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ON MINIMAL g^*p-CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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Abstract: In this paper a new class of maps called minimal $g^{\star}p$-continuous, maximal $g^{\star}p$-continuous, minimal $g^{\star}p$-irresolute, maximal $g^{\star}p$-irresolute, minimal-maximal $g^{\star}p$-continuous and maximal-minimal-$g^{\star}p$ continuous maps in topological spaces and study their relations with various types of continuous maps.

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Key Words: minimal $g^{\star}p$-continuous, maximal $g^{\star}p$-continuous, minimal $g^{\star}p$-irresolute, maximal $g^{\star}p$-irresolute, minimal-maximal $g^{\star}p$-continuous and maximal-minimal-$g^{\star}p$ continuous maps

1. Introduction

In the years 2001 and 2003, F. Nakaoka and N. Oda [8, 9] and [10] introduced and studied minimal open (resp. minimal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively.

Recently, A. Vadivel and A. Swaminathan [14] have introduced minimal $g^{\star}p$-open (resp. minimal $g^{\star}p$-closed) sets, which are subclasses of $g^{\star}p$-open (resp. $g^{\star}p$-closed) sets. In this paper, a new class of maps called minimal $g^{\star}p$-continuous maps and maximal $g^{\star}p$-continuous maps in topological spaces are introduced. They are subclasses of $g^{\star}p$-open (resp. $g^{\star}p$-closed) sets respectively.

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Some of the properties obtained here.

2. Preliminaries

Definition 2.1.. A subset A of a topological space (X, τ) is called:

(i) regular open set if $A = intcl(A)$ [13] and regular closed set [13] if $A = clint(A)$.

(ii) pre-open set if $A \subseteq intcl(A)$ [7] and pre-closed set [7] if $clint(A) \subseteq A$.

(iii) α-open set [11] if $A \subseteq int(cl(int(A)))$ and α-closed set [11] if $cl(int(cl(A))) \subseteq A$.

(iv) semi-pre-open set [2](=β-open set [1]) if $A \subseteq cl(int(cl(A)))$ and semi-pre-closed set [2] (=β-closed set [1]) if $int(cl(int(A))) \subseteq A$.

Definition 2.2.. A subset A of a space $(X,\ \tau)$ is said to be generalized closed set (briefly, g-closed) [5] if $cl\,(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of a g-closed set is said to be g-open.

Definition 2.3.. A subset A of a topological space $(X,\ \tau)$ is said to be strongly generalized pre closed set (briefly, $g^{\square}p$-closed set) [15] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X.

Definition 2.4.. (see [8]) A proper nonempty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is ϕ or U.

Definition 2.5.. (see [9]) A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.

Definition 2.6.. (see [10]) A proper nonempty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is ϕ or F.

Definition 2.7.. (see [10]) A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.

Definition 2.8.. (see [14]) A proper nonempty $g^{\square}p$-open subset U of a topological space X is said to be a minimal $g^{\square}p$-open set if any $g^{\square}p$-open set which is contained in U is ϕ or U.

Definition 2.9.. (see [14]) A proper nonempty $g^{\square}p$-open subset U of a topological space X is said to be a maximal $g^{\square}p$-open set if any $g^{\square}p$-open set which contains U is X or U.

Definition 2.10.. (see [14]) A proper nonempty $g^{\square}p$-closed subset F of a topological space X is said to be a minimal $g^{\square}p$-closed set if any $g^{\square}p$-closed set which is contained in F is ϕ or F.

Definition 2.11.. (see [14]) A proper nonempty $g^{\square}p$-closed subset F of a topological space X is said to be a maximal $g^{\square}p$-closed set if any $g^{\square}p$-closed set which contains F is X or F.

Definition 2.12.. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is called:

(i) g-continuous [3] if $f^{-1}(V)$ is g-open set of (X, τ) for every open set V of (Y, σ).

(ii) almost continuous [12] if $f^{-1}(V)$ is open set of (X, τ) for every regular open set V of (Y, σ).

(iii) precontinuous [7] if $f^{-1}(V)$ is preopen set of (X, τ) for every open set V of (Y, σ).

(iv) α-continuous [6] if $f^{-1}(V)$ is α-open set of (X, τ) for every open set V of (Y, σ).

(v) β-continuous [1] if $f^{-1}(V)$ is β-open set of (X, τ) for every open set V of (Y, σ).

