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COMMON FIXED POINT THEOREMS FOR WEAKLY COMMUTING MAPPINGS WITH PROPERTY (E.A.) IN INTUITIONISTIC FUZZY METRIC SPACES

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Abstract: In this paper, employing the property (E.A.), we prove common fixedt heorem for weakly commuting mappings via an implicit relation inintuitionistic fuzzy metric spaces. Our results generalize the results of Sedghi et al. [19, Theorem 2.3] in intuitionistic fuzzy metric spaces.

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Key Words: weakly commuting mapping, property (E.A.), implicit relation, common fixed point, intuitionistic fuzzy metric space

1. Introduction

In 1986, Jungck [8] introduced the notion of compatible mappings for a pair of self-mappings. However, the study of common fixed points of non-compatible maps is also very interesting (see [16]). Aamri and Moutawakil [1] generalized the concept of non-compatibility by defining the notion of property

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(E.A.) proved common fixed point theorems under strict contractive conditions. Jungck and Rhoades [9] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. In the literature, many results have been proved for contraction mappings satisfying property (E.A.)in different settings such as probabilistic metric spaces ([5], [12]) and fuzzy metric spaces ([11], [13], [15]).

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [20] and later there has been much progress in the study of intuitionistic fuzzy sets ([4], [7]).

In 2004, Park [17] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [6]. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc.

Recently, Sedghi et al. [19] established a common fixed point theorem in fuzzy metric space using property (E.A.).

In this paper, employing the property (E.A.), we prove common fixed theorem for weakly commuting mappings via an implicit relationin intuitionistic fuzzy metric spaces. Our results generalize the results of Sedghi et al. [19, Theorem 2.3] in intuitionistic fuzzy metric space.

2. Preliminaries

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use characterization fuzzy intersections and union, respectively. These concepts were originally introduced by Menger [14] instudy of statistical metric spaces.

Definition 2.1. ([18]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is *continuous t-norm* if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Definition 2.2. ([18]) A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is *continuous t-conorm* if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative and associative;
(ii) $\diamond$ is continuous;

(iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;
(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all$a, b, c, d \in [0, 1]$.

Alaca et al. [2] using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michálek [10] as follows.

Definition 2.3. ([2]) A 5-tuple $(X, M, N, *, \diamond)$ is said to be an *intuitionistic fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous;
(vii) $\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \to [0, 1]$ is right continuous;
(xiii) $\lim_{t \to \infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an *intuitionistic fuzzy metric* on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4. ([2]) Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated as $x \diamond y = 1 - ((1 - x) * (1 - y))$ for all $x, y \in X$.

Remark 2.5. ([2]) In an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.

Also, Alaca et al. [2] introduced the following notions.

Definition 2.6. ([2]) Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be

(i) *convergent* to a point $x \in X$ if

$$\lim_{n \to \infty} M(x_n, x, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(x_n, x, t) = 0$$

for all $t > 0$,

(ii) a *Cauchy sequence* if

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \quad \text{and} \quad \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$$

for all $t > 0$ and $p > 0$.

Definition 2.7. ([2]) An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be *complete* if and only if every Cauchy sequence in X is convergent.

Definition 2.8. ([1]) Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. Then a pair (A, S) is said to satisfy the *property* $(E.A.)$ if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z \in X$.

Definition 2.9. ([9]) Let A and S be self-mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$. Then a pair (A, S) is said to be *weakly commuting* if $M(ASx, SAx, t) \geq M(Ax, Sx, t)$ and $N(ASx, SAx, t) \leq N(Ax, Sx, t)$ for all $x, y \in X$ and $t > 0$.

3. Main Results

Implicit relations play important role in establishing of common fixed point results. Let Φ be the set of all functions $\phi : [0,1]^3 \to [0,1]$ and $\psi : [0,1]^3 \to [0,1]$ satisfying the following conditions:

(A) $\phi(x, y, z)$ and $\psi(x, y, z)$ is continuous in each coordinate variable for all $x, y, z \in [0,1]$,

(B) $\phi(1,1,1) = 1$ and $\psi(0,0,0) = 0$,

(C) $\phi(u,1,1) > u$ or $\phi(1,u,1) > u$ or $\phi(1,1,u) > u$ for all $u \neq 1$ and $\psi(u,0,0) < u$ or $\psi(0,u,0) < u$ or $\psi(0,0,u) < u$ for all $u \neq 0$.

