

**POISSON APPROXIMATION FOR THE NUMBER
OF ISOLATED COMPLETE GRAPHS IN
A RANDOM INTERSECTION GRAPH**

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Abstract: Let $W_{n,r}$ be the number of isolated complete graphs of order r in a random intersection graph $\mathbb{G}(n, m, p)$. In this paper, we demonstrate that $W_{n,r}$ can be approximated by Poisson distribution and give the bound of this approximation by using the Stein-Chen method.

Key Words: random intersection graph, isolated complete graphs, Stein-Chen method

1. Introduction

Given a set V with n vertices and another universal set U with m elements, define a bipartite graph $B(n, m, p)$ with independent vertex sets V and U and edges between $v \in V$ and $u \in U$ existing independently with probability p . The random intersection graph $\mathbb{G}(n, m, p)$, derived from $B(n, m, p)$, is defined on the vertex set V with vertices $v_1, v_2 \in V$ adjacent if and only if there exist some $u \in U$ such that both v_1 and v_2 are adjacent to u in $B(n, m, p)$. Also define

S_i be a random subset of U such that each element of S_i is adjacent to $i \in V$, in which case two vertices $i, j \in V$ are adjacent if and only if $S_i \cap S_j \neq \phi$, and edge set $E(\mathbb{G})$ is define as

$$E(\mathbb{G}) = \{\{i, j\} : i, j \in V, S_i \cap S_j \neq \phi\}.$$

The properties of $\mathbb{G}(n, m, p)$ were studied in [2,3] contrasted with the well known random graph model $\mathbb{G}(n, p)$, in which vertices are made adjacent to each other independently and with probability p , and showed that for a fixed $\alpha > 0$, the number of elements m is taken to be $m = \lfloor n^\alpha \rfloor$. In 1999, Karon-ski, Scheinerman and Singer-Cohen[2] showed that the total variation distance between the distribution of $\mathbb{G}(n, m, p)$ and $\mathbb{G}(n, p)$ converges to 0 when $\alpha > 0$ and p is defined appropriately. Without loss of generality we consider the independent set V . For $i = 1, 2, 3, \dots, n$, let

$$X_i = \begin{cases} 1 & \text{if vertex } i \text{ is an isolated in } \mathbb{G}(n, m, p) \\ 0 & \text{otherwise,} \end{cases}$$

and

$$X = \sum_{i \in \Gamma} X_i.$$

Clearly, X is the number of isolated vertices in $\mathbb{G}(n, m, p)$.

In 2011, Yilum Shang[6] proved that the distribution function of X can be approximated by Poisson distribution with parameter

$$\begin{aligned} \lambda := EW &= nP(X_i = 1) = n \sum_{s=0}^m \binom{m}{s} p^s (1-p)^{m-s} (1-p)^{(n-1)s} \\ &= n[1-p + p(1-p)^{(n-1)}]^m. \end{aligned}$$

In 2013, M. Donganont[9] showed the another proof of Poisson approximation for the number of isolated vertices in $\mathbb{G}(n, m, p)$ by Stein-Chen and coupling method. The results as the following,

Theorem 1.1. *Let W be the number of isolated vertices in a random intersection graph $\mathbb{G}(n, m, p)$.*

For $A \subseteq \{0, 1, 2, \dots, n\}$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, we have

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{nC_{\lambda,A}}{e^{pm(1-(1-p)^{n-2})}}$$

where $C_{\lambda,A} = \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$,

$$\Delta(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

When $C_w = \{0, 1, \dots, w\}$

Corollary 1.1. *Let W be the number of isolated vertices in a random intersection graph $\mathbb{G}(n, m, p)$. Let $A \subseteq \{0, 1, 2, \dots, n\}$, $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, $q = 1 - p$, and $p = \frac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-q)^{n-2}-1}},$$

where $C(\lambda, A) = \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

Let

$$\Gamma_{n,k} = \{i =: \{i_1, i_2, \dots, i_k\} \mid 1 \leq i_1 < \dots < i_k \leq n\}$$

be the set of all possible combinations of k vertices. we note that T_k is a tree of order k in $\mathbb{G}(n, m, p)$ and say that T_k is isolated in $\mathbb{G}(n, m, p)$ if there is no edge in $\mathbb{G}(n, m, p)$ between a vertex in the tree and the other outside of the tree.