(vi) $g^{\square}p$-continuous [15] if $f^{-1}(V)$ is $g^{\square}p$-open set of (X, τ) for every open set V of (Y, σ).

(vii) $g^{\square}p$-irresolute [15] if $f^{-1}(V)$ is $g^{\square}p$-open set in X for every $g^{\square}p$-open set V in Y.

Definition 2.13.. Let X and Y be the topological spaces. A map $f : (X,\ \tau) \to (Y,\ \sigma)$ is called:

(i) minimal continuous [4] (briefly min-continuous) if $f^{-1}(M)$ is an open set in X for every minimal open set M in Y.

(ii) maximal continuous [4] (briefly max-continuous) if $f^{-1}(M)$ is an open set in X for every maximal open set M in Y.

(iii) minimal irresolute [4] (briefly min-irresolute) if $f^{-1}(M)$ is minimal open set in X for every minimal open set M in Y.

(iv) maximal irresolute [4] (briefly max-irresolute) if $f^{-1}(M)$ is maximal open set in X for every maximal open set M in Y.

(v) minimal-maximal continuous [4] (briefly min-max continuous) if $f^{-1}(M)$ is maximal open set in X for every minimal open set M in Y.

(vi) maximal-minimal continuous [4] (briefly max-min continuous) if $f^{-1}(M)$ is minimal open set in X for every maximal open set M in Y.

3. Minimal g^*p-Continuous Maps and Maximal g^*p-Continuous Maps

Definition 3.1.. Let X and Y be the topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is called:

(i) minimal $g^\star p$-continuous (briefly min-$g^\star p$-continuous) if $f^{-1}(M)$ is a $g^\star p$-open set in X for every minimal $g^\star p$-open set M in Y.

(ii) maximal $g^\star p$-continuous (briefly max-$g^\star p$-continuous) if $f^{-1}(M)$ is a $g^\star p$-open set in X for every maximal $g^\star p$-open set M in Y.

(iii) minimal $g^\star p$-irresolute (briefly min-$g^\star p$-irresolute) if $f^{-1}(M)$ is minimal $g^\star p$-open set in X for every minimal $g^\star p$-open set M in Y.

(iv) maximal $g^\star p$-irresolute (briefly max-$g^\star p$-irresolute) if $f^{-1}(M)$ is maximal $g^\star p$-open set in X for every maximal $g^\star p$-open set M in Y.

(v) minimal-maximal $g^\star p$-continuous (briefly min-max $g^\star p$-continuous) if $f^{-1}(M)$ is maximal $g^\star p$-open set in X for every minimal $g^\star p$-open set M in Y.

(vi) maximal-minimal $g^\star p$-continuous (briefly max-min $g^\star p$-continuous) if $f^{-1}(M)$ is minimal $g^\star p$-open set in X for every maximal $g^\star p$-open set M in Y.

Theorem 3.1.. *Every $g^\star p$-continuous map is minimal $g^\star p$-continuous but not conversely.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a $g^\star p$-continuous map. To prove that f is minimal $g^\star p$-continuous. Let N be any minimal $g^\star p$-open set in Y. Since every minimal $g^\star p$-open set is a $g^\star p$-open set, N is a $g^\star p$-open set in Y. Since f is $g^\star p$-continuous, $f^{-1}(N)$ is a $g^\star p$-open set in X. Hence f is a minimal $g^\star p$-continuous. $\square$

Example 3.1. Consider $X = Y = \{a, b, c\}$ with topologies $\tau = \{X, \phi, \{a\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{b\}, \{a, b\} \{b, c\}\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be an identity map. Then this function is minimal $g^\star p$-continuous but it is not a $g^\star p$-continuous, since for the open set $\{b, c\}$ in Y, $f^{-1}(\{b, c\}) = \{b, c\}$ which is not a $g^\star p$-open set in X.

Theorem 3.2.. *Every $g^{\square}p$-continuous map is maximal $g^{\square}p$-continuous but not conversely.*

Proof. Similar to that of Theorem 3.1. □

Example 3.2. Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\},\ \{a,\ b\}\}$ and $\sigma = \{Y,\ \phi,\ \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an identity map. Then f is a maximal $g^{\square}p$-continuous map but it is not a $g^{\square}p$-continuous, since for the open set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ which is not a $g^{\square}p$-open set in X.