Example 3.1. Define $\phi : [0,1]^3 \to [0,1]$ and $\psi : [0,1]^3 \to [0,1]$ as $\phi(x, y, z) = \max\{x, y, z\}$ and $\psi(x, y, z) = \min\{x, y, z\}$ for all $x, y, z \in [0,1]$. Clearly ϕ and ψ satisfies conditions (A), (B) and (C).

Now, we prove a common fixed point theorem for three mappings in an intuitionistic fuzzy metric space.

Theorem 3.2. *Let A, S and T be self mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, where $*$ is t-norm defined by $a*b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0,1]$ satisfying*

(C1) *the pairs (A, S) and (A, T) satisfies property (E.A.),*

(C2) *the pairs (A, S) and (A, T) are weakly commuting and A is continuous,* (C3) *for any* $x, y \in X$ *and* $t > 0$,

$$M(Sx, Ty, t) \geq \phi(M(Ax, Sx, t), M(Ay, Ty, t), M(Ax, Tx, t))$$

and

$$N(Sx, Ty, t) \leq \psi(N(Ax, Sx, t), N(Ay, Ty, t), N(Ax, Tx, t)),$$

where $\phi, \psi \in \Phi$.
Then A, S *and* T *have a unique common fixed point in* X.

Proof. Since the pairs (A, S) and (A, T) satisfies propery (E.A.), there exists sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} M(Ax_n, Sx_n, t) = \lim_{m\to\infty} M(Ax_m, Sx_m, t) = 1$$

and

$$\lim_{n\to\infty} N(Ax_n, Sx_n, t) = \lim_{m\to\infty} N(Ax_m, Sx_m, t) = 0.$$

Now, we prove $\{Ax_n\}$ is a Cauchy sequence. By (C1) and the definition of an intuitionistic fuzzy metric space, we have

$$\begin{aligned}
&M(Ax_n, Ax_m, t) \\
&\geq M(Ax_n, Sx_n, t/3) * M(Sx_n, Tx_m, t/3) * M(Tx_m, Ax_m, t/3) \\
&\geq M(Ax_n, Sx_n, t/3) * \phi(M(Ax_n, Sx_n, t/3), M(Ax_m, Tx_m, t/3), \\
&\qquad M(Ax_n, Tx_n, t/3)) * M(Tx_m, Ax_m, t/3)
\end{aligned}$$

and

$$\begin{aligned}
&N(Ax_n, Ax_m, t) \\
&\leq N(Ax_n, Sx_n, t/3) \diamond N(Sx_n, Tx_m, t/3) \diamond N(Tx_m, Ax_m, t/3) \\
&\leq N(Ax_n, Sx_n, t/3) \diamond \psi(N(Ax_n, Sx_n, t/3), N(Ax_m, Tx_m, t/3), \\
&\qquad N(Ax_n, Tx_n, t/3)) \diamond N(Tx_m, Ax_m, t/3).
\end{aligned}$$

Taking $n, m \to \infty$, by the definition of ϕ and ψ, we have

$$\lim_{n\to\infty} M(A_n, Ax_m, t) \geq 1 * \phi(1, 1, 1) * 1 = 1 * 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(A_n, Ax_m, t) \leq 0 \diamond \psi(0, 0, 0) \diamond 0 = 0 \diamond 0 \diamond 0 = 0.$$

This gives $\lim_{n,m\to\infty} M(A_n, Ax_m, t) = 1$ and $\lim_{n,m\to\infty} N(A_n, Ax_m, t) = 0$. Hence $\{Ax_n\}$ is a Cauchy sequence in X, and since X is complete, $\{Ax_n\}$ converges to z in X. Since A is continuous, it follows that $\lim_{n\to\infty} A^2x_n = Az$.

$$M(Sx_n, z, t) \geq M(Sx_n, Ax_n, t/2) * M(Ax_n, z, t/2)$$

and

$$N(Sx_n, z, t) \leq M(Sx_n, Ax_n, t/2) \diamond M(Ax_n, z, t/2).$$

Also as $n \to \infty$,

$$\lim_{n\to\infty} M(Sx_n, z, t) \geq 1 * 1 = 1$$

and

$$\lim_{n\to\infty} N(Sx_n, z, t) \leq 0 \diamond 0 = 0.$$

This gives $\lim_{n\to\infty} Sx_n = z$. As A is continuous, it follows that $\lim_{n\to\infty} ASx_n = Az$.

