

A NEW DECISION MAKING THEORY IN SOFT MATRICES

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Abstract: In 1999, Molodtsov introduced the concept of soft sets as a general mathematical tool for the dealing with uncertainty. The solution of such problems involves the use of mathematical principles based on uncertainty and imprecision. N. Cagman and S. Enginoglu [6] introduced the notion of soft matrices in 2010 from the theory of soft sets. They also constructed a soft max-min decision making method which can be successfully applied to the decision making for two person in soft matrix approach. Inspired by the soft matrices we are interested to find a new decision making theory which involves more than two persons or three persons. As an outcome in this paper, we apply the decision making for three persons in soft matrix approach and provide some interesting results on it.

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1. Introduction

In 1999, Molodstov [13] initiated a novel concept know as Soft Set as a new

mathematical tool for dealing with uncertainties. He pointed out that the important existing theories viz. Probability Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Sets, Rough Set Theory etc., which can be considered as mathematical tools for dealing with uncertainties, have their own difficulties. These theories cannot be successfully used to solve complicated problems in the fields of Engineering, Social Science, Economics, Medical Science etc. He further pointed out that the reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory. The Soft Set Theory introduced by Molodtsov [13] is free of the difficulties present in these theories. The absence of any restrictions on the approximate description in Soft Set Theory makes this theory very convenient and easily applicable. It has been demonstrated that Soft Set Theory brings about a rich potential for applications in many fields like functions smoothness, Riemann integration, decision making, measurement theory, game theory, etc.

By using the rough sets, P. K. Maji [10] first defined some operations on soft sets and introduced the soft set into the decision making problems [11]. D. Chen [7] introduced a new definition of Soft Set parameterization reduction. Kong et al. [8] introduced the definition of normal parameter reduction into Soft Sets. Ali [3] gave some new operations in Soft Set Theory. Cagman and Enginoglu [5] defined the new operations of soft sets and constructed the uni-int decision making method which selects a set of optimum elements from the alternatives. Cagman [6] defined soft matrices which were a matrix representation of the soft sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the soft sets represented by them in a computer. They also construct a soft decision making which is more practical and can be successfully applied to many problems that contain uncertainties without using the rough sets and Fuzzy Soft Sets.

In this paper, we first recall the definition of soft set theory and the example of soft sets theory. In section 3, we define the soft matrix theory and $max-max$ decision making method for two persons and also three persons.

2. Preliminaries

We recall some basic notions in soft set theory. Molodtsov [13] defined soft set in the following way. Throughout the paper, let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A, B, C \subseteq E$.

Definition 2.1. [13] A soft set F_A over U is a set defined by a function

f_A representing a mapping $f_A: E \rightarrow P(U)$ such that $f_A = \phi$ if $x \notin A$. Here, f_A is called approximate function of the soft set F_A . A soft set over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}.$$

Note that the set of all soft sets over U will be denoted by $S(U)$.

Remark 2.2. The subscript A in the notation f_A indicates that f_A is the approximate function of F_A . It can be defined more than one soft set in a subset A of the set of parameters E . In this time the approximate functions of F_A, G_A, H_A etc. must be f_A, g_A, h_A etc., respectively.

From now on, each soft set is often defined on a different subset of the parameters set whenever there is no confusion.

Example 2.3. A soft set F_A describes the various companies mobile. Suppose that there are five mobiles in the universe $U = \{m_1, m_2, m_3, m_4, m_5\}$ under consideration, and $E = \{x_1, x_2, x_3, x_4\}$ is the set of parameters. The $x_i (i = 1, 2, 3, 4)$ stand for the parameters expensive, beautiful, branded, cheap, respectively. In this case, to define a soft set means to point out expensive mobile, beautiful mobile, and so on.