Remark 3.1.. Minimal $g^{\square}p$-continuous and maximal $g^{\square}p$-continuous maps are independent of each other.

Example 3.3. In Example 3.1, f is a minimal $g^{\square}p$-continuous but it is not a maximal $g^{\square}p$-continuous. In Example 3.2, f is a maximal $g^{\square}p$-continuous but it is not a minimal $g^{\square}p$-continuous.

Remark 3.2.. Minimal (resp. maximal) $g^{\square}p$-continuous and almost continuous maps are independent of each other.

Example 3.4. Let $X = Y = \{a,\ b,\ c\}$ be with $\tau = \{X,\ \phi,\ \{a\},\ \{a,\ b\}\}$ and $\sigma = \{Y,\ \phi,\ \{b\}, \{b, c\}\}$. Define a map $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = a$, $f(b) = b$ and $f(c) = c$. Then f is a almost continuous but not minimal (resp. maximal) $g^{\square}p$-continuous.

Example 3.5. In Example 3.2, define a map $f : (X,\ \tau) \to (Y,\ \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is a minimal (resp. maximal) $g^{\square}p$-continuous but it is not a almost continuous.

Remark 3.3.. (i) Minimal $g^{\square}p$-continuous and precontinuous (resp. α-continuous, β-continuous) maps are independent of each other.

(ii) Maximal $g^{\square}p$-continuous and precontinuous (resp. α-continuous, β-continuous) maps are independent of each other.

Example 3.6. (i) Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{b\},\ \{a,\ b\},\ \{b, c\}\}$ and $\sigma = \{Y,\ \phi,\ \{b\},\ \{b, c\}\}$. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an identity map. Then f is a precontinuous (resp. α-continuous, β-continuous) but it is not a minimal $g^{\square}p$-continuous, since for the minimal $g^{\square}p$-open set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ which is not a $g^{\square}p$-open set in X.

(ii) Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\},\ \{c\},\ \{a, c\}\}$ and $\sigma = \{Y,\ \phi,\ \{c\}\}$. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be an identity map. Then f is a precontinuous (resp. α-continuous, β-continuous) but it is not a maximal $g^{\square}p$-continuous, since for the maximal $g^{\square}p$-open set $\{b, c\}$ in Y, $f^{-1}(\{b, c\}) = \{b, c\}$

which is not a $g^{\star}p$-open set in X.

Example 3.7. (i) In Example 3.1, f is a minimal $g^{\star}p$-continuous but it is not a precontinuous (resp. α-continuous, β-continuous), since for the open set $\{b, c\}$ in Y, $f^{-1}(\{b, c\}) = \{b, c\}$ which is not a preopen (resp. α-open, β-open).

(ii) In Example 3.2, f is a maximal $g^{\star}p$-continuous but it is not a precontinuous (resp. α-continuous, β-continuous), since for the open set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ which is not a preopen (resp. α-open, β-open).

Remark 3.4.. Minimal (resp. maximal) $g^{\star}p$-continuous and g-continuous maps are independent of each other.

Example 3.8. In Example 3.1, f is a minimal $g^{\star}p$-continuous but it is not a g-continuous, since for the open set $\{b, c\}$ in Y, $f^{-1}(\{b, c\}) = \{b, c\}$ which is not a g-open in X.
(ii) In Example 3.2, f is a maximal $g^{\star}p$-continuous but it is not a g-continuous, since for the open set $\{a, c\}$ in Y, $f^{-1}(\{a, c\}) = \{a, c\}$ which is not a g-open in X.

Example 3.9. (i) Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\}\}$ and $\mu = \{Y,\ \phi,\ \{c\}\}$.
(i) Let $f : (X,\ \tau) \to (Y,\ \mu)$ be an identity map. Then f is a g-continuous but it is not a minimal $g^{\star}p$-continuous, since for the minimal $g^{\star}p$-open set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{c\}$ which is not a $g^{\star}p$-open in X.

(ii) Consider the Example 3.6(ii), f is a g-continuous but it is not a maximal $g^{\star}p$-continuous, since for the maximal $g^{\star}p$-open set $\{b,\ c\}$ in Y, $f^{-1}(\{b,\ c\}) = \{b,\ c\}$ which is not a $g^{\star}p$-open in X.

Remark 3.5.. From the above discussion and known results we have the following implications.

