International Journal of Pure and Applied Mathematics
Volume 94 No. 2 2014, 155-161
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v94i2.3

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A NOTE ON PERFECT FUZZY MATCHING

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Abstract: The notion of matching in a fuzzy graph could be defined using the concept of effective edges [8] or by fractional matching [4]. In this paper, we derive a necessary condition for a fuzzy graph on a cycle or a complete graph or a stargraph to have a perfect fuzzy matching. Also we discuss perfect fuzzy matching on strong regular fuzzy graphs.

AMS Subject Classification: 05C72, 03E72
Key Words: fuzzy graph, fuzzy matching, perfect fuzzy matching

1. Introduction

Zadeh introduced the notion of Fuzzy sets and Fuzzy relations to deal with the problems of uncertainty in real physical world. In 1975, Rosenfeld [5] introduced the concept of fuzzy graphs. Using the concept of effective edges, Somasundaram [8] defined matching in a fuzzy graph. This matching is defined only for those graphs having effective edges. Ramakrishnan P.V and Vaidyanathan M [4] introduced matching in a fuzzy graph using the concept of fractional matching. The notion of fractional matching given by Scheinerman [1], in 1997, involves the vertex weight, the edge weight and the incidence of

Received: December 20, 2013

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edges. In this paper, we derive the necessary condition for the fuzzy graph on a cycle or a complete graph or a star graph to have perfect fuzzy matching. Further, we prove that a strong regular fuzzzy graph will have no perfect fuzzy matching.

2. Preliminaries

Definition 2.1. *A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ with $\mu(u,v) \leq \sigma(u) \wedge \sigma(v),\ \forall u,v \in V$, where V is a finite nonempty set and $\wedge$ denote minimum.*

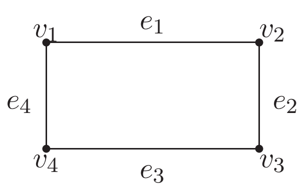
Definition 2.2. *A fuzzy graph $G = (\sigma, \mu)$ is defined to be a strong fuzzy graph if*
$\mu(u,v) = \sigma(u) \wedge \sigma(v),\ \forall (u,v) \in E.$

Definition 2.3. *A fuzzy graph $G = (\sigma, \mu)$ is defined to be a complete fuzzy graph if*
$\mu(u,v) = \sigma(u) \wedge \sigma(v),\ \forall\ u,v\ \in V.$

Definition 2.4. *Let $G = (\sigma, \mu)$ be a fuzzy graph on $G^* = (V,E)$. The fuzzy degree of a node $u \in V$ is defined as $(fd)(u) = \sum_{u \neq v, v \in V} \mu(u,v)$.*
G is said to be regular fuzzy graph if each vertex has same fuzzy degree.
If $(fd)(v) = k,\ \forall v \in V, G$ is said to be $k-$regular fuzzy graph.

Definition 2.5. *Let $G = (\sigma, \mu)$ be a fuzzy graph on (V,E). where V is the vertex set and E is the set of edges with non-zero weights. A subset M os E is called a Fuzzy matching if for each vertex u, we have $\sum_{v \in V} \mu(u,v) \leq \sigma(u)$.*

Example 2.6. Let $G = (\sigma, \mu)$ be a fuzzy graph on (V,E) where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$ with $e_1 = v_1v_2, e_2 = v_2v_3, e3 = v_3, v_4$ *and* $e_4 = v_4v_1$



$\sigma(v_1) = 1, \sigma(v_2) = 0.5, \sigma(v_3) = 0.4, \sigma(v_4) = 0.7$
$\mu(e_1) = 0.3, \mu(e_2) = 0.4, \mu(e_3) = 0.2, \mu(e_4) = 0.5$
$\sum_{v_2 \in V, (v_1,v_2) \in M} \mu(v_1, v_2) = 0.3+\ 0.5 = 0.8 \leq 1 = \sigma(v_1).$

$\sum_{v_3 \in V, (v_2, v_3) \in M} \mu(v_2, v_3) = 0.4+ 0.3 = 0.7 \leq 0.5 = \sigma(v_2)$.
$\sum_{v_1 \in V, (v_1, v_4) \in M} \mu(v_1, v_4) = 0.5+ 0.2 = 0.7 \leq 0.7 = \sigma(v_4)$.
Thus $M = \{e_1, e_2, e_4\}$ is a fuzzy matching in G.

Definition 2.7. *A fuzzy matching M is called a Perfect Fuzzy Matching if for each vertex u, $\sum_{v \in V, (u,v) \in M} \mu(u, v) = \sigma(u)$.*

Example 2.8. Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ with $e_1 = v_1v_2, e_2 = v_2v_3, e3 = v_3, v_4, e_4 = v_4v_1$ *and* $e_5 = v_4v_2$
$\sigma(v_1) = 1, \sigma(v_2) = 0.9, \sigma(v_3) = 0.8, \sigma(v_4) = 0.9$
$\mu(e_1) = 0.4, \mu(e_2) = 0.5, \mu(e_3) = 0.3, \mu(e_4) = 0.6, \mu(e_5) = 0.8$
Here $\sum_{v_2 \in V, (v_1, v_2) \in M} \mu(v_1, v_2) = 0.4+ 0.6 = 1 = \sigma(v_1)$.
$\sum_{v_3 \in V, (v_2, v_3) \in M} \mu(v_2, v_3) = 0.4+ 0.5 = 0.9 = \sigma(v_2)$.
$\sum_{v_3 \in V, (v_3, v_4) \in M} \mu(v_3, v_4) = 0.5+ 0.3 = 0.8 = \sigma(v_3)$.
$\sum_{v_4 \in V, (v_1, v_4) \in M} \mu(v_1, v_4) = 0.3+ 0.6 = 0.9 = \sigma(v_4)$.
Thus $M = \{v_1v_2, v_2v_3, v_3v_4, v_4v_1\}$ is a perfect fuzzy matching in G.

Definition 2.9. *Let $G = (\sigma, \mu)$ be a fuzzy graph and M be a fuzzy matching. Then fuzzy matching number $\Gamma(G)$ is defined to be $\Gamma(G) = \sum_{(u,v) \in M} \mu(u, v)$.*

Example 2.10. In example 2.6, $\Gamma(G) = 1.2$. In example 2.8, $\Gamma(G) = 1.8$.

Theorem 2.11. *[6] Let $G = (\sigma, \mu)$ be a fuzzy graph on $K_{n,n}$ with bipartition (X, Y) where $X = \{u_1, u_2, \cdots, u_n\}$ and $Y = \{v_1, v_2, \cdots, v_n\}$. Then G is strong regular if and only if*

1. $\sigma(v_j) = k$
2. $\sigma(u_i) = k$.
3. $\mu(u_i, v_j) = k$, *for* $i, j = 1, 2, \cdots n$ *and for some* k.

3. Perfect Fuzzy Matching

In this section we discuss some necessary conditions for some of the fuzzy graphs to have a perfect matching.

Theorem 3.1. *Let $G = (\sigma, \mu)$ be a regular fuzzy graph on the cycle (V, E). If $\sigma(u) = k$, which is constant for all $u \in V$ and $\mu(u, v) = \frac{k}{2}$ for all $(u, v) \in E$, then E is a perfect fuzzy matching for G.*

Proof

Since only two edges are incident wtih each vertex for cycles, for any vertex $v \in V$

$$\begin{aligned}\sum_{v\in V,(u,v)\in E} \mu(u,v) \ where\ v,w \in V &= \mu(u,v)+\mu(u,w)\\ &= \frac{k}{2}+\frac{k}{2}\\ &= k=\sigma(u)\end{aligned}$$

Then E is a perfect fuzzy matching in G.

The converse of the above theorem need not be true. This can be seen using the following example.

Example 3.2. Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$ where $e_1 = v_1v_2, e_2 = v_2v_3, e3 = v_3, v_4\ and\ e_4 = v_4v_1$

Define $\sigma(v_1) = 0.6, \sigma(v_2) = 0.6, \sigma(v_3) = 0.5, \sigma(v_4) = 0.5$ and
$\mu(e_1) = 0.4, \mu(e_2) = 0.2, \mu(e_3) = 0.3, \mu(e_4) = 0.2$.

Then the graph G has perfect fuzzy matching but the conditions of the above theorem are not satisfied.

Theorem 3.3. *Let $G = (\sigma, \mu)$ be a complete fuzzy graph K_n on (V, E). If $\sigma(u) = k$ which is constant for all $u \in V$ and $\mu(u, v) = [\frac{k}{n}] = k_1$, for all (u, v) on the cycle C_n and $\mu(u, v) = \frac{k-2k_1}{n-3}$ for all the interior edges $(u, v) \in E$, then E is a perfect fuzzy matching for G.*

Proof

For any complete fuzzy graph K_n, two edges are incident with each vertex of the cycle and remaining $(n - 3)$ edges are incident with interior vertices. Hence

$$\begin{aligned}\sum_{v\in V,(u,v)\in E} \mu(u,v) &= 2k_1+(n-3)(\frac{k-2k_1}{n-3}\\ &= 2k_1+k-2k_1\\ &= k\\ &= \sigma(u), for\ each\ vertex\ u.\end{aligned}$$

Therefore E is a perfect fuzzy matching for G.

The converse of the above theorem is not true. This can be seen using the following example.

Example 3.4. Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E) where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$
where $e_1 = v_1v_2, e_2 = v_2v_3, e3 = v_3, v_4, e_4 = v_4v_1, e_5 = v_4v_2$ *and* $e_6 = v_1v_3$
Define $\sigma(v_1) = 0.9, \sigma(v_2) = 0.8, \sigma(v_3) = 0.7, \sigma(v_4) = 0.8$ and
$\mu(e_1) = 0.2, \mu(e_2) = 0.3, \mu(e_3) = 0.1, \mu(e_4) = 0.4, \mu(e_5) = 0.3, \mu(e_6) = 0.3.$
Then E is a perfect matching for G but the conditions of the above theorem are not satisfied.

Theorem 3.5. *Let $G = (\sigma, \mu)$ be a strong fuzzy graph on the star graph $S_n = (V, E)$ with $V = \{v, v_1, v_2, \cdots, v_{n-1}\}$. If $\sigma(v_i) = k, \forall i = 1, 2, \cdots, (n-1)$ and if $\sigma(v) = (n-1)k$, then E is a perfect fuzzy matching in G.*

Proof
Since G is strong,

$$\begin{aligned} \mu(v, v_i) &= \sigma(v) \wedge \sigma(v_i), \ \forall v_i \in V \\ &= (n-1)k \wedge k \\ &= k \end{aligned}$$

Now, for any $v \in V$,

$$\begin{aligned} \sum_{v_i \in V, (v, v_i) \in E} \mu(v, v_i) &= \sum_{(v, v_i) \in E} k \\ &= (n-1)k, \ \textit{since } (n-1) \textit{ edges are incident with } v. \\ &= \sigma(v) \end{aligned}$$

Therefore, E is a perfect fuzzy matching for G.
The converse of the above theorem is also not true. This can be seen from the following example.

Example 3.6. Let $G = (\sigma, \mu)$ be a fuzzy graph on (V, E) where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$ where $e_1 = vv_1, e_2 = vv_2, e_3 = vv_3, e_4 = vv_4$ *and* $e_5 = vv_5$.
Define $\sigma(v_1) = 0.2, \sigma(v_2) = 0.1, \sigma(v_3) = 0.2, \sigma(v_4) = 0.1, \sigma(v_5) = 0.2, \sigma(v) = 0.8$
$\mu(e_1) = 0.2, \mu(e_2) = 0.1, \mu(e_3) = 0.2, \mu(e_4) = 0.1, \mu(e_5) = 0.2.$
Then E is a perfect matching for G but the conditions of the above theorem are not satisfied.

The following theorem establishes that a strong regular fuzzy graph need not have a perfect fuzzy matching.

Theorem 3.7. *If $G = (\sigma, \mu)$ is a strong regular fuzzy graph on (V, E) with each vertex is of degree atleast two, then E is not a perfect fuzzy matching in G.*

Proof If possible, let E be a perfect fuzzy matching for G. Then
$\sum_{v\in V,(u,v)\in E} \mu(u, v) = \sigma(u), \forall u \in V,$ ———(1) by definition.
Since G is regular, $(fd)(u)$ = constant = k(say),$\forall u \in V$.
Therefore, $\sum_{v\in V,(u,v)\in E} \mu(u, v) = k, \forall u \in V$, which imply
$\sigma(u) = k, \forall u \in V$. (Using (1)).
Now from (1) $\sum_{v\in V,(u,v)\in E} \mu(u, v) = k \Rightarrow \mu(u, v) < k$, since atleast two edges incident with u, for some (u, v).
Since G is strong, $\sigma(u) \wedge \sigma(v) < k \Rightarrow k \wedge k < k \Rightarrow k < k$, which is a contradiction.
Therefore, E is not perfect fuzzy matching for G.
In particular, we have the following

Corollary 3.8. *If $G = (\sigma, \mu)$ is strong regular fuzzy graph on $K_{n,n}$, then E is not a perfect fuzzy matching.*

Proof Suppose E is a perfect fuzzy matching for G.
Then $\sum_{v\in V,(u,v)\in E} \mu(u, v) = \sigma(u)$, for each $u \in V$.
Let $\sigma(u) = k, \forall u \in X$.
Therefore,$\sum_{v\in V} \mu(u, v) = k, \forall u \in X$.
Hence $\mu(u, v) < k$, for some $v \in V$, which is a contradiction to theorem 2.11
Therefore, E is not a perfect fuzzy matching for G.

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