Assume that $A = x_1, x_2, x_3$ and $B = x_2, x_3, x_4 \subseteq E$.

$$f_A(x_1) = \{m_1, m_3, m_4, m_5\}, f_A(x_2) = \{m_2, m_3, m_5\}, f_A(x_3) = \{m_4\},$$

$$g_A(x_1) = \{\phi\}, g_A(x_2) = \{m_1, m_3, m_4\}, g_A(x_3) = \{U\},$$

$$f_B(x_2) = \{m_1, m_3, m_5\}, f_B(x_3) = \{m_2, m_3\}, f_B(x_4) = \{m_2\}.$$

Then we can view the soft sets F_A, G_A and F_B as consisting of the following collection of approximations:

$$F_A = \{(x_1, \{m_1, m_3, m_4, m_5\}), (x_2, \{m_2, m_3, m_5\}), (x_3, \{m_4\})\},$$

$$G_A = \{(x_2, \{m_1, m_3, m_4\}), (x_3, \{U\})\},$$

$$F_B = \{(x_2, \{m_1, m_3, m_5\}), (x_3, \{m_2, m_3\}), (x_4, \{m_2\})\}.$$

Here, $g_A(x_1) = \phi$ means that there is no element in U related to the parameter $x_1 \in E$. Therefore, we do not display such elements in the soft sets.

Using operation with soft sets defined by Molodtstov [13], the Cartesian product and relation between two soft sets are defined below.

Definition 2.4. [11] Let (F,A) and (G,B) be two soft sets over a common universe U , then the Cartesian product of these two soft sets is denoted by $(F, A) \times (G, B)$ and is defined by $(F, A) \times (G, B) = (H, A \times B)$ where $A \times B$ is the Cartesian product of sets A and B . $H(\alpha, \beta) = F(\alpha) \times G(\beta), \alpha \in A, \beta \in B$, $A \times B$ is the Cartesian product of sets A and B .

Definition 2.5. [11] Let (F, A) and (G, B) be two soft sets then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.6. [11] Let (F, A) and (G, B) be two soft sets then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cup G(\beta)$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.7. [11] A soft relation may be defined as a soft set over the power set of the cartesian product of two crisp sets. If X and Y are two non-empty crisp sets of some universal set and E is a set of parameters then a soft relation denoted as (R, E) is defined as a mapping from E to $P(X \otimes Y)$.

Definition 2.8. [11] Let (F, A) and (G, B) be two soft sets over a common universe U , then the relation R of these two soft sets is defined as $(R, A \times B) \subset (F, A) \times (G, B)$ such that $F(\alpha) \times G(\beta) \in (R, A \times B)$ implies that $F(\alpha)$ is related with $G(\beta)$ where $\alpha \in A, \beta \in B$.

Using operation with soft sets defined by Molodtstov [13], and Maji [10], the soft matrix and its operations defined below:

Definition 2.9. [6] Let F_A be a soft set over U . Then, a subset of $U \times E$ is uniquely defined by $R_A = \{(m, x) : x \in A, m \in f_A(x)\}$ which is called a relation from of F_A . The characteristic function of R_A is written by $\imath R_A : U \times E \rightarrow \{0, 1\}$,

$$\imath R_A(m, x) = \begin{cases} 1, & (m, x) \in R_A \\ 0, & (m, x) \notin R_A \end{cases}$$

If $U = \{m_1, m_2, m_3, m_4, \dots, m_n\}, E = \{x_1, x_2, x_3, \dots, x_p\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form

R_A	x_1	x_2	\dots	x_p
m_1	$\imath R_A(m_1, x_1)$	$\imath R_A(m_1, x_2)$	\dots	$\imath R_A(m_1, x_p)$
m_2	$\imath R_A(m_2, x_1)$	$\imath R_A(m_2, x_2)$	\dots	$\imath R_A(m_2, x_p)$
\dots	\dots	\dots	\dots	\dots
m_n	$\imath R_A(m_n, x_1)$	$\imath R_A(m_n, x_2)$	\dots	$\imath R_A(m_n, x_p)$

If $a_{ij} = \imath R_A(m_i, x_j)$, we define a matrix $SM_{m \times p} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1p} \\ a_{21} & a_{22} & \dots & a_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{np} \end{pmatrix}$,

which is called an $n \times p$ soft matrix of the soft set F_A over U .

Example 2.10. Let $F_A = \{((x_1), \{m_1, m_3, m_5\}), ((x_2), \{m_2, m_3\}), ((x_3), \{m_4\})\}$. Then, the relation form of F_A is written by $R_A = \{(m_1, x_1), (m_2, x_2), (m_3, x_1), (m_3, x_2), (m_4, x_3), (m_5, x_1)\}$.