For each $i \in \Gamma_{n,k}$, we define the indicator random variable

$$X_i = \begin{cases} 1 & \text{if there is an isolated tree in } \mathbb{G}(n, m, p) \text{ that spans the vertices} \\ & i = (i_1, \dots, i_k), \\ 0 & \text{otherwise,} \end{cases}$$

and set

$$W_{n,k} = \sum_{i \in \Gamma_{n,k}} X_i.$$

Then $W_{n,k}$ is the number of isolated trees in $\mathbb{G}(n, m, p)$.

In 2013, Mana[10] shows that if $m = \lfloor n^\alpha \rfloor$; $\alpha > 0$, then $W_{n,k}$ can be approximated by Poisson approximation with parameter

$$\begin{aligned} \lambda := \mathbb{E}(W_{n,k}) &= \binom{n}{k} P(X_i = 1) = \binom{n}{k} \sum_{s=0}^m \binom{m}{s} p^{s+k-1} (1-p)^{m-s} (1-p)^{(n-k)s} \\ &= \binom{n}{k} p^{k-1} \left[1 - p + p(1-p)^{(n-k)} \right]^m. \end{aligned} \tag{1}$$

By using Stein-Chen and Coupling Method. The result is following,

Theorem 1.2. *Let W be the number of isolated trees in a random intersection graph $\mathbb{G}(n, m, p)$.*

For $A \subseteq \{0, 1, 2, \dots, n\}$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, we have

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C_{(\lambda, A)}(1 + \frac{n^k}{k!}pk^2)p^{k-1}}{e^{mp(1-(1-p)^{n-2k})}}$$

where $C_{(\lambda, A)} = \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$,

$$\Delta(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

When $C_w = \{0, 1, \dots, w\}$

Corollary 1.2. *Let W be the number of isolated trees in a random intersection graph $\mathbb{G}(n, m, p)$. Let $A \subseteq \{0, 1, 2, \dots, n\}$, $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, and $p = \frac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$*

1. *If $\gamma > k > 0$, $q = 1 - p$, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-\gamma)}},$$

where $C(\lambda, A) = 2 \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

2. *If $k > \gamma > 0$, $q = 1 - p$, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2k})+(k\gamma-k)}},$$

where $C(\lambda, A) = 2 \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

In this paper, we use this idea to show that the number of isolated complete graphs can be approximated by Poisson distribution.

Now, define,

$$\Gamma_{n,r} = \{i =: \{i_1, i_2, \dots, i_r\} \mid 1 \leq i_1 < \dots < i_r \leq n\}$$

be the set of all possible combinations of r vertices. we note that K_r is a complete graph of order r in $\mathbb{G}(n, m, p)$ and say that K_r is isolated if there is no edge between a vertex in the complete graph and the other outside of that one.

For each $i \in \Gamma_{n,r}$, we define the indicator random variable

$$X_i = \begin{cases} 1 & \text{if there is an isolated complete graph in } \mathbb{G}(n, m, p) \text{ that spans} \\ & \text{the vertices } i = (i_1, \dots, i_r), \\ 0 & \text{otherwise,} \end{cases}$$

and set

$$W_{n,r} = \sum_{i \in \Gamma_{n,r}} X_i.$$

Thus, $W_{n,r}$ is the number of isolated complete graphs in $\mathbb{G}(n, m, p)$.

