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FIXED POINTS FOR THREE CONTRACTIVE MAPPINGS IN THREE FUZZY METRIC SPACES

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Abstract: In the present paper, we establish a common fixed point theorem in three complete fuzzy metric spaces. An analogous result for compact metric spaces is also proved. We also give example in support of our result. As an application to our results, we obtain the corresponding common fixed point theorems in metric spaces. Our results improves the results of Fisher [2], [3] in two metric spaces and Jain et al. [8] in three metric spaces.

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Key Words: complete fuzzy metric space, compact fuzzy metric space, common fixed point

1. Introduction and Preliminaries

In 1965, Zadeh [18] introduced the concept of fuzzy set as a new way to represent vagueness in our everyday life. However, when the uncertainty is due to fuzziness rather than randomness, as sometimes in the measurement of an ordinary length, it seems that the concept of a fuzzy metric space is more suitable. We can divide them into following two groups.

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The first group involves those results in which a fuzzy metric on a set X is treated as a mapping, where X represents the totality of all fuzzy points of a set and satisfy some axioms which are analogous to the ordinary metric axioms. Thus, in such an approach numerical distances are set up between fuzzy objects.

On the other hand in second group, we keep those results in which the distance between objects is fuzzy and the objects themselves may or may not be fuzzy.

Kramosil and Michálek [9] have introduced the conceptof fuzzy metric spaces in different ways. Many authors ([1], [5]-[7], [9]-[17]) have proved fixed point theorems in fuzzy metric spaces.

In this section, we recall some useful facts from the fuzzy metric spaces.

Definition 1.1. ([14]) A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is *continuous t-norm* if $*$ satisfies

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $*(a,1) = a$ for all $a \in [0,1]$;

(iv) $*(a,b) \leq *(c,d)$ whenever $a \leq c$ and $b \leq d$ for all $a,b,c,d \in [0,1]$.

Definition 1.2. ([4]) A 3-tuple $(X, M, *)$ is said to be a *fuzzy metric space* if X is an arbitrary set, $*$ is a continuous t-norm, and M is fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions for all $x, y, x \in X$ and $s, t > 0$

(i) $M(x,y,0) > 0$;

(ii) $M(x,y,t) = 1$ for all $t > 0$ if and only if $x = y$;

(iii) $M(x,y,t) = M(y,x,t)$;

(iv) $*(M(x,y,t), M(y,z,s)) \leq M(x,z,t+s)$;

(v) $M(x,y,\cdot) : [0,\infty) \to [0,1]$ is left continuous.

The function $M(x,y,t)$ denote the degree of nearness between x and y with respect to t, respectively.

Lemma 1.3. ([4]) *In fuzzy metric space $(X, M, *)$, $M(x,y,\cdot)$ is non-decreasing for all $x, y \in X$.*

Definition 1.4. ([4]) Let (X, M, T) be a fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be

(a) *convergent* to a point $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.

(b) *Cauchy sequence* if $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for all $t > 0$ and $p > 0$ or if for each $\epsilon > 0$, there exist $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for each $n, m \geq n_0$.

Definition 1.5. ([4]) A fuzzy metric space $(X, M, *)$ is said to be *complete* if and only if every Cauchy sequence in X is convergent.

Example 1.6. ([6]) Let $X = \{1/n : n \in \mathbb{N}\} \cup \{0\}$ and $*$ be the continuous t-norm defined by $*(a, b) = ab$ (or $*(a, b) = \min\{a, b\}$) for all $a, b \in [0, 1]$. For each $x, y \in X$ and $t > 0$, define $M(x, y, t)$ by

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & t > 0, \\ 0, & t = 0. \end{cases}$$

Clearly, $(X, M, *)$ is a complete fuzzy metric space.

Lemma 1.7. ([4], [9]) *Let $(X, M, *)$ be a fuzzy metric space. Then M is continuous function on $X^2 \times (0, \infty)$.*

Lemma 1.8. ([15]) *Let $(X, M, *)$ be a fuzzy metric space. If a sequence $\{x_n\}$ in X is such that for every $n = 1, 2, 3, \cdots$,*

$$M(x_n, x_{n+1}, t) \geq 1 - c^n(1 - M(x_0, x_1, t))$$

for every $t > 0$, where $c \in (0, 1)$, then $\{x_n\}$ is a Cauchy sequence in X.