Hence, the soft matrix $[a_{ij}] = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

Definition 2.11. [6] Let $[a_{ij}] \in SM_{n \times p}$. Then $[a_{ij}]$ is called a zero soft matrix, denoted by $[0]$, if $a_{ij} = 0$ for all i and j .

Definition 2.12. [6] Let $[a_{ij}] \in SM_{n \times p}$. Then $[a_{ij}]$ is called an a universal soft matrix, denoted by $[1]$, if $a_{ij} = 1$ for all i and j .

Definition 2.13. [16] Let (F, A) be a soft set defined on the universe U . Then a soft matrix over (F, A) is denoted by $[M(F, A)]$ is a matrix whose elements are the elements of the soft set (F, A) . Mathematically, $[M(F, A)] = (m_{ij})$ where $m_{ij} = F(x)$ for some $x \in A$.

Example 2.14. Let us consider $A = E = \{x_1, x_2, x_3, x_4\}$ and $U = \{m_1, m_2, m_3, m_4, m_5\}$ where $(F, A) = \{F(x_1) = \text{expensive mobiles} = \{m_1, m_3, m_5\}, F(x_2) = \text{beautiful mobiles} = \{m_1, m_3, m_4, m_5\}, F(x_3) = \text{branded mobiles} = \{m_2, m_3, m_4\}, F(x_4) = \text{cheapest mobiles} = \{m_2\}\}$. Let

$$[M(F, A)] = \begin{bmatrix} \text{expensive mobiles} = \{m_1, m_3, m_5\} & \text{branded mobiles} = \{m_2, m_3, m_4\} \\ \text{beautiful mobiles} = \{m_1, m_3, m_4, m_5\} & \text{cheapest mobiles} = \{m_2\} \end{bmatrix}$$

Here we see that all the elements of the matrix $[M(F, A)]$ are of the soft set (F, A) . Hence the above matrix is a soft matrix.

3. Product Soft Matrices

In this section, using the theory of soft matrix introduced by N. Cagman[5] and Cartesian product of two soft sets introduced by M. Pal[16], we define a

new notion of soft matrices called **product soft matrices**.

Definition 3.1. Let U be the universal set. Let (F, A) and (G, B) be a two soft sets over common universe U . Then the Cartesian product soft sets $(F, A) \times (G, B) = (H, A \times B)$.

Example 3.2. Let us consider two soft sets A and B over a common universe U . Let $U = \{m_1, m_2, m_3, m_4, m_5\}$, $A = \{x_1, x_2\} \subseteq E$ and $B = \{x_2, x_3\} \subseteq E$. Then $F(x_1) = \{m_1, m_3, m_4\}$, $F(x_2) = \{m_2, m_3, m_5\}$, $G(x_2) = \{m_1, m_3\}$ and $G(x_3) = \{m_2, m_3\}$. Here $A \times B = \{(x_1, x_2), (x_1, x_3), (x_2, x_2), (x_2, x_3)\}$. Then, the relation form of

$$R_{(H,A \times B)} = \{((x_1, x_2), \{(m_1, m_1), (m_1, m_3), (m_3, m_1), (m_3, m_3), (m_4, m_1), (m_4, m_3)\}), ((x_1, x_3), \{(m_1, m_2), (m_1, m_3), (m_3, m_2), (m_3, m_3), (m_4, m_2), (m_4, m_3)\}), ((x_2, x_2), \{(m_2, m_1), (m_2, m_3), (m_3, m_1), (m_3, m_3), (m_5, m_1), (m_5, m_3)\}), ((x_2, x_3), \{(m_2, m_2), (m_2, m_3), (m_3, m_2), (m_3, m_3), (m_5, m_2), (m_5, m_3)\})\}.$$

Definition 3.3. Let U be the universal set. Let (F, A) and (G, B) be a two soft sets over common universe. Then the Cartesian product soft sets $(F, A) \times (G, B) = (H, A \times B)$. We define the relation of $(H, A \times B)$ is $R_{(H,A \times B)} = \{(h, e) : h \in H(\alpha, \beta), e \in A \times B\}$. The special function of $R_{(H,A \times B)}$ is written by

$$\tilde{C}R_{(H,A \times B)} : U \times E \rightarrow \{0, \frac{1}{2}, 1\}, \tilde{C}R_{(H,A \times B)} = \begin{cases} 1, & \text{if } (h, e) \in A \cap B \\ \frac{1}{2} = 0.5, & \text{if } (h, e) \in A \Delta B \\ 0, & \text{if } (h, e) \notin A \cup B \end{cases}$$

If $(d_{ij}) = d(h, e)$, we define a matrix $(d_{ij})_{(n \times p)} = \begin{pmatrix} d_{11} & d_{12} & \dots & d_{1p} \\ d_{21} & d_{22} & \dots & d_{2p} \\ \vdots & \vdots & \vdots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{np} \end{pmatrix}$ is

called as a **product soft matrices**, where n is the number of elements is U

and p is the product of the number of element in the set A and the number of element in the set B .

Definition 3.4. Let $(F, A) = (a_{ij})$ and $(G, B) = (b_{ij})$ are two soft matrices. Then $(F, A)AND(G, B)$ is denoted by $[(F, A)] \wedge [(G, B)] = [H, A \times B] = (d_{ij})$ is a soft matrix, is defined by $(d_{ij}) = (a_{ij}) \cap (b_{ij})$.

Example 3.5. Let $U = \{m_1, m_2, m_3, m_4, m_5\}$ and $E = \{x_1, x_2, x_3, x_4\}$. Assume that $A = \{x_1, x_2, x_3\}$ and $B = \{x_2, x_3, x_4\} \subseteq E$. Let

$$(F, A) = \{((x_1), \{m_1, m_3, m_4, m_5\}), ((x_2), \{m_2, m_3, m_5\}), ((x_3), \{m_4\})\}$$

$$(F, B) = \{((x_2), \{m_1, m_3, m_5\}), ((x_3), \{m_2, m_3\}), ((x_4), \{m_2\})\}.$$

Hence

$$(F, A) \wedge (F, B) = \{((x_1, x_2), \{m_1, m_3, m_5\}), ((x_1, x_3), \{m_3\}), ((x_1, x_4), \{\phi\}), ((x_2, x_2), \{m_3, m_5\}), ((x_2, x_3), \{m_2, m_3\}), ((x_2, x_4), \{m_2\}), (x_3, x_2), \{\phi\}), ((x_3, x_3), \{\phi\}), ((x_3, x_4), \{\phi\})\}.$$

Hence, the product soft matrices is $(d_{ij})_{(5 \times 9)} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Definition 3.6. Let $(F, A) = (a_{ij})$ and $(G, B) = (b_{ij})$ are two soft matrices. Then $(F, A)OR(G, B)$ is denoted by $[(F, A)] \vee [(G, B)] = [H, A \times B] = (d_{ij})$ is a soft matrix, is defined by $(d_{ij}) = (a_{ij}) \cup (b_{ij})$.

Example 3.7. Let $U = \{m_1, m_2, m_3, m_4, m_5\}$ and $E = \{x_1, x_2, x_3, x_4\}$. Assume that $A = \{x_1, x_2, x_3\}$ and $B = \{x_2, x_3, x_4\} \subseteq E$. Let

$$(F, A) = \{((x_1), \{m_1, m_3, m_4, m_5\}), ((x_2), \{m_2, m_3, m_5\}), ((x_3), \{m_4\})\}$$

$$(G, B) = \{((x_2), \{m_1, m_3, m_5\}), ((x_3), \{m_2, m_3\}), ((x_4) = \{m_2\})\}.$$

Hence

$$(F, A) \cup (G, B) = \{((x_1, x_2), \{m_1, m_3, m_4, m_5\}), ((x_1, x_3), \{m_1, m_2, m_3, m_4, m_5\}), \\ ((x_1, x_4), \{m_1, m_2, m_3, m_4, m_5\}), ((x_2, x_2), \{m_1, m_2, m_3, m_5\}), \\ ((x_2, x_3), \{m_2, m_3, m_5\}), ((x_2, x_4), \{m_2, m_3, m_5\}), ((x_3, x_2), \\ \{m_1, m_3, m_4, m_5\}), ((x_3, x_3), \{m_2, m_3, m_4\}), ((x_3, x_4), \{m_2, m_4\})\}.$$

Hence, the product soft matrices is

$$(d_{ij})_{(5 \times 9)} = \begin{pmatrix} 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.5 & 1 & 1 & 0.5 & 0.5 & 0.5 \\ 1 & 1 & 0.5 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 1 & 0.5 & 0.5 & 1 & 0.5 & 0.5 & 0.5 & 0 & 0 \end{pmatrix}$$

Definition 3.8. The Choice value of an Object $d_j \in U$ is defined by $d_j = \max\{\max(d(h,e))\}$, where $d(h,e)$ are the entries of d_{ij} .

Definition 3.9. Let $U = \{m_1, m_2, m_3, \dots, m_n\}$ be an initial universe and $\max\{\max(d(h,e))\} = [u_{i1}]$. Then a subset of U can be obtained by using $[u_{i1}]$ as in the following way $opt_{[u_{i1}]} = \{u_i : u_i \in U, u_{i1} = \max(1 \text{ or } \frac{1}{2})\}$, which is called an optimum set of U .

Assume that a set of alternative and a set of parameters are given. Now, we can construct a soft $max - max$ decision making method by the following algorithm.

- Step 1: Choose feasible subsets of the set of parameters,
- Step 2: Construct the Cartesian product soft set,
- Step 3: Find a Cartesian or product of the soft matrices,
- Step 4: Compute the $max - max$ decision matrix of the product.
- Step 5: Find an optimum set of U .

Example 3.10. Assume that a mobile shop has a set of different types of mobiles $U = \{m_1, m_2, m_3, m_4, m_5\}$ which may be characterized by a set of parameters $E = \{x_1, x_2, x_3, x_4\}$. For $j = 1, 2, 3, 4$, the parameters x_j stand for expensive, beautiful, branded, cheap, respectively.

Step 1: Suppose that the two friends *Mr.X* and *Mr.Y* have the choose the sets of their parameters, $A = \{x_1, x_2, x_3\}$ and $B = \{x_2, x_3, x_4\}$, respectively.

Step 2: The Cartesian product soft set are

$$(F, A) = \{((x_1), \{m_1, m_3, m_4, m_5\}), ((x_2), \{m_2, m_3, m_5\}), ((x_3), \{m_4\})\}$$

$$(G, B) = \{((x_2), \{m_1, m_3, m_5\}), ((x_3), \{m_2, m_3\}), ((x_4), \{m_2\})\}.$$

The “OR product” of cartesian set is defined as

$$\begin{aligned} (F, A) \vee (G, B) = & \{((x_1, x_2), \{m_1, m_3, m_4, m_5\}), ((x_1, x_3), \{m_1, m_2, m_3, m_4, m_5\}), \\ & ((x_1, x_4), \{m_1, m_2, m_3, m_4, m_5\}), ((x_2, x_2), \{m_1, m_2, m_3, m_5\}), \\ & ((x_2, x_3), \{m_2, m_3, m_5\}), ((x_2, x_4), \{m_2, m_3, m_5\}), ((x_3, x_2), \\ & \{m_1, m_3, m_4, m_5\}), ((x_3, x_3), \{m_2, m_3, m_4\}), ((x_3, x_4), \{m_2, m_4\})\}. \end{aligned}$$

Step 3: Now, we can find a cartesian product of the soft matrix A and B by using *Or – product* as follows

$$(d_{ij})_{(5 \times 9)} = \begin{pmatrix} 1 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.5 & 1 & 1 & 0.5 & 0.5 & 0.5 \\ 1 & 1 & 0.5 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 1 & 0.5 & 0.5 & 1 & 0.5 & 0.5 & 0.5 & 0 & 0 \end{pmatrix}$$

Step 4: We can find a max-max decision soft matrix as

$$\begin{aligned} & \text{Max}\{\text{Max values in every column}\} \\ & = \text{Max}\{\{m_1, m_3, m_5\}, \{m_3\}, \{m_1, m_2, m_3, m_4, m_5\}, \end{aligned}$$

$$\{m_3, m_5\}, \{m_2, m_3\}, \{m_2\}, \{m_1, m_2, m_3, m_4, m_5\}, \{m_2, m_3, m_4\}, \{m_2, m_4\} = m_3.$$

And $(d_j) = (0 \ 0 \ 1 \ 0 \ 0)^T$.

Step 5: Finally, we can find an optimal set of U according to

$$Max\{Max(d(h, e))\} = [u_{i1}] = m_3,$$

where m_3 is an optimum mobile to buy for $Mr.X$ and $Mr.Y$.

Definition 3.11. Let $(F, A) = (a_{ij}), (G, B) = (b_{ij})$ and (M, C) are three soft matrices. Then $(F, A)OR(G, B)OR(M, C)$ is denoted by $[(F, A)] \vee [(G, B)] \vee [(M, C)] = [H, A \times B \times C] = (d_{ij})$ is a soft matrix, is defined by $(d_{ij}) = (a_{ij}) \cup (b_{ij}) \cup (c_{ij})$.

Example 3.12. Assume that a mobile shop has a set of different types of mobiles $U = \{m_1, m_2, m_3, m_4, m_5\}$ which may be characterized by a set of parameters $E = \{x_1, x_2, x_3, x_4\}$. For $j = 1, 2, 3, 4$ the parameters x_j stand for expensive, beautiful, branded, cheap, respectively.

Step 1: The three friends $Mr.X, Mr.Y$ and $Mr.Z$ have the choose the sets of their parameters, $A = \{x_1, x_2, x_3\}, B = \{x_2, x_3, x_4\}$ and $C = \{x_1\}$ respectively.

Step 2: The cartesian product of the three soft sets are

$$\begin{aligned} (F, A) &= \{((x_1), \{m_1, m_3, m_4, m_5\}), ((x_2), \{m_2, m_3, m_5\}), ((x_3), \{m_4\})\} \\ (G, B) &= \{((x_2), \{m_1, m_3, m_5\}), ((x_3), \{m_2, m_3\}), ((x_4), \{m_2\})\} \text{ and} \\ (H, C) &= \{((x_1), \{m_1, m_3\})\}. \end{aligned}$$

The Or product of cartesian set is defined as $(F, A) \vee (G, B) \vee (H, C)$

$$\begin{aligned} &= \{((x_1, x_2, x_1), \{m_1, m_3, m_4, m_5\}), ((x_1, x_3, x_1), \{m_1, m_2, m_3, m_4, m_5\}), \\ &((x_1, x_4, x_1), \{m_1, m_2, m_3, m_4, m_5\}), ((x_2, x_2, x_1), \{m_1, m_2, m_3, m_5\}), \\ &((x_2, x_3, x_1), \{m_1, m_2, m_3, m_5\}), ((x_2, x_4, x_1), \{m_1, m_2, m_3, m_5\}), \\ &((x_3, x_2, x_1), \{m_1, m_3, m_4, m_5\}), ((x_3, x_3, x_1), \{m_1, m_2, m_3, m_4\}), \end{aligned}$$

$$((x_3, x_4, x_1), \{m_1, m_2, m_3, m_4\})\}$$

Step 3: Now, we can find a cartesian product of the soft matrix A , B and C by using OR-product as follows

$$(d_{ij})_{(5 \times 9)} = \begin{pmatrix} 1 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0.5 & 0.5 \\ 1 & 1 & 0.5 & 1 & 1 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0 \end{pmatrix}$$

Note: Let p_1 be the maximum element in the sets $\{A,B$ and $C\}$ and let p_2 be the next maximum element in the sets $\{A,B$ and $C\}$. Let $p = p_1 \times p_2$.

Step 4: We can find a max-max decision soft sets as

$$\begin{aligned} \text{Max \{Max values in every column\}} &= \text{Max \{ \{m_1, m_3\}, \{m_3\}, \{m_1, m_2, m_3, m_4,} \\ & m_5\}, \{m_3\}, \{m_3\}, \{m_1, m_2, m_3, m_5\}, \{m_1, m_3, m_4, m_5\}, \{m_1, m_2, m_3, m_4\}, \\ & \{m_1, m_2, m_3, m_4\} \}} = m_3, \text{ and } (d_j) = (0 \ 0 \ 1 \ 0 \ 0)^T. \end{aligned}$$

Step 5: Finally, $Mr.X$, $Mr.Y$, and $Mr.Z$ combinations buy the Mobile m_3 .

