I_0\}$ and Y is fuzzy nearly compact.

Definition 4.28. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Then f is said to be

(1) fuzzy semi-open [22] if the image of every fuzzy open set of X is fuzzy semiopen in Y.

(2) fuzzy α-open [19] if the image of every fuzzy open set of X is fuzzy α-open in Y.

(3) fuzzy preopen [22] if the image of every fuzzy open set of X is fuzzy preopen in Y.

(4) fuzzy γ-open [9] if the image of every fuzzy open set of X is fuzzy γ-open in Y.

(5) fuzzy (LC, s) if the image of every fuzzy open set of X is fuzzy LC set in Y.

Theorem 4.29. *For a fuzzy function $f : X \to Y$, where Y is a fuzzy extremally disconnected space, the following properties are equivalent.*

(1) f is fuzzy open.

(2) f is fuzzy α-open and a fuzzy (LC, s).

(3) f is fuzzy preopen and a fuzzy (LC, s).

(4) f is fuzzy semi-open and a fuzzy (LC, s).

(5) f is fuzzy γ-open and a fuzzy (LC, s).

Definition 4.30. Let $f : X \to Y$ be a fuzzy function. Then f is said to be

(1) fuzzy almost continuous [1] if the inverse image of every fuzzy regular open set of Y is fuzzy open in X.

(2) fuzzy almost α-continuous [15] if the inverse image of every fuzzy regular open set of Y is fuzzy α-open in X.

(3) fuzzy almost semicontinuous [13] if the inverse image of every fuzzy regular open set of Y is fuzzy semi-open in X.

(4) fuzzy almost precontinuous [21] if the inverse image of every fuzzy regular open set of Y is fuzzy preopen in X.

(5) fuzzy almost γ-continuous if the inverse image of every fuzzy regular open set of Y is fuzzy γ-open in X.

(6) fuzzy almost (LC, s)-continuous if the inverse image of every fuzzy regular open set of Y is a fuzzy LC set in X.

Theorem 4.31. *For a fuzzy function $f : X \to Y$, where X is a fuzzy extremally disconnected space, the following properties are equivalent.*

(1) f is a fuzzy almost continuous.

(2) f is fuzzy almost α-continuous and a fuzzy (LC, s)-continuous.

(3) f is fuzzy almost precontinuous and a fuzzy (LC, s)-continuous.

(4) f is fuzzy almost semicontinuous and a fuzzy (LC, s)-continuous.

(5) f is fuzzy almost γ-continuous and a fuzzy (LC, s)-continuous.

Definition 4.32. Let $f : X \to Y$ be a fuzzy function. Then f is said to be fuzzy almost contra (LC, s)-continuous if the inverse image of every fuzzy regular closed set of Y is fuzzy LC set in X.

Theorem 4.33. *For a fuzzy function $f : X \to Y$, where X is a fuzzy extremally disconnected space, the following properties are equivalent.*

(1) f is fuzzy almost contra continuous.

(2) f is fuzzy almost contra α-continuous and fuzzy contra (LC, s)-continuous.

(3) f is fuzzy almost contra precontinuous and fuzzy contra (LC, s)-continuous.

(4) f is fuzzy almost contra semicontinuous and fuzzy contra (LC, s)-continuous.

(5) f is fuzzy almost contra γ-continuous and fuzzy contra (LC, s)-continuous.

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