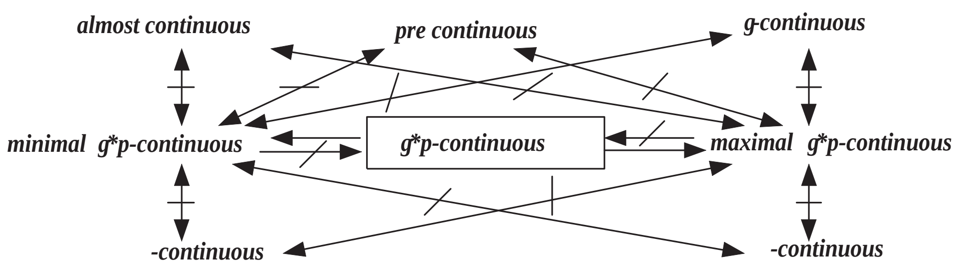

fig-1

Theorem 3.3.. *Let X and Y be the topological spaces. A map $f : X \to Y$ is minimal $g^{\square}p$-continuous if and only if the inverse image of each maximal $g^{\square}p$-closed set in Y is a $g^{\square}p$-closed set in X.*

Proof. The proof follows from the definition and fact that the complement of minimal $g^{\square}p$-open set is maximal $g^{\square}p$-closed set. $\square$

Theorem 3.4.. *Let X and Y be the topological spaces and A be a nonempty subset of X. If $f : X \to Y$ is minimal $g^{\square}p$-continuous then the restriction map $f_A : A \to Y$ is a minimal $g^{\square}p$-continuous.*

Proof. Let $f : X \to Y$ is minimal $g^{\square}p$-continuous map. To prove that $f_A : A \to Y$ is a minimal $g^{\square}p$-continuous. Let N be any minimal $g^{\square}p$-open set in Y. Since f is minimal $g^{\square}p$-continuous, $f^{-1}(N)$ is a $g^{\square}p$-open set in X. But $f_A^{-1}(N) = A \cap f^{-1}(N)$ and $A \cap f^{-1}(N)$ is a $g^{\square}p$-open set in A. Therefore f_A is a minimal $g^{\square}p$-continuous. $\square$

Theorem 3.5.. *If $f : X \to Y$ is $g^{\square}p$-irresolute map and $g : Y \to Z$ is minimal $g^{\square}p$-continuous map, then $g \circ f : X \to Z$ is a minimal $g^{\square}p$-continuous.*

Proof. Let N be any minimal $g^{\square}p$-open set in Z. Since g is minimal $g^{\square}p$-continuous, $g^{-1}(N)$ is a $g^{\square}p$-open set in Y. Again since f is $g^{\square}p$-irresolute, $f^{-1}(g^{-1}(N)) = (g \circ f)^{-1}(N)$ is $g^{\square}p$-open set in X. Hence $g \circ f$ is a minimal $g^{\square}p$-continuous. $\square$

Theorem 3.6.. *Let X and Y be the topological spaces. A map $f : X \to Y$ is maximal $g^{\square}p$-continuous if and only if the inverse image of each minimal $g^{\square}p$-closed set in Y is a $g^{\square}p$-closed set in X.*

Proof. The proof follows from the definition and fact that the complement of maximal $g^{\square}p$-open set is minimal $g^{\square}p$-closed set. $\square$

Theorem 3.7.. *Let X and Y be the topological spaces and let A be a nonempty subset of X. If $f : X \to Y$ is maximal $g^{\square}p$-continuous then the restriction map $f_A : A \to Y$ is a maximal $g^{\square}p$-continuous.*

Proof. Similar to that of Theorem 3.4. $\square$

Theorem 3.8.. *If $f : X \to Y$ is $g^{\square}p$-irresolute map and $g : Y \to Z$ is maximal $g^{\square}p$-continuous map, then $g \circ f : X \to Z$ is a maximal $g^{\square}p$-continuous.*

Proof. Similar to that of Theorem 3.5. $\square$

Theorem 3.9.. *Every minimal $g^{\square}p$-irresolute map is minimal $g^{\square}p$-continuous map but not conversely.*