If $m = \lfloor n^\alpha \rfloor$; $\alpha > 0$, we show that $W_{n,r}$ can be approximated by Poisson approximation with parameter

$$\begin{aligned} \lambda := \mathbb{E}(W_{n,r}) &= \binom{n}{r} P(X_i = 1) = \binom{n}{r} \sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-r)s} \\ &= \binom{n}{r} p^{2r-2} \left[1 - p + p(1-p)^{(n-r)} \right]^m. \end{aligned} \tag{2}$$

By using Stein-Chen and Coupling Method. The main result is following,

Theorem 1.3. *Let W be the number of isolated complete graphs of order r in a random intersection graph $\mathbb{G}(n, m, p)$. For $A \subseteq \{0, 1, 2, \dots, n\}$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C_{(\lambda,A)}(1 + \frac{n^r}{r!}r^2)p^{2r-2}}{e^{mp(1-(1-p)^{n-2r})}}$$

where $C_{(\lambda,A)} = \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$,

$$\Delta(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

When $C_w = \{0, 1, \dots, w\}$

Corollary 1.3. *Let W be the number of isolated complete graphs of order r in a random intersection graph $\mathbb{G}(n, m, p)$. If $A \subseteq \{0, 1, 2, \dots, n\}$, $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$, $q = 1 - p$ and $p = \frac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2r})+(2r-2)\gamma-r}},$$

where $C(\lambda, A) = 2 \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

2. Stein-Chen and Coupling Method

In 1972, Stein [1] gave a new technique to find a bound in the normal approximation to a distribution of a sum of dependent random variables. His technique was relied instead on the elementary differential equation. In 1975, Chen [4, 5] applied Stein’s idea to the Poisson case. The central idea of the Stein-Chen method is the difference equation

$$I_A(j) - \mathcal{P}_\lambda(A) = \lambda g_{\lambda,A}(j + 1) - j g_{\lambda,A}(j), j \in \mathbb{N} \cup \{0\} \tag{3}$$

where $\lambda > 0$ and $A \subseteq \mathbb{N} \cup \{0\}$ and $I_A : \mathbb{N} \cup \{0\} \rightarrow \mathbb{R}$ is defined by

$$I_A(w) = \begin{cases} 1 & ; w \in A, \\ 0 & ; w \notin A. \end{cases}$$

The equation (3) is called Stein’s equation for Poisson distribution function and its solution is

$$g_{\lambda,A}(w) = \begin{cases} (w - 1)! \lambda^{-w} e^\lambda [\mathcal{P}_\lambda(I_A \cap C_{w-1}) - \mathcal{P}_\lambda(I_A) \mathcal{P}_\lambda(C_{w-1})] & ; w \geq 1, \\ 0 & ; w = 0 \end{cases}$$

where

$$C_{w-1} = \{0, 1, \dots, w - 1\} \text{ and } \mathcal{P}_\lambda(I_A) = e^{-\lambda} \sum_{l=0}^{\infty} I_A(l) \frac{\lambda^l}{l!}. \tag{7}$$

By substituting j and λ in (3) by any integer-valued random variable W and $\lambda = \mathbb{E}(W)$, we have

$$P(W \in A) - Poi_\lambda(A) = \mathbb{E}(\lambda g_{\lambda,A}(W + 1)) - \mathbb{E}(W g_{\lambda,A}(W)). \tag{4}$$

So far W could be $\sum_{i \in \Gamma} X_i$ and $\lambda = \mathbb{E}(W) = \sum_{i \in \Gamma} p_i$ where $p_i = \mathbb{E}(X_i) = P(X_i = 1)$.

In 1992, Barbour, Holst and Janson[7] constructed coupling random variable W_i and used Stein-Chen method to find the bound in Poisson approximation of W . They assumed that for each i the distribution $L(W_i)$ of W_i equals to the conditional distribution $L(W - X_i | X_i = 1)$ and gave the fundamental theorem as follows:

Theorem 2.1. *If W and W_i are defined as above, then*

$$|P(W \in A) - Poi_\lambda(A)| \leq \|g_{\lambda,A}\| \sum_{i \in \Gamma} p_i \mathbb{E}(|W - W_i|) \tag{5}$$

where $\|g_{\lambda,A}\| := \sup_w [g_{\lambda,A}(w + 1) - g_{\lambda,A}(w)]$.