2. Main Results

Now, we prove a fixed point theorem involving three complete fuzzy metric spaces.

Theorem 2.1. *Let $(X, M_1, *_1)$, $(Y, M_2, *_2)$ and $(Z, M_3, *_3)$ be three complete fuzzy metric spaces. If T is a continuous mapping of X into Y, S is a continuous mapping of Y into Z and R is a mapping of Z into X satisfying*

$$\begin{aligned} &M_1(RSTx, RSTx', t) \\ &\geq 1 - c[1 - \min\{M_1(x, RSTx, t), M_1(x', RSTx', t), \\ &\qquad M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\}] \end{aligned} \tag{2.1}$$

for all $x, x' \in X$,

$$\begin{aligned} &M_2(TRSy, TRSy', t) \\ &\geq 1 - c[1 - \min\{M_2(y, TRSy, t), M_2(y', TRSy', t), \\ &\qquad M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\}] \end{aligned} \tag{2.2}$$

for all $y, y' \in Y$ *and*

$$\begin{aligned} &M_3(STRz, STRz', t) \\ &\geq 1 - c[1 - \min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \\ &\qquad M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\}] \end{aligned} \tag{2.3}$$

for all $z, z' \in Z$, *where* $0 \leq c < 1$.

Then RST *has a unique fixed point* $u \in X$, TRS *has a unique fixed point* $v \in Y$ *and* STR *has a unique fixed point* $w \in Z$. *Further,* $Tu = v$, $Sv = w$ *and* $Rw = u$.

Proof. Let x_0 be an arbitrary point in X. Define a sequence $\{x_n\}$ in X, $\{y_n\}$ in Y and $\{z_n\}$ in Z such that

$$(RST)^n x_0 = x_n, \quad y_n = Tx_{n-1}, \quad z_n = Sy_n, \quad n = 1, 2, 3, \dots. \tag{2.4}$$

Using inequality (2.2) we have by using (2.4)

$$\begin{aligned} &M_2(y_n, y_{n+1}, t) \\ &= M_2(TRSy_{n-1}, TRSy_n, t) \\ &\geq 1 - c[1 - \min\{M_2(y_{n-1}, TRSy_{n-1}, t), M_2(y_n, TRSy_n, t), \\ &\qquad M_2(y_{n-1}, y_n, t), M_3(Sy_{n-1}, Sy_n, t), M_1(RSy_{n-1}, RSy_n, t)\}] \\ &= 1 - c[1 - \min\{M_2(y_{n-1}, y_n, t), M_2(y_n, y_{n+1}, t), \\ &\qquad M_2(y_{n-1}, y_n, t), M_3(z_{n-1}, z_n, t), M_1(x_{n-1}, x_n, t)\}] \\ &= 1 - c[1 - \min\{M_2(y_{n-1}, y_n, t), M_3(z_{n-1}, z_n, t), M_1(x_{n-1}, x_n, t)\}]. \end{aligned} \tag{2.5}$$

On using inequality (2.3) we have by using (2.5)

$$\begin{aligned} &M_3(z_n, z_{n+1}, t) \\ &= M_2(STRz_{n-1}, STRz_n, t) \\ &\geq 1 - c[1 - \min\{M_3(z_{n-1}, RSTz_{n-1}, t), M_3(z_n, RSTz_n, t), \\ &\qquad M_3(z_{n-1}, z_n, t), M_1(Rz_{n-1}, Rz_n, t), M_2(TRz_{n-1}, TRz_n, t)\}] \\ &= 1 - c[1 - \min\{M_3(z_{n-1}, z_n, t), M_1(x_{n-1}, x_n, t), M_2(y_{n-1}, y_n, t)\}]. \end{aligned} \tag{2.6}$$

Similarly, on using inequality (2.1) we have by using (2.5) and (2.6),

$$\begin{aligned} &M_1(x_n, x_{n+1}, t) \\ &\geq 1 - c[1 - \min\{M_1(x_{n-1}, x_n, t), M_2(y_{n-1}, y_n, t), M_3(z_{n-1}, z_n, t)\}]. \end{aligned} \tag{2.7}$$

By repeated application on inequalities (2.5)-(2.7), we get

$$\begin{aligned}&M_1(x_n, x_{n+1}, t)\\&\geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}],\end{aligned}$$

$$\begin{aligned}&M_2(y_n, y_{n+1}, t)\\&\geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}],\end{aligned}$$

$$\begin{aligned}&M_3(z_n, z_{n+1}, t)\\&\geq 1 - c^{n-1}[1 - \min\{M_1(x_1, x_2, t), M_2(y_1, y_2, t), M_3(z_1, z_2, t)\}],\end{aligned}$$

where $n = 1, 2, 3, \ldots$. By Lemma 1.8, as $0 \leq c < 1$, we get $\{x_n\}$ is a Cauchy sequence in X, $\{y_n\}$ is a Cauchy sequence in Y and $\{z_n\}$ is a Cauchy sequence in Z. As X, Y and Z are complete fuzzy metric space, therefore, $\{x_n\}$ has a limit $u \in X$, $\{y_n\}$ has a limit $v \in Y$ and $\{z_n\}$ has a limit $w \in Z$. Also, as T and S are continuous

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Tx_n = Tu = v, \quad \lim_{n\to\infty} z_n = \lim_{n\to\infty} Sx_n = Sv = w.$$

Further, by using inequality (2.1), we get

$$\begin{aligned}&M_1(RSTu, x_n, t)\\&= M_1(RSTu, RSTx_{n-1}, t)\\&\geq 1 - c[1 - \min\{M_1(u, RSTu, t), M_1(x_{n-1}, RSTx_{n-1}, t),\\&\qquad M_1(u, x_{n-1}, t), M_2(Tu, Tx_{n-1}, t), M_3(STu, STx_{n-1}, t)\}].\end{aligned}$$

On letting $n \to \infty$, we have

$$\begin{aligned}&M_1(RSTu, u, t)\\&\geq 1 - c[1 - \min\{M_1(u, RSTu, t), M_1(u, u, t),\\&\qquad M_1(u, u, t), M_2(Tu, v, t), M_3(STu, STu, t)\}]\\&\geq 1 - c[1 - M_1(u, RSTu, t)],\end{aligned}$$

therefore, we have $RSTu = u$. Hence RST has fixed point $u \in X$.

Similarly, we can easily show by using inequality (2.2) that TRS has fixed point $v \in Y$, that is, $TRSv = TRSTu = Tu = v$ and by using inequality (2.3) that STR has fixed point $w \in Z$, that is, $STRw = STRSv = Sv = w$.

Finally, for uniqueness, suppose that RST has another fixed point $a \in X$, then by inequality (2.1), we have

$$\begin{aligned}
&M_1(u, a, t) \\
&= M_1(RSTu, RSTa, t) \\
&\geq 1 - c[1 - c\min\{M_1(u, RSTu, t), M_1(a, RSTa, t), \\
&\qquad M_1(u, a, t), M_2(Tu, Ta, t), M_3(STu, STa, t)\}] \\
&= 1 - c[1 - c\min\{M_2(Tu, Ta, t), M_3(STu, STa, t)\}].
\end{aligned} \tag{2.8}$$

Also, by inequality (2.2), we have

$$\begin{aligned}
&M_2(Tu, Ta, t) \\
&= M_1(TRSTu, TRSTa, t) \\
&\geq 1 - c[1 - c\min\{M_2(Tu, TRSTu, t), M_2(Ta, TRSTa, t), \\
&\qquad M_2(Tu, Ta, t), M_3(STu, STa, t), M_3(RSTu, RSTa, t)\}] \\
&= 1 - c[1 - c\min\{M_3(STu, STa, t), M_1(u, a, t)\}].
\end{aligned} \tag{2.9}$$

By using (2.9) in (2.8), we have

$$M_1(u, a, t) \geq 1 - c[M_3(STu, STa, t)]. \tag{2.10}$$

Further, by using inequality (2.3), we have

$$\begin{aligned}
&M_2(STu, STa, t) \\
&= M_3(STRSTu, STRSTa, t) \\
&\geq 1 - c[1 - c\min\{M_3(STu, RSTSTu, t), M_3(STa, RSTSTa, t), \\
&\qquad M_3(STu, STa, t), M_1(u, a, t), M_2(STu, STa, t)\}] \\
&= 1 - c[1 - c\min\{M_1(u, a, t), M_2(Tu, Ta, t)\}].
\end{aligned}$$

Using in (2.10), we get

$$M_1(u, a, t) \geq 1 - c^2[M_1(u, a, t)].$$

Since $c < 1$, this implies $u = a$. Hence RST has unique fixed point in X.

Similarly, we prove that TRS has a unique common fixed point $v \in Y$ and STR has a unique common fixed point $w \in Z$. This completes the proof. □

In Theorem 2.1, by taking $X = Y = Z$, $M_1 = M_2 = M_3 = M$ and $*_1 = *_2 = *_3 = *$, we have following corollary.

Corollary 2.2. *Let $(X, M, *)$ be a complete fuzzy metric space. If T and S are continuous mappings of X into X and R is a mapping of X into X satisfying*

$$\begin{aligned}&M(RSTx, RSTx', t)\\&\geq 1 - c[1 - \min\{M(x, RSTx, t), M(x', RSTx', t),\\&\qquad M(x, x', t), M(Tx, Tx', t), M(STx, STx', t)\}]\end{aligned}$$

for all $x, x' \in X$,

$$\begin{aligned}&M(TRSy, TRSy', t)\\&\geq 1 - c[1 - \min\{M(y, TRSy, t), M(y', TRSy', t),\\&\qquad M(y, y', t), M(Sy, Sy', t), M(RSy, RSy', t)\}]\end{aligned}$$

for all $y, y' \in X$ and

$$\begin{aligned}&M(STRz, STRz', t)\\&\geq 1 - c[1 - \min\{M(z, RSYz, t), M(z', RSTz', t),\\&\qquad M(z, z', t), M(Rz, Rz', t), M(TRz, TRz', t)\}]\end{aligned}$$

for all $z, z' \in X$, where $0 \leq c < 1$.

Then RST has a unique fixed point $u \in X$, TRS has a unique fixed point $v \in Y$ and STR has a unique fixed point $w \in Z$. Further, $Tu = v$, $Sv = w$ and $Rw = u$ and if $u = v = w$, u is a unique common fixed point of R, S and T.

Next, we prove a fixed point theorem involving three compact fuzzy metric spaces.

Theorem 2.3. *Let $(X, M_1, *_1)$, $(Y, M_2, *_2)$ and $(Z, M_3, *_3)$ be three compact fuzzy metric spaces. If T is a continuous mapping of X into Y, S is a continuous mapping of Y into Z and R is a continuous mapping of Z into X satisfying*

$$\begin{aligned}&M_1(RSTx, RSTx', t)\\&> \min\{M_1(x, RSTx, t), M_1(x', RSTx', t),\\&\qquad M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\}\end{aligned} \tag{2.11}$$

for all $x, x' \in X$,

$$\begin{aligned}&M_2(TRSy, TRSy', t)\\&> \min\{M_2(y, TRSy, t), M_2(y', TRSy', t),\\&\qquad M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\}\end{aligned} \tag{2.12}$$

for all $y, y' \in Y$ *and*

$$\begin{aligned} &M_3(STRz, STRz', t) \\ &> \min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \\ &\qquad M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\} \end{aligned} \tag{2.13}$$

for all $z, z' \in Z$.

Then RST *has a unique fixed point* $u \in X$, TRS *has a unique fixed point* $v \in Y$ *and* STR *has a unique fixed point* $w \in Z$. *Further,* $Tu = v$, $Sv = w$ *and* $Rw = u$.

Proof. Suppose first of all there exists $u, u' \in X$ such that

$$\begin{aligned} &\min\{M_1(x, RSTx, t), M_1(x', RSTx', t), \\ &\qquad M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\} \\ &= 1, \end{aligned}$$

this gives $u = RSTu$, $u' = RSTu'$, $u = u'$ and $Tu = Tu'$.

Let $Tu = v$. Then we have $RSv = u$, thisgives $TRSv = Tu = v$ and $STRSv = STRw = STu = Sv = w$. Then RST has a fixed point $u \in X$, TRS has a fixed point $v \in Y$ and STR hasa fixed point $w \in Z$.

Similarly, we holds if there exist $v, v' \in Y$ and $w, w' \in Z$ such that

$$\begin{aligned} &\min\{M_2(y, TRSy, t), M_2(y', TRSy', t), \\ &\qquad M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\} \\ &= 1 \end{aligned}$$

or

$$\begin{aligned} &\min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \\ &\qquad M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\} \\ &= 1. \end{aligned}$$

Hence, this proves the results.

Now, suppose that $u, u' \in X$, $v, v' \in Y$ and z, z' do not exist. Then

$$\begin{aligned} &1 - \min\{M_1(x, RSTx, t), M_1(x', RSTx', t), \\ &\qquad M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\} \\ &\neq 0 \end{aligned}$$

for all $x, x' \in X$ and so real valued function defined by

$$F(x, x', t) = \frac{M_1(RSTx, RSTx', t)}{1 - \mathcal{O}},$$

where
$$\mathcal{O} = \min\{M_1(x, RSTx, t), M_1(x', RSTx', t), M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\}$$
is continous. Since X is compact. F attains its maximum value, say c_1. By inequality (2.11) we have $0 \leq c_1 < 1$ and so
$$\begin{aligned} &M_1(RSTx, RSTx', t) \\ &\geq 1 - c_1[1 - \min\{M_1(x, RSTx, t), M_1(x', RSTx', t), \\ &\qquad M_1(x, x', t), M_2(Tx, Tx', t), M_3(STx, STx', t)\}] \end{aligned}$$
for all $x, x' \in X$. Similarly, By inequalities (2.12) and (2.13), there exists c_2, where $0 \leq c_2 < 1$ such that
$$\begin{aligned} &M_3(TRSy, TRSy', t) \\ &\geq 1 - c_2[1 - \min\{M_2(y, TRSy, t), M_2(y', TRSy', t), \\ &\qquad M_2(y, y', t), M_3(Sy, Sy', t), M_1(RSy, RSy', t)\}] \end{aligned}$$
for all $y, y' \in Y$ and there exists c_3, where $0 \leq c_3 < 1$ such that
$$\begin{aligned} &M_2(STRz, STRz', t) \\ &\geq 1 - c_3[1 - \min\{M_3(z, RSYz, t), M_3(z', RSTz', t), \\ &\qquad M_3(z, z', t), M_1(Rz, Rz', t), M_2(TRz, TRz', t)\}] \end{aligned}$$
for all $z, z' \in Z$. It follows that all conditions of Theorem 2.1 are satisfied with $c = \min\{c_1, c_2, c_3\}$. Therefore, RST has a unique fixed point $u \in X$, TRS has a unique fixed point $v \in Y$ and STR has a unique fixed point $w \in Z$. Further, $Tu = v$, $Sv = w$ and $Rw = u$. This completes the proof. □

In Theorem 2.3, by taking $X = Y = Z$, $M_1 = M_2 = M_3 = M$ and $*_1 = *_2 = *_3 = *$, we have following corollary.

Corollary 2.4. *Let $(X, M, *)$ be a compact fuzzy metric space. If T, S and R are continuous mappings of X into X satisfying*
$$\begin{aligned} &M(RSTx, RSTx', t) \\ &> \min\{M(x, RSTx, t), M(x', RSTx', t), M(x, x', t), \\ &\qquad M(Tx, Tx', t), M(STx, STx', t)\}] \end{aligned}$$
for all $x, x' \in X$,
$$\begin{aligned} &M(TRSy, TRSy', t) \\ &> \min\{M(y, TRSy, t), M(y', TRSy', t), M(y, y', t), \\ &\qquad M(Sy, Sy', t), M(RSy, RSy', t)\}] \end{aligned}$$

for all $y, y' \in Y$ *and*

$$\begin{aligned}&M(STRz, STRz', t)\\&> \min\{M(z, RSYz, t), M(z', RSTz', t), M(z, z', t),\\&\qquad M(Rz, Rz', t), M(TRz, TRz', t)\}\end{aligned}$$

for all $z, z' \in X$.

Then RST *has a unique fixed point* $u \in X$, TRS *has a unique fixed point* $v \in Y$ *and* STR *has a unique fixed point* $w \in Z$. *Further,* $Tu = v$, $Sv = w$ *and* $Rw = u$ *and if* $u = v = w$, u *is a unique common fixed point of* R, S *and* T.

3. Corresponding Results in Metric Spaces

Every metric space (X, d) can be realized as a fuzzy metric space $(X, M, *)$ by taking

$$M(x, y, t) = \begin{cases}\frac{t}{t+d(x,y)}, & t > 0,\\ 0, & t = 0.\end{cases}$$

Hence results in a fuzzy metric space can be used to prove existence of fixed point in a metric space.

In this section, as a sample, we utilize Theorem 2.1 to derive corresponding common fixed point theorem in metric spaces.

Theorem 3.1. *Let* (X, d_1), (Y, d_2) *and* (Z, d_3) *be three complete metric spaces. If* T *is a continuous mapping of* X *into* Y, S *is a continuous mapping of* Y *into* Z *and* R *is a mapping of* Z *into* X *satisfying*

$$\begin{aligned}&d_1(RSTx, RSTx')\\&\le c\max\{d_1(x, RSTx), d_1(x', RSTx'),\\&\qquad d_1(x, x'), d_2(Tx, Tx'), d_3(STx, STx')\}]\end{aligned} \tag{2.14}$$

for all $x, x' \in X$,

$$\begin{aligned}&d_2(TRSy, TRSy')\\&\le c\max\{d_2(y, TRSy), d_2(y', TRSy'),\\&\qquad d_2(y, y'), d_3(Sy, Sy'), d_1(RSy, RSy')\}]\end{aligned} \tag{2.15}$$

for all $y, y' \in Y$ *and*

$$\begin{aligned}&M_2(STRz, STRz', t)\\&\le c\max\{d_3(z, RSYz), d_3(z', RSTz'),\\&\qquad d_3(z, z'), d_1(Rz, Rz'), d_2(TRz, TRz')\}]\end{aligned} \tag{2.16}$$

for all $z, z' \in Z$, where $0 \le c < 1$.

Then RST has a unique fixed point $u \in X$, TRS has a unique fixed point $v \in Y$ and STR has a unique fixed point $w \in Z$. Further, $Tu = v$, $Sv = w$ and $Rw = u$.

Proof. Define

$$M_1(x,y,t) = \begin{cases} \frac{t}{t+d_1(x,y)}, & t > 0, \\ 0, & t = 0. \end{cases}, \quad M_2(x,y,t) = \begin{cases} \frac{t}{t+d_2(x,y)}, & t > 0, \\ 0, & t = 0, \end{cases}$$

and

$$M_3(x,y,t) = \begin{cases} \frac{t}{t+d_3(x,y)}, & t > 0, \\ 0, & t = 0 \end{cases}$$

and $*_1(a,b) = *_2(a,b) = *_3(a,b) = \min\{a,b\}$ for all $a, b \in [0,1]$. Then metric spaces (X, d_1), (Y, d_2) and (Z, d_3) can be realized as a fuzzy metric spaces $(X, M_1, *_1)$, $(Y, M_2, *_2)$ and $(Z, M_3, *_3)$, respectively. It is straightforward to notice that Theorem 3.1 satisfies all the conditions of Theorem 2.1. Also inequalities (2.14)-(2.16) of Theorem 3.1 implies inequalities (2.1)-(2.3) of Theorem 2.1. Thus, all the conditions of Theorem 2.1 are satisfied so that RST has a unique fixed point $u \in X$, TRS has a unique fixed point $v \in Y$ and STR has a unique fixed point $w \in Z$. Further, $Tu = v$, $Sv = w$ and $Rw = u$. □

Remark 3.2. The similar results to Theorem 3.1 can also be outlined in respects of Corollary 2.2, Theorem 2.3 and Corollary 2.4.

Remark 3.3. Theorem 3.1 improves the results of Jain et al. [8] in three metric spaces.

Remark 3.4. By taking $Y = Z$ and $d_3 = d_2$ and R is the identity mapping, Theorem 3.1 inproves the results of Fisher [2], [3].

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