Similarly, we prove that $\lim_{m\to\infty} Tx_m = z$ and by the continuity of A, we have $\lim_{m\to\infty} ATx_m = Az$.

Now, by weakly commutativity of A and S, we have

$$\begin{aligned} M(SAx_n, Az, t) &\geq M(SAx_n, ASx_n, t/2) * M(ASx_n, Az, t/2) \\ &\geq M(Sx_n, Ax_n, t/2) * M(ASx_n, Az, t/2). \end{aligned}$$

As $n \to \infty$,

$$\begin{aligned} \lim_{n\to\infty} M(SAx_n, Az, t) &\geq M(z, z, t/2) * M(Az, Az, t/2) \\ &= 1 * 1 = 1 \end{aligned}$$

and

$$\begin{aligned} N(SAx_n, Az, t) &\leq N(SAx_n, ASx_n, t/2) \diamond N(ASx_n, Az, t/2) \\ &\leq N(Sx_n, Ax_n, t/2) \diamond N(ASx_n, Az, t/2). \end{aligned}$$

As $n \to \infty$,

$$\begin{aligned} \lim_{n\to\infty} N(SAx_n, Az, t) &\leq N(z, z, t/2) \diamond N(Az, Az, t/2) \\ &= 0 \diamond 0 = 0. \end{aligned}$$

This gives $\lim_{n\to\infty} SAx_n = Az$. Similarly $\lim_{n\to\infty} TAx_n = Az$.

Now, we prove that $Az = z$. By (C1) and the definition of ϕ and ψ, we have

$$\begin{gathered} M(Sx_n, TAx_n, t) \geq \phi(M(Ax_n, Sx_n, t), M(AAx_n, TAx_n, t), \\ M(Ax_n, Tx_n, t)) \end{gathered}$$

and

$$N(Sx_n, TAx_n, t) \le \psi(N(Ax_n, Sx_n, t), N(AAx_n, TAx_n, t), N(Ax_n, Tx_n, t)).$$

As $n \to \infty$,

$$\begin{aligned} M(z, Az, t) &\ge \phi(M(z, z, t), M(Az, Az, t), M(z, z, t)) \\ &= \phi(1, 1, 1) = 1 \end{aligned}$$

and

$$\begin{aligned} N(z, Az, t) &\le \psi(N(z, z, t), N(Az, Az, t), N(z, z, t)) \\ &= \psi(0, 0, 0) = 0. \end{aligned}$$

This gives $Az = z$.

Now, we show that $Tz = z$. By (C1), (B) and (C), we have

$$\begin{aligned} M(z, Tz, t) &= M(Az, Tz, t) \\ &\ge M(Az, SAx_n, \epsilon t) * M(SAx_n, Tz, (1-\epsilon)t) \\ &\ge M(Az, SAx_n, \epsilon t) * \phi(M(AAx_n, SAx_n, (1-\epsilon)t), \\ &\quad M(Az, Tz, (1-\epsilon)t), M(AAx_n, TAx_n, (1-\epsilon)t)). \end{aligned}$$

As $n \to \infty$,

$$\begin{aligned} M(z, Tz, t) &\ge M(Az, Az, \epsilon t) * \phi(M(Az, Az, (1-\epsilon)t), \\ &\quad M(Az, Tz, (1-\epsilon)t), M(Az, Az, (1-\epsilon)t)) \\ &\ge 1 * \phi(1, M(z, Tz, (1-\epsilon)t), 1) \\ &= \phi(1, M(z, Tz, (1-\epsilon)t), 1) \\ &> M(z, Tz, (1-\epsilon)t). \end{aligned}$$

As $\epsilon \to 0$, we have

$$M(z, Tz, t) \ge M(z, Tz, t)$$

and

$$\begin{aligned} N(z, Tz, t) &= N(Az, Tz, t) \\ &\le N(Az, SAx_n, \epsilon t) \diamond N(SAx_n, Tz, (1-\epsilon)t) \\ &\le N(Az, SAx_n, \epsilon t) \diamond \psi(N(AAx_n, SAx_n, (1-\epsilon)t), \\ &\quad N(Az, Tz, (1-\epsilon)t), N(AAx_n, TAx_n, (1-\epsilon)t)). \end{aligned}$$

As $n \to \infty$,

$$\begin{aligned}
N(z,Tz,t) &\leq N(Az,Az,\epsilon t) \diamond \psi(N(Az,Az,(1-\epsilon)t), \\
&\qquad N(Az,Tz,(1-\epsilon)t), N(Az,Az,(1-\epsilon)t)) \\
&\leq 0 \diamond \psi(0, N(z,Tz,(1-\epsilon)t), 0) \\
&= \psi(0, M(z,Tz,(1-\epsilon)t), 0) \\
&< N(z,Tz,(1-\epsilon)t).
\end{aligned}$$

As $\epsilon \to 0$, we have

$$N(z,Tz,t) \leq N(z,Tz,t).$$

Hence $Tz = z$. Similarly, we prove that $Sz = z$ by (C1), (B) and (C). Hence $Az = Sz = Tz = z$, that is, z is a common fixed point of A, S and T.

Finally, for uniqueness, let w ($z \neq w$) be another common fixed point of A, S and T. By (C1) and definition of ϕ and ψ, we have

$$\begin{aligned}
M(z,w,t) &= M(Sz,Tw,t) \\
&\geq \phi(M(Az,Sz,t), M(Aw,Tw,t), M(Az,Tz,t)) \\
&= \phi(M(z,z,t), M(w,w,t), M(z,z,t)) \\
&= \phi(1,1,1) = 1
\end{aligned}$$

and

$$\begin{aligned}
N(z,w,t) &= N(Sz,Tw,t) \\
&\leq \psi(N(Az,Sz,t), N(Aw,Tw,t), N(Az,Tz,t)) \\
&= \psi(N(z,z,t), N(w,w,t), N(z,z,t)) \\
&= \psi(0,0,0) = 0.
\end{aligned}$$

Hence $z = w$. Therefore z is a unique common fixed point of A, S and T. This completes the proof. □

Corollary 3.3. *Let A, S and T be self-mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, where $*$ is t-norm defined by $a * b = \min\{a,b\}$ and $a \diamond b = \max\{a,b\}$ for all $a, b \in [0,1]$ satisfying* (C1), (C2) *and*

(C4) *for any $x, y \in X$ and $t > 0$,*

$$M(Sx,Ty,t) \geq \phi(M(Ax,Sx,t), M(Ay,Ty,t), M(Ax,Tx,t))$$

and

$$N(Sx,Ty,t) \leq \psi(N(Ax,Sx,t), N(Ay,Ty,t), N(Ax,Tx,t)),$$

where $\phi : [0,1]^3 \to [0,1]$ and $\psi : [0,1]^3 \to [0,1]$ as $\phi(x,y,z) = \max\{x,y,z\}$ and $\psi(x,y,z) = \min\{x,y,z\}$ for all $x, y, z \in [0,1]$.
Then A, S and T have a unique common fixed point in X.

From Corollary 3.3, if S and T are the identity mappings, then we obtain the following.

Corollary 3.4. *Let A, S and T be self-mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, where $*$ is t-norm defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$ satisfying*

(C5) *for every for any $x, y \in X$ and $t > 0$,*

$$M(x, y, t) \geq \phi(M(Ax, x, t), M(Ay, y, t), M(Ax, x, t))$$

and

$$N(x, y, t) \leq \psi(N(Ax, x, t), M(Ay, y, t), N(Ax, x, t)),$$

where $\phi : [0, 1]^3 \to [0, 1]$ and $\psi : [0, 1]^3 \to [0, 1]$ as $\phi(x, y, z) = \max\{x, y, z\}$ and $\psi(x, y, z) = \min\{x, y, z\}$ for all $x, y, z \in [0, 1]$.

Suppose that there exists a sequence $\{x_n\}$ in X suthat that

$$\lim_{n \to \infty} M(Ax_n, x_n, t) = 1 \quad \text{and} \quad \lim_{n \to \infty} N(Ax_n, x_n, t) = 0.$$

Then A have a unique fixed point in X.

As an application of Theorem 3.2, we prove a common fixed point theorem for two finite families of mappings in an intuitionistic fuzzy metric space.

Theorem 3.5. *Let A, S_i $(i \in \mathbb{N})$ and T_j $(j \in \mathbb{N})$ be self-mappings of a complete intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, where $*$ is t-norm defined by $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$ for all $a, b \in [0, 1]$ satisfying*

(C6) *for any $x, y \in X$ and $t > 0$,*

$$M(S_i x, T_j y, t) \geq \phi(M(Ax, S_i x, t), M(Ay, T_j y, t), M(Ax, T_j x, t))$$

and

$$N(S_i x, T_j y, t) \leq \psi(N(Ax, S_i x, t), N(Ay, T_j y, t), N(Ax, T_j x, t)),$$

where $\phi, \psi \in \Phi$.

(C7) *there exist $i_0, j_0 \in \mathbb{N}$ such that the pairs (A, S_{i_0}) and (A, T_{j_0}) satisfies property (E.A.),*

(C8) *the pairs (A, S_i) and (A, T_j) are weakly commuting and A is continuous.*

Then A, S_i and T_j have a unique common fixed point in X.

Proof. By Theorem 3.2, A, S_{i_0} and T_{j_0} for some $i_0, j_0 \in \mathbb{N}$ have a unique common fixed point in X, that is, $Az = S_{i_0}z = T_{j_0}z = z$. Suppose that there exist $i \in \mathbb{N}$ such that $i \neq i_0$. Then we have

$$\begin{aligned} M(S_i z, z, t) &= M(S_i z, T_{j_0} z, t) \\ &\geq \phi(M(Az, S_i z, t), M(Az, T_{j_0} z, t), M(Az, T_{j_0} z, t)) \\ &= \phi(M(z, S_i z, t), M(z, z, t), M(z, z, t)) \\ &= \phi(M(z, S_i z, t), 1, 1)) > M(z, S_i z, t) \end{aligned}$$

and

$$\begin{aligned} N(S_i z, z, t) &= N(S_i z, T_{j_0} z, t) \\ &\leq \psi(N(Az, S_i z, t), N(Az, T_{j_0} z, t), N(Az, T_{j_0} z, t)) \\ &= \psi(N(z, S_i z, t), N(z, z, t), N(z, z, t)) \\ &= \psi(N(z, S_i z, t), 0, 0)) < N(z, S_i z, t), \end{aligned}$$

which is a contradiction. This gives $S_i z = z$ for all $i \in \mathbb{N}$. Similarly, for all $j \in \mathbb{N}$, we get $T_j z = z$. Therefore, for every $i, j \in \mathbb{N}$, we have $Az = S_i = T_j z = z$, that is z is a common fixed point of A, S_i and T_j.

Finally, for uniqueness, it easily follows from (C6). Therefore, A, S_i and T_j have a unique common fixed point in X. This completes the proof. □

Remark 3.6. Theorem 3.5 is a partial generalization of Theorem 3.2 as two finite families.

Remark 3.7. Our results generalize the results of Sedghi et al. [19, Theorem 2.3] in an intuitionistic fuzzy metric space.

Example 3.8. ([19]) Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, where $X = [0,1]$ and define $\phi : [0,1]^3 \to [0,1]$ and $\psi : [0,1]^3 \to [0,1]$ as $\phi(x,y,z) = \max\{x,y,z\}$ and $\psi(x,y,z) = \min\{x,y,z\}$ for all $x, y, z \in [0,1]$. Clearly ϕ and ψ satisfies conditions (A), (B) and (C). Therefore, $\phi, \psi \in \Phi$. Define A, S and T by

$$Ax = \frac{x}{8}, \quad Sx = \frac{x}{x+16} \quad \text{and} \quad Tx = 0,$$

respectively, and

$$M(x,y,t) = \frac{t}{t+|x-y|}, \quad M(x,y,t) = \frac{|x-y|}{t+|x-y|}$$

for all $x, y \in X = [0,1]$ and $t > 0$. Then, clearly, with sequences $\{x_n\} = \{\frac{1}{n}\}$ in X, we have

$$\lim_{n\to\infty} M(Ax_n, Sx_n, t) = \lim_{m\to\infty} M(Ax_m, Tx_m, t) = 1$$

and

$$\lim_{n\to\infty} N(Ax_n, Sx_n, t) = \lim_{m\to\infty} N(Ax_m, Tx_m, t) = 0,$$

which shows that pairs (A, S) and (A, T) satisfies the property (E.A.). By a routine calculation, one can verify that all conditions of Theorem 3.2 satisfied and $x = 0$ is the unique common fixed point of A, S and T.