In 2006, Santiwipanon and Teerapabolarn [8] proved that for any subset A of $\{0, 1, \dots, n\}$,

$$\|g_{\lambda,A}\| \leq \lambda^{-1} \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A + 1} \right\} \tag{6}$$

where

$$\Delta(\lambda) = \begin{cases} e^\lambda + \lambda - 1 & \text{if } \lambda^{-1}(e^\lambda - 1) \leq M_A, \\ 2(e^\lambda - 1) & \text{if } \lambda^{-1}(e^\lambda - 1) > M_A, \end{cases}$$

and

$$M_A = \begin{cases} \max\{w \mid C_w \subseteq A\} & \text{if } 0 \in A, \\ \min\{w \mid w \in A\} & \text{if } 0 \notin A. \end{cases}$$

In next section, we will use Theorem 2.1 and (6) to prove our main result by constructing the random variable W_i .

3. Proof of Theorem 1.3

Let $A \subseteq \mathbb{N}$. By (5), it is enough to bound $\mathbb{E}|W - W_i|$ for $i \in \Gamma_{n,r}$ where the distribution of W_i equals to the conditional distribution of $W - X_i$ given $X_i = 1$.

Let W_i be the number of isolated complete graphs of order r in a random intersection graph $\mathbb{G}(n, m, p) - i$, $\mathbb{G}(n, m, p) - i$ obtained from $\mathbb{G}(n, m, p)$ by dropping the set $i \subseteq V$ and all the edges containing any of these vertices. Then for $w_0 \in \{0, 1, \dots, \lfloor \frac{n-r}{r} \rfloor\}$, we have

$$\begin{aligned} P(W_i = w_0) &= \binom{n-r}{w_0} \left[\sum_{s=0}^{m} \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s} \right]^{w_0} \\ &= \binom{n-r}{w_0} p^{(2r-2)w_0} [1 - p + p(1-p)^{(n-2r)}]^{mw_0} \end{aligned} \tag{7}$$

and

$$\begin{aligned}
 & P(W - X_i = w_0 \mid X_i = 1) \\
 = & \frac{P(W - X_i = w_0, X_i = 1)}{P(X_i = 1)} \\
 = & \frac{P(W = w_0 + 1, X_i = 1)}{P(X_i = 1)} \\
 = & \frac{\binom{n-r}{w_0} \left[\sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s} \right]^{w_0} \sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s}}{\sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s}} \\
 = & \binom{n-r}{w_0} \left[\sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s} \right]^{w_0} \\
 = & \binom{n-r}{w_0} p^{(2r-2)w_0} [1 - p + p(1-p)^{(n-2r)}]^{mw_0}. \tag{8}
 \end{aligned}$$

From (7) and (8), the distribution of W_i equals to the conditional distribution of $(W - X_i \mid X_i = 1)$.

For $i, j \in \Gamma_{n,r}$ such that $i \neq j$, we define the indicator random variable $X_j^{(i)}$ and E_{ij} , as follow

$$X_j^{(i)} = \begin{cases} 1 & \text{if there is an isolated complete graph in } \mathbb{G}(n, m, p) - i \text{ that} \\ & \text{spans the vertices } i = \{i_1, \dots, i_r\}, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$E_{ij} = \begin{cases} 1 & \text{if there exists adjacent between } i_r \in i \text{ and } j_l \in j, \\ 0 & \text{otherwise.} \end{cases}$$

We observe that in case $X_i = 1$, that is we have an isolated complete graph in $\mathbb{G}(n, m, p)$ that span vertices $i = \{i_1, \dots, i_r\}$. Thus the number of isolated complete graphs in a random intersection graph $\mathbb{G}(n, m, p) - i$ equals to the number of isolated complete graphs in a random intