International Journal of Pure and Applied Mathematics
Volume 92 No. 1 2014, 99-108
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v92i1.8

ijpam.eu

ON THE ADJACENT STRONG EDGE COLORING OF $P_n \vee P_n$ $P_n \vee C_n$ AND $C_n \vee C_n$

Zhao Chuancheng[1 §], Ren Zhiguo[2], Yao Shuxia[3], Liu Jun[4]
[1,2,4]Institute of Information and Engineering
Lanzhou City University
Lanzhou, 730070, P.R. CHINA
[3]Department of Mathematics
Lanzhou City University
Lanzhou, 730070, P.R. CHINA

Abstract: In this paper, we discuss the adjacent strong edge coloring of join-graphs about $P_n \vee P_n$ and $P_n \vee C_n$ and $C_n \vee C_n$.

AMS Subject Classification: 05C15
Key Words: path, cycle, join-graph, adjacent strong edge coloring

1. Introduction

The coloring problem of graphs is widely applied in practice. In [1], some conditional coloring problems as introduced. Some network problem can be converted to the strong edge coloring (see [2]-[5]) and adjacent strong edge coloring, see [6].

Definition 1. (see [2]-[5]) For a graph $G(V, E)$, if a proper coloring f is satisfied with $C(u) \neq C(v)$ for $\forall u, v \in V(G)(u \neq v)$, then f is called *k-strong edge coloring of G*, is abbreviated k-SEC, and

$$\chi_s^{'}(G) = \min\{k|k - SEC\ of\ G\}$$

is called the strong edge chromatic number of G. And for $\forall uv \in E(G)$,$C(u) \neq$

Received: December 31, 2013

§Correspondence author

$C(v)$, then f is called k-adjacent strong edge coloring of G, is abbreviated k-ASEC, and

$$\chi'_{as}(G) = \min\{k|k - ASEC\ of\ G\}$$

is called the adjacent strong edge chromatic number of G[6].Where

$$C(u) = \{f(uv)|uv \in E(G)\}.$$

Conjecture. (see [6]) *For a connected graph with order $p \geq 3$, and $G \neq C_5$ (5-cycle),*

$$\chi'_{as}(G) \leq \Delta(G) + 2.$$

Where $p = |V(G)|$, $\Delta(G)$ is maximal degree of G.

There are many references proof this conjecture is true, for example [7],[8], for $\Delta(G) \leq 3$, this conjecture is true; For a connected graph with $|V(G)| \geq 3$:

(1) If G is a bipartite graph with no isolate edges, then

$$\chi'_{as}(G) \leq \Delta(G) + 2.$$

(2)If G is a k-chromatic graph with no isolate edges, then

$$\chi'_{as}(G) \leq \Delta(G) + O(log\ k).$$

Definition 2. (see [9]) For graph G and graph H, $V(G) \cap V(H) = \emptyset$, and

$$\begin{cases} V(G \vee H) = V(G) \cup V(H) \\ E(G \vee H) = E(G) \cup E(H) \cup \{uv|u \in V(G), v \in V(H)\} \end{cases}$$

then $G \vee H$ is called join-graph of G and H.

For $m > n \geq 1$, there are many adjacent strong chromatic number of $P_m \vee P_n, P_m \vee C_n, C_m \vee C_n$. In this paper we have the adjacent strong chromatic number of $P_n \vee P_n, P_n \vee C_n, C_n \vee C_n$, the others terminologies refer to [9]-[10].

2. Adjacent Strong Edge Coloring of $P_n \vee P_n$

Lemma 1. (see [6]) *If G is a connected graph with $|V(G)| \geq 3$, and $uv \in E(G)$, $d(u)$=$d(v)$=$\Delta(G)$, then*

$$\chi'_{as}(G) \geq \Delta(G) + 1.$$

Theorem 1. *For $n \geq 2$, then*

$$\chi'_{as}(P_n \vee P_n) = \begin{cases} n+3, & if\ 2 \leq n \leq 6; \\ n+4, & if\ n \geq 7. \end{cases}$$

Proof. There are four cases to be considered.

Case 1. When n=2, then $P_2 \vee P_2 = K_4$ (complete graph with order 4), it's true by [6].

Case 2. When $3 \leq n \leq 6$, $\Delta(P_n \vee P_n) = n + 2$. By Lemma 1, we need to prove exists $(n+3)$-ASEC.
Let P_n and P_n be $u_1u_2 \cdots u_n$ and $v_1v_2 \cdots v_n$

Subcase 2.1. When n=3, a mapping f from $E(P_3 \vee P_3)$ to $\{1, 2, 3, 4, 5, 6\}$ is defined as follows:

$$f(u_2v_2) = f(u_3v_1) = 1; f(u_2v_3) = f(v_1v_2) = 2; f(u_1v_3) = f(u_2u_3) = 3;$$

$$f(u_1u_2) = f(u_3v_2) = 4; f(u_1v_2) = f(u_2v_1) = f(u_3v_3) = 5;$$

$$f(u_1v_1) = f(v_2v_3) = 6.$$

Obviously, f ia 6-ASEC of $P_3 \vee P_3$.

Subcase 2.2. When n=4, a mapping f from $E(P_4 \vee P_4)$ to $\{1, 2, 3, 4, 5, 6, 7\}$ is defined as follows:

$$f(u_1v_i) = i, i = 1, 2, 3, 4; f(u_iv_j) = i + j + 1, i = 2, 3; j = 1, 2, 3, 4;$$

$$f(u_4v_1) = 5; f(u_4v_2) = 3; f(u_4v_3) = 7; f(u_4v_4) = 1;$$

$$f(u_1u_2) = f(v_1v_2) = 7; f(u_2u_3) = f(v_2v_3) = 1; f(u_3u_4) = f(v_3v_4) = 2.$$

For such f, we have:

$$\overline{C}(u_1) = \{5, 6\}; \overline{C}(u_4) = \{3, 6\}; \overline{C}(v_1) = \{2, 6\}; \overline{C}(v_4) = \{3, 5\};$$

$$\overline{C}(u_2) = \{2\}; \overline{C}(u_3) = \{3\}; \overline{C}(v_2) = \{6\}; \overline{C}(v_3) = \{4\};$$

Where $\overline{C}(w) = \{1, 2, 3, 4, 5, 6, 7\} \backslash C(w)$, $w \in \{u_1, u_2, u_3, u_4\} \cup \{v_1, v_2, v_3, v_4\}$. So f is 7-ASEC of $P_4 \vee P_4$. The conclusion is true.

Subcase 2.3. When n=5, a mapping f from $E(P_5 \vee P_5)$ to $\{1, 2, 3, 4, 5, 6, 7, 8\}$ is given as follows:

$$f(u_1v_1) = 6;\ f(u_1v_j) = j, j = 2, 3, 4;\ f(u_1v_5) = 5;$$

$$f(u_iv_j) = i + j - 1, i = 2, 3, 4;\ j = 1, 2, 3, 4, 5;$$

$$f(u_5v_1) = 5;\ f(u_1u_2) = f(u_5v_2) = 7;\ f(u_2u_3) = f(v_1v_2) = f(u_5v_3) = 8;$$

$$f(u_3u_4) = f(v_2v_3) = f(u_5v_4) = 1;\ f(u_5v_5) = 4;$$

$$f(u_4u_5) = f(v_3v_4) = 2;\ f(v_4v_5) = 3.$$

For such f, we have:

$$\overline{C}(u_1) = \{1, 8\}; \overline{C}(u_i) = \{i - 1\}, i = 2, 3, 4; \overline{C}(u_5) = \{3, 6\}; \overline{C}(v_1) = \{1, 7\};$$

$$\overline{C}(v_i) = \{i + 4\}, i = 2, 3, 4; \overline{C}(v_5) = \{1, 2\}.$$

Where $\overline{C}(w) = \{1, 2, 3, 4, 5, 6, 7, 8\} \backslash C(w)$,$w \in \{u_i | i = 1, 2, 3, 4, 5\} \cup \{v_j | j = 1, 2, 3, 4, 5\}$. So f is 8-ASEC of $P_5 \vee P_5$. The conclusion is true.

Subcase 2.4. When $n = 6$, a mapping f from $E(P_6 \vee P_6)$ to $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is given as follows:

$$f(u_1v_1) = 5; f(u_1v_2) = 7; f(u_1v_3) = 2; f(u_1v_4) = 9; f(u_1v_5) = 4; f(u_1v_6) = 6;$$

$$f(u_2v_1) = 9; f(u_3v_1) = 8; f(u_4v_1) = 4; f(u_5v_1) = 3; f(u_6v_1) = 6;$$

$$f(u_iv_j) = i + j - 1, i = 2, 3, 4, 5; j = 2, 3, 4, 5;$$

$$f(u_6v_2) = 8; f(u_6v_3) = 3; f(u_6v_4) = 2; f(u_6v_5) = 1; f(u_6v_6) = 7;$$

$$f(u_2v_6) = 8; f(u_3v_6) = 9; f(u_4v_6) = 3; f(u_5v_6) = 5;$$

$$f(u_1u_2) = f(u_3u_4) = f(v_1v_2) = f(v_3v_4) = 1;$$

$$f(u_2u_3) = f(u_4u_5) = f(v_5v_6) = 2; f(v_4v_5) = 3;$$

$$f(u_5u_6) = 4; f(v_2v_3) = 9.$$

For the f, we have

$$\overline{C}(u_1) = \{3, 8\}; \overline{C}(u_2) = \{7\}; \overline{C}(u_3) = \{3\}; \overline{C}(u_4) = \{9\};$$

$$\overline{C}(u_5) = \{1\}; \overline{C}(u_6) = \{5, 9\}; \overline{C}(v_1) = \{2, 7\}; \overline{C}(v_2) = \{2\};$$
$$\overline{C}(v_3) = \{8\}; \overline{C}(v_4) = \{4\}; \overline{C}(v_5) = \{5\}; \overline{C}(v_6) = \{1, 4\}.$$

So, the f is a 9-ASEC of $P_6 \vee P_6$, the Theorem 2 is true when n=6.

Case 3. When $n = 7$, first we prove $\chi'_{as}(P_n \vee P_n) \geq 11$. Otherwise, $\chi'_{as}(P_n \vee P_n) = 10$ by Lemma 1.

Let $C = \{1, 2, \cdots, 10\}$,$\overline{C}(w) = C \backslash C(w)$, suppose f of 10-ASEC of $P_7 \vee P_7$. We have

$$\begin{cases} |\overline{C}(u_i)| = |\overline{C}(v_i)| = 2, i = 1, 7; \\ |\overline{C}(u_i)| = |\overline{C}(v_i)| = 1, i = 2, 3, 4, 5, 6. \end{cases} \tag{1}$$

Obviously, we have

$$\begin{cases} \overline{C}(u_i) \neq \overline{C}(v_i), & i = 1, 2, 3, 4, 5, 6, 7; \\ \overline{C}(u_i) \neq \overline{C}(u_{(i+1)}, \overline{C}(u_i) \neq \overline{C}(u_{(i+1)}, & i = 1, 2, 3, 4, 5, 6. \end{cases} \tag{2}$$

and

$$\begin{cases} \overline{C}(u_2) \in \overline{C}(u_1) \cup \overline{C}(v_1) \cup \overline{C}(v_7); \\ \overline{C}(u_6) \in \overline{C}(u_7) \cup \overline{C}(v_1) \cup \overline{C}(v_7); \\ \overline{C}(v_2) \in \overline{C}(v_1) \cup \overline{C}(u_1) \cup \overline{C}(u_7); \\ \overline{C}(v_6) \in \overline{C}(v_7) \cup \overline{C}(u_1) \cup \overline{C}(u_7); \\ \overline{C}(u_i) \in \overline{C}(v_1) \cup \overline{C}(v_7), i = 3, 4, 5; \\ \overline{C}(v_i) \in \overline{C}(u_1) \cup \overline{C}(u_7), i = 3, 4, 5; \end{cases} \tag{3}$$

and

$$\bigcup_{i=2}^{6} (\overline{C}(u_i) \cup \overline{C}(v_i)) = \overline{C}(u_1) \cup \overline{C}(u_7) \cup \overline{C}(v_1) \cup \overline{C}(v_7). \tag{4}$$

Using (1) and (2), we have

$$|\bigcup_{i=2}^{6} (\overline{C}(u_i) \cup \overline{C}(v_i))| = 10 > |\overline{C}(u_1) \cup \overline{C}(u_7) \cup \overline{C}(v_1) \cup \overline{C}(v_7)| \leq 8 \tag{5}$$

(4) and (5) are contradiction, hence

$$\chi'_{as}(P_7 \vee P_7) \geq 11.$$

Similar for $n \geq 8$, we can be to prove

$$\chi'_{as}(P_n \vee P_n) \geq n + 4 (n \geq 8).$$

Case 4. when $n \geq 8$, we now give a (n+4)-ASEC of $P_n \vee P_n$ f as follows:

$$C = \{1, 2, \cdots, n + 3, 0\}, \overline{C}(w) = C \backslash C(w),$$

Let f be:

$$f(u_1v_i) = i, i = 1, 2, \cdots, n;$$

$$f(u_iv_j) = i + j(mod\ (n+4)), i = 2, 3, \cdots, n; j = 1, 2, \cdots, n;$$

$$f(u_iu_{(i+1)}) = f(v_iv_{(i+1)}) = n + i + 2(mod\ (n+4)), i = 1, 2, \cdots, n-1$$

For f, we have

$$\overline{C}(u_1) = \{n+1, n+2, 0\}; \overline{C}(u_i) = \{i-1, i\}, i = 2, 3, \cdots, n-1;\ \overline{C}(u_n) = \{n-2, n-1, n\}$$

$$\overline{C}(v_1) = \{2, n+2, 0\}; \overline{C}(v_i) = \{i-1, i+1\}, i = 2, 3, \cdots, n-1;\ \overline{C}(v_n) = \{n-2, n-1, n+1\}.$$

So f is a (n+4)-ASEC of $P_n \vee P_n$, hence the theorem 1 is true.

3. Adjacent Strong Edge Coloring of $P_n \vee C_n$

Theorem 2. *For $n \geq 3$,*

$$\chi'_{as}(P_n \vee C_n) = \begin{cases} 6, & n = 3; \\ n+4, & n \geq 4. \end{cases}$$

Proof. There are three cases to be considered.
Let $P_n = v_1v_2\cdots v_n$, $C_n = u_1u_2\cdots u_nu_1$.

Case 1. When n+3, to be need only exists 6-ASEC of $P_3 \vee C_3$. We now give 6-ASEC f as follows:

$$f(u_1u_2) = f(u_3v_1) = f(v_2v_3) = 1;\ f(u_2u_3) = f(v_1v_2) = 2;$$

$$f(u_1u_3) = f(u_2v_1) = 3;\ f(u_1v_2) = f(u_3v_3) = 4;$$

$$f(u_1v_3) = f(u_2v_2) = 5;\ f(u_1v_1) = f(u_2v_3) = f(u_3v_2) = 6.$$

Obviously, f is a 6-ASEC of $P_3 \vee C_3$. So that $\chi'_{as}(P_3 \vee C_3) = 6$.

Case 2. When n=4, we prove $\chi'_{as}(P_4 \vee C_4) \geq 8$,firstly. Otherwise, if $\chi'_{as}(P_4 \vee C_4) = 7$, then $\forall u \in V(C_4)$ *and* $v_2, v_3 \in V(P_4)$, we have

$$|C(u)| = |C(v_2)| = |C(v_3)| = 6,$$

and

$$|C(v_1)| = |C(v_4)| = 5$$

We have

$$|\overline{C}(u)| = |\overline{C}(v_2)| = |\overline{C}(v_3)| = 1,$$

and

$$|\overline{C}(v_1) = |\overline{C}(v_4)| = 2$$

Same Theorem 1 with n=7,

$$\overline{C}(v_1) \cup \overline{C}(v_4) = \bigcup_{i=2}^{4} \overline{C}(u_i) \cup \overline{C}(v_2) \cup \overline{C}(v_3)$$

$$4 \geq |\overline{C}(v_1)| \cup \overline{C}(v_4)| < |\bigcup_{i=2}^{4} \overline{C}(u_i) \cup \overline{C}(v_2) \cup \overline{C}(v_3)| = 6$$

Obviously, it is impossible, hence $\chi'_{as}(P_4 \vee C_4) \geq 8$.

Case 3. Similar we can prove $\chi'_{as}(P_n \vee C_n) \geq n+4$ when $n \geq 5$.
We now give a (n+4)-ASEC of $P_n \vee C_n (n \geq 4)$,
Let $C = \{1, 2, \cdots, n+3, 0\}, \overline{C}(w) = C \backslash C(w)$,
When n=4, f as follows:

$$f(u_iv_j) = i+j-1(mod\, 8), i = 1,2,3,4; j = 1,2,3,4;\ f(u_iu_{(i+1)}) = i+5, i = 1,2;$$

$$f(u_3u_4) = 1;\ f(u_4u_1) = 0;\ f(v_iv_{(i+1)}) = i+6(mod\ 8), i = 1,2,3;$$

For f, we have

$$\overline{C}(u_1) = \{5,7\}, \overline{C}(u_2) = \{0,1\}, \overline{C}(u_3) = \{0,2\}, \overline{C}(u_4) = \{2,3\}$$

$$\overline{C}(v_1) = \{5,6,0\}, \overline{C}(v_2) = \{1,6\}, \overline{C}(v_3) = \{2,7\}, \overline{C}(v_4) = \{2,3,0\}$$

So f is a 8-ASEC of $P_4 \vee C_4$, it's theorem 2 is true when n=4.
When n=5, f as follows:

$$f(u_iv_j = i+j-1(mod\ 9), i = 1,2,3,4; j = 1,2,3,4,5;$$

$$f(u_5v_i) = 6+i(mod\ 9), i = 1,2,3,4,5;$$

$$f(u_iu_{(i+1)}) = i+7(mod\ 9), i = 1,2,3;\ f(u_4u_5) = 3;\ f(u_5u_1) = 6;$$

$$f(v_iv_{(i+1)}) = i+8(mod\ 9), i = 1,2,3,4;$$

It is easy to see that f is 9-ASEC of $P_5 \vee C_5$. Hence, the conclusion is true.
When $n \geq 6$, f as follows:

$$f(u_iv_j) = i + j - 1(mod\ n + 4), i = 1, 2, \cdots, n - 1; j = 1, 2, \cdots, n;$$

$$f(u_nv_i) = n + i + 1(mod\ n + 4), i = 1, 2, \cdots, n;$$

$$f(u_iu_{(i+1)} = n + i + 2(mod\ n + 4), i = 1, 2, \cdots, n - 2;$$

$$f(u_{(n-1)}u_n) = n - 3;\ f(u_nu_1) = n + 1;$$

$$f(v_iv_{(i+1)}) = n + i + 3(mod\ n + 4), i = 1, 2, \cdots, n - 1;$$

For f, we have

$$\overline{C}(u_1) = \{n + 2, 0\};\ \overline{C}(u_i) = \{i - 1, n + i(mod\ n + 4)\}, i = 2, 3, \cdots, n - 2;$$

$$\overline{C}(u_{(n-1)}) = \{n - 3, 2n - 1(mod\ n + 4\};\ \overline{C}(u_n) = \{n - 1, n\}$$

$$\overline{C}(v_1) = \{n, n + 1, n + 3\};\ \overline{C}(v_n) = \{n - 1, 2n - 1(mod\ n + 4), 2n(mod\ n + 4)\};$$

$$\overline{C}(v_i) = \{n + i - 1(mod\ n + 4), n + i(mod\ n + 4)\}, i = 2, 3, \cdots, n - 1.$$

So f is (n+4)-ASEC of $P_n \vee C_n (n \geq 6)$. Hence the conclusion is true.

4. Adjacent Strong Edge Coloring of $C_n \vee C_n$

Theorem 3. *For $n \geq 3$, then*

$$\chi'_{as}(C_n \vee C_n) = n + 4.$$

Proof. Same as Theorem 2, first can be to prove $\chi'_{as}(C_n \vee C_n) \geq n + 4$.

Supposing the two cycles are $u_1u_2 \cdots u_nu_1$ and $v_1v_2 \cdots v_nv_1$ with separately.

When n=3, $C_3 \vee C_3 = K_6$(*complete graph with order* 6), can be to see appendix.

When n=4, it's easy to see that $\chi'_{as}(C_n \vee C_n) \geq n + 4$

We now give a 8-ASEC of $C_4 \vee C_4$, f as follows,

Let $C = \{1, 2, \cdots, 7, 0\}, \overline{C}(w) = C \backslash C(w)$,

$$f(u_iv_j) = i + j - 1(mod\ 8), i = 1, 2, 3, 4; j = 1, 2, 3, 4;$$

$$f(u_iu_{(i+1)}) = i + 5, i = 1, 2;$$

$$f(u_3u_4) = 1;\ f(u_4u_1) = 0;\ f(v_1v_2) = 0;\ f(v_2v_3) = 2;\ f(v_4v_1) = 0;$$

For f, we have

$$\overline{C}(u_1) = \{5,7\}, \overline{C}(u_2) = \{0,1\}, \overline{C}(u_3) = \{0,2\}, \overline{C}(u_4) = \{2,3\}$$

$$\overline{C}(v_1) = \{5,6,0\}, \overline{C}(v_2) = \{1,6\}, \overline{C}(v_3) = \{1,7\}, \overline{C}(v_4) = \{1,3\}$$

So f is a 8-ASEC of $C_4 \vee C_4$, it's theorem 3 is true when n=4.
When n=5, we now give a 9-ASEC of $C_5 \vee C_5$, f as follows,

$$f(u_iv_j) = i + j - 1(mod\ 9), i = 1,2,3,4; j = 1,2,3,4,5;$$

$$f(u_5v_i) = i + 6(mod\ 9), i = 1,2,3,4,5;$$

$$f(u_1u_2) = 8;\ f(u_2u_3) = 0;\ f(u_3u_4) = 3;\ f(u_4u_5) = 3;\ f(u_5u_1) = 6;$$

$$f(v_1v_2) = 6;\ f(v_2v_3) = 7;\ f(v_3v_4) = 2;\ f(v_4v_5) = 3;\ f(v_5v_1) = 0;$$

It's easy to see f is 9-ASEC of $C_5 \vee C_5$. Hence the conclusion is true.
When $n \geq 6$, let f be as follows:
the edges of $u_1u_2, u_2u_3, \cdots, u_{n-1}u_n$ are coloring colors with n+3,n+4 rotate;
the edges of $v_1v_2, v_2v_3, \cdots, v_{n-1}v_n$ are coloring colors with n+4,n+3 rotate;

$$f(u_1u_n) = f(v_1v_n) = n + 1;$$

$$f(u_1v_j) = j, j = 1,2\cdots,n;$$

$$f(u_iv_j) = i+j(when\ i+j > n+2,\ then\ mod\, n+2), i = 2,3,\cdots,n-1; j = 1,2,\cdots,n;$$

$$f(u_nv_1) = n + 2;\ f(u_nv_j) = j - 1, j = 2,3,\cdots,n.$$

For the f, we have

$$\overline{C}(u_1) = \{n+2, n+4\};\ \overline{C}(u_i) = \{i-1, i\}, i = 2,3\cdots,n-1;$$

$$\overline{C}(u_n) = \{n, n+4\}(n \equiv 0(mod\ 2))\ or\ \overline{C}(u_n) = \{n, n+3\}(n \equiv 1(mod\ 2));$$

$$\overline{C}(v_1) = \{2, n+3\};\ \overline{C}(v_i) = \{i+1, n+i\}(when\ i+j > n+2, take\ mod\ n+2;$$

$$\overline{C}(v_n) = \{n-2, n+3\}(n \equiv 0(mod\ 2))\ or\ \overline{C}(v_n) = \{n, n+4\}(n \equiv 1(mod\ 2));$$

So, the f is a (n+4)-ASEC of $C_n \vee C_n$, the theorem 3 is true.

Acknowledgments

This research is supported by Lanzhou City University PhD Research Fund (LZCU-BS2013-09, LZCU-2013-12).

References

[1] F. Harary, Conditional colorability in graphs, In: *Graphs and Applications, Proc. Graph theory Fist Colorado Symp*, John Wilh & Sons, New York (1985).

[2] A.C. Burrisand, R.H. Schelp, Vertex-distinguishing proper edge-colorings, *J. of Graph Theory*, **26**, No. 2 (1997), 73-82.

[3] C. Bazgan, A. Harkat-Benhamdine, H. Li, et al, On the vertex-distinguishing proper edge-coloring of graph, *J. Combin. Theory., Ser. B*, **75** (1999), 288-301.

[4] P.N. Balister, B. Bollobás, R.H. Schelp, Vertex distinguishing coloring of graphs with $\Delta(G)=2$, *Discrete Mathematics*, **252**, No. 2 (2002), 17-29.

[5] P.N. Balister, O.M. Riordanand, R.H. Schelp, Vertex-distinguishing edge coloring of graphs, *J. Graph Theory*, **42** (2003), 95-109.

[6] Zhongfu Zhang, Linzhong Liu, Jianfang Wang, Adjacent strong edge coloring of graphs, *Applied Mat. Hmatics Letters*, **15** (2002), 623 626.

[7] Wang Shudong, Li Chongming, Xu Ji, et al, On the adjacent strong edge coloring of some graphs, *J. of Mathematical Research and Exposition*, **22**, No. 4 (2002), 412-417.

[8] P.N. Balister, Gyori, R.H. Schelp, On the adjacent-strong edge coloring of graphs with $\Delta(G) \leq 3$, *J.G.T.*. To Appear.

[9] J.A. Bondyand, U.S. Marty, *Graph Theory with Applications*, The Macmillan Press Ltd, New York (1976).

[10] G. Chartrand, L.F. Linda, *Graphs and Diagraphs*, Ind. Edition Wadsirth Brokks/Cole, Monterey, CA (1986).