References

[1] M. Aamri, D.El Moutawakil, Some new common fixed point theorems under strict contractive conditions, *J. Math. Anal. Appl.*, **270** (2002), 181-188, **doi:** 10.1016/S0022-247X(02)00059-8

[2] C. Alaca, D. Türkoğlu, C. Yildiz, Fixed points in intuitionistic fuzzy metric spaces, *Chaos Solitons Fractals*, **29** (2006), 1073-1078, **doi:** 10.1016/j.chaos.2005.08.066

[3] K.T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, **20** (1986), 87-96.

[4] D. Çoker, An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and System*, **88** (1997), 81-89.

[5] J.X. Fang, Y. Gao, Common fixed point theorems under strict contractive conditions in Menger spaces, *Nonlinear Anal.*, **70** (2009), 184-193, **doi:** 10.1016/j.na.2007.11.045

[6] A. George, P. Veeramani, On some results in fuzzy metric spaces, *Fuzzy Sets and Systems*, **64** (1994), 395-399, **doi:** 10.1016/0165-0114(94)90162-7

[7] J.G. García, S.E. Rodabaugh, Order-theoretic, topological, categorical redundancies of interval-valued sets, grey sets, vague sets, interval-valued "intuitionistic" sets, "intuitionistic" fuzzy sets and topologies, *Fuzzy Sets and Systems*, **156** (2005), 445-484, **doi:** 10.1016/j.fss.2005.05.023

[8] G. Jungck, Compatible mappings and common fixed points, *Int. J. Math. Math. Sci.*, **9** (1986), 771-779, **doi:** 10.1155/S0161171286000935

[9] G. Jungck, B.E. Rhoades, Fixed points for set valued functions without continuity, *Indian J. Pure Appl. Math.*, **29** (1998), 227-238.

[10] I. Kramosil, J. Michálek, Fuzzy metric and statistical spaces, *Kybernetika*, **11** (1975), 336-344.

[11] S. Kumar, B. Fisher, A common fixed point theorem in fuzzy metric space using property (E.A.) and implicit relation, *Thai. J. Math.*, **8** (2010), 439-446.

[12] S. Kumar, B.D. Pant, Common fixed point theorems in probabilistic metric spaces using implicit relation and property (E.A.), *Bull. Allahabad Math. Soc.*, **25** (2010), 223-235.

[13] S. Manro, S.S. Bhatia, S. Kumar, Common fixed point theorems in fuzzy metric spaces, *Ann. Fuzzy Math. Inform.*, **3** (2012), 151-158.

[14] K. Menger, Statistical metrics, *Proc. Natl. Acad. Sci. USA*, **28** (1942), 535-537, **doi:** 10.1073/pnas.28.12.535

[15] D. Miheţ, Fixed point theorems in fuzzy metric spaces using property E.A., *Nonlinear Anal.*, **73** (2010), 2184-2188, **doi:** 10.1016/j.na.2010.05.044

[16] R.P. Pant, Common fixed points of contractive maps, *J. Math. Anal. Appl.*, **226** (1998), 251-258, **doi:** 10.1006/jmaa.1998.6029

[17] J.H. Park, Intuitionistic fuzzy metric spaces, *Chaos Solitons Fractals*, **22** (2004), 1039-1046, **doi:** 10.1016/j.chaos.2004.02.051

[18] B. Schweizer, A. Sklar, Probabilistic Metric Spaces, North Holland Amsterdam, 1983.

[19] S. Sedghi, C. Alaca, N. Shobe, On fixed points of weakly commuting mappings with property (E.A.), *J. Adv. Stud. Topol.*, **3** (2012), 11-17.

[20] L.A. Zadeh, Fuzzy sets, *Infor. and Control.*, **8** (1965), 338-353.