Proof. Let $f : X \to Y$ be a minimal $g^{\square}p$-irresolute map. Let N be any minimal $g^{\square}p$-open set in Y. Since f is minimal $g^{\square}p$-irresolute, $f^{-1}(N)$ is a minimal $g^{\square}p$-open set in X. That is $f^{-1}(N)$ is a $g^{\square}p$-open set in X. Hence f is a minimal $g^{\square}p$-continuous. □

Example 3.10. Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{b\},\ \{a,\ b\},\ \{b,\ c\}\}$ and $\sigma = \{Y,\ \phi,\ \{c\}\}$. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a map defined as $f(a) = a$, $f(b) = f(c) = c$. Then f is a minimal $g^{\square}p$-continuous map but it is not a minimal $g^{\square}p$-irresolute, since for the minimal $g^{\square}p$-open set $\{c\}$ in Y, $f^{-1}(\{c\}) = \{b, c\}$ which is not a minimal $g^{\square}p$-open set in X.

Theorem 3.10.. *Every maximal $g^{\square}p$-irresolute map is maximal $g^{\square}p$-continuous map but not conversely.*

Proof. Similar to that of Theorem 3.9. □

Example 3.11. Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\},\ \{c\},\ \{a,\ c\}\}$ and $\sigma = \{Y,\ \phi,\ \{b\},\ \{a,\ b\},\ \{b,\ c\}\}$. Let $f : (X,\ \tau) \to (Y,\ \sigma)$ be a map defined as $f(a) = b$, $f(b) = c$ and $f(c) = b$. Then f is a maximal $g^{\square}p$-continuous map but it is not a maximal $g^{\square}p$-irresolute in X.

Remark 3.6.. Minimal $g^{\square}p$-irresolute and $g^{\square}p$-continuous (resp. maximal $g^{\square}p$-continuous) maps are independent of each other.

Example 3.12. (i) Consider $X = Y = \{a,\ b,\ c\}$ with topologies $\tau = \{X,\ \phi,\ \{a\}, \{c\}, \{a, c\}\}$ and $\mu = \{Y,\ \phi,\ \{a\}, \{a, b\}\}$. Let $f : (X,\ \tau) \to (Y,\ \mu)$ be defined as $f(a) = a$, $f(b) = f(c) = b$. Then f is a $g^{\square}p$-continuous (resp. maximal $g^{\square}p$-continuous) but it is not a minimal $g^{\square}p$-irresolute.

(ii) In Example 3.11, a map defined as $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is a minimal $g^{\square}p$-irresolute but it is not a $g^{\square}p$-continuous (resp. maximal $g^{\square}p$-continuous).

Remark 3.7.. Maximal $g^{\square}p$-irresolute and $g^{\square}p$-continuous (resp. minimal $g^{\square}p$-continuous) maps are independent of each other.

Example 3.13. (i) Consider the Example 3.12(i), let $f : (X,\ \tau) \to (Y,\ \mu)$ be defined as $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is a $g^{\square}p$-continuous (resp. minimal $g^{\square}p$-continuous) but it is not a maximal $g^{\square}p$-irresolute.

(ii) Consider $X = Y = \{a,\ b,\ c\}$ with topologies

$$\tau = \{X,\ \phi,\ \{b\}, \{a, b\}, \{b, c\}\}$$

and

$$\mu = \{Y,\ \phi,\ \{a\}, \{c\}, \{a, c\}\}.$$

Let $f : (X,\ \tau) \to (Y,\ \mu)$ be defined as $f(a) = a$, $f(b) = c$ and $f(c) = b$. Then f is a maximal $g^{\star}p$-irresolute but it is not a $g^{\star}p$-continuous (resp. minimal $g^{\star}p$-continuous).

Remark 3.8.. Maximal $g^{\star}p$-irresolute and minimal $g^{\star}p$-irresolute maps are independent of each other.

Example 3.14. (i) In Example 3.13(ii), f is a maximal $g^{\star}p$-irresolute but it is not a minimal $g^{\star}p$-irresolute.

(ii) In Example 3.12(ii), f is a minimal $g^{\star}p$-irresolute but it is not a maximal $g^{\star}p$-irresolute.

Theorem 3.11.. *Let X and Y be the topological spaces. A map $f : X \to Y$ is minimal $g^{\star}p$-irresolute if and only if the inverse image of each maximal $g^{\star}p$-closed set in Y is a maximal $g^{\star}p$-closed set in X.*

Proof. The proof follows from the definition and fact that the complement of minimal $g^{\star}p$-open set is maximal $g^{\star}p$-closed set. □

Theorem 3.12.. *If $f : X \to Y$ and $g : Y \to Z$ are minimal $g^{\star}p$-irresolute maps, then $g \circ f : X \to Z$ is a minimal $g^{\star}p$-irresolute map.*

Proof. Let N be any minimal $g^{\star}p$-open set in Z. Since g is minimal $g^{\star}p$-irresolute, $g^{-1}(N)$ is a minimal $g^{\star}p$-open set in Y. Again since f is minimal $g^{\star}p$-irresolute, $f^{-1}(g^{-1}(N)) = (g \circ f)^{-1}(N)$ is a minimal $g^{\star}p$-open set in X. Therefore $g \circ f$ is a minimal $g^{\star}p$-irresolute. □

Theorem 3.13.. *Let X and Y be topological spaces. A map $f : X \to Y$ is maximal $g^{\star}p$-irresolute if and only if the inverse image of each minimal $g^{\star}p$-closed set in Y is a minimal $g^{\star}p$-closed set in X.*

Proof. The proof follows from the definition and fact that the complement of maximal $g^{\star}p$-open set is minimal $g^{\star}p$-closed set. □

Theorem 3.14.. *If $f : X \to Y$ and $g : Y \to Z$ are maximal $g^{\star}p$-irresolute maps, then $g \circ f : X \to Z$ is a maximal $g^{\star}p$-irresolute map.*

Proof. Similar to that of Theorem 3.12. □

Theorem 3.15.. *Every minimal-maximal $g^{\star}p$-continuous map is minimal $g^{\star}p$-continuous map but not conversely.*

Proof. Let $f : X \to Y$ be a minimal-maximal $g^{\star}p$-continuous map. Let N be any minimal $g^{\star}p$-open set in Y. Since f is minimal-maximal $g^{\star}p$-continuous, $f^{-1}(N)$ is a maximal $g^{\star}p$-open set in X. Since every maximal $g^{\star}p$-open set is a $g^{\star}p$-open set, $f^{-1}(N)$ is a $g^{\star}p$-open set in X. Hence f is a minimal $g^{\star}p$-

continuous. □

Example 3.15. Consider $X = Y = \{a, b, c\}$ with topologies $\tau = \{\phi, X, \{c\}\}$ and $\mu = \{\phi, Y, \{a\}\}$. Let $f : X \to Y$ be a map defined as $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is minimal $g^{\star}p$-continuous but it is not a minimal-maximal $g^{\star}p$-continuous, since for the minimal $g^{\star}p$-open set $\{a\}$ in Y, $f^{-1}(\{a\}) = \{c\}$, which is not a maximal $g^{\star}p$-open set in X.

Theorem 3.16.. *Every maximal-minimal $g^{\star}p$-continuous map is maximal $g^{\star}p$-continuous map but not conversely.*

Proof. Similar to that of Theorem 3.15. □

Example 3.16. In Example 3.15, f is maximal $g^{\star}p$-continuous but it is not a maximal-minimal $g^{\star}p$-continuous, since for the maximal $g^{\star}p$-open set $\{a, b\}$ in Y, $f^{-1}(\{a, b\}) = \{b, c\}$, which is not a minimal $g^{\star}p$-open set in X.

Remark 3.9.. From the above discussion and known results we have the following implications.

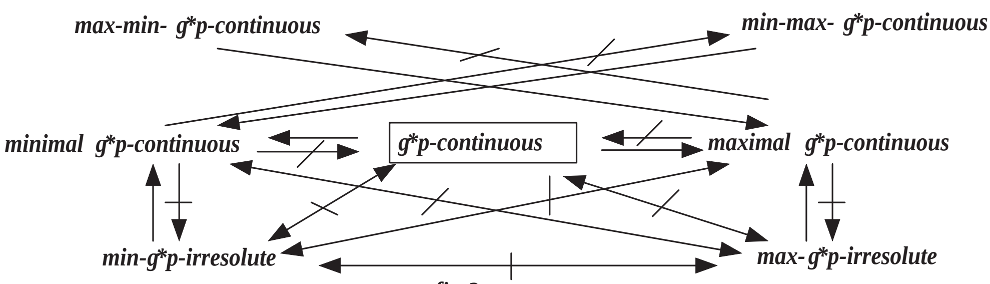

fig-2

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