4. Conclusion

Thus, we have presented a new algorithm using the product soft marix and proposed a new "OR" operations of soft matrix to solve soft matrix based decision making problems. The advantage of this new suggested method is that it is very convenient and easy to apply when compared with the other methods. This new method is also used to apply for soft sets and soft matrices based on decision making problems involving any number of decision makers.

References

- [1] Abdul Razak Shalleh, Shawkat Alkhazaleh, Nasruddin Hassan and Abd Ghafur Ahmad, Multiparameterized soft set, *Journal of Mathematics and Statistics*, **8**, No. 1 (2012), 92-97.
- [2] H. Aktas and N. Cagman, Soft Sets and Soft Groups, *Information Sciences*, **1** (77) (2007), 2726-2735.
- [3] M.I. Ali, F. Feng, X. Liu, W.K. Min and M. Shabir, *On Some new operations in Soft Set theory*, Computers and Mathematics with Applications, Vol. 57 pp. 1547-1553,2009.
- [4] Arindam Chaudhuri, Dr. Kajal De, Dr. Dipak Chatterjee, *Solution of the Decision Making Problems Using Fuzzy Soft Relations*, International Journal of Information Technology, Vol. 15, pp. No.1, 2009.
- [5] N. Cagman and S. Enginoglu, *Soft set theory and uni-int decision making*, European Journal of Operational Research, Vol. 207, pp. 848-855, 2010.
- [6] N. Cagman and S. Enginoglu, *Soft matrix theory and its decision making*, Computers and Mathematics with Applications, Vol. 59, pp. 3308-3314, 2010.
- [7] Degang Chen, E.C.C. Tsang And Daniel S. Yeung, Xizhao Wang, *The Parameterization Reduction of Soft Sets and its Applications*, Computers and Mathematics with Applications, Vol. 49, pp. 757-763,2005.
- [8] Z. Kong, L. Gao, L. Wang and S. Li, *The normal parameter reduction of Soft Sets and its algorithm*, Computers and Mathematics with Applications, Vol. 56, pp. 3029-3037,2008.
- [9] D.V. Kovkov, V.M. Kolbanov and D.A. Molodtsov, *Soft Set Theory - Based optimization*, Journal of Computer and System Sciences International, Vol. 46(6), pp. 872-880,2007.
- [10] P. K. Maji, R. Bismas, A.R. Roy, *Soft Set Theory* , Computers and Mathematics with Applications, Vol. 45, pp. 555-562,2003.
- [11] P. K. Maji, A. R. Roy, *An Application of Soft Set in Decision Making Problem*, Computers and Mathematics with Applications, Vol. 44, pp. 1077-1083, 2002.

- [12] P.K. Maji, A.R. Roy, *A Fuzzy Soft Set Theoretic Approach to decision making problem*, Journal of computational and Applied Mathematics, Vol. 203, pp. 412-418,2007.
- [13] D.A.Molodstov, *Soft Set Theory - First Result*, Computers and Mathematics with Applications, Vol. 37, pp. 19-31,1999.
- [14] T.K. Neog and D.K. Sut, *A New Approach to the Theory of Soft Sets*, International Journal of Computer Applications, Vol. 32, No.2, pp. 1-6,2011.
- [15] Z. Pawlak, *Rough Sets*, International Journal of Information and Computer Sciences, 11 (1982) 431-356.
- [16] Sanjib Mondal and Madhumangal Pal, *Soft matrices*, African Journal of Mathematics and Computer Science Research, Vol. 4(13), pp. 379-388, December 2011.
- [17] Shawkat Alkhazaleh, Abdul Razak Salleh, Nasruddin Hassan, *Soft Multisets Theory*, Applied Mathematical Sciences, Vol. 5(72), pp. 3561-3573, 2011.
- [18] Xun Ge and Yang, *Investigations on Some Operations of Soft Sets*, World Academy of Science, Engineering and Technology, Vol. 75, pp. 1113-1116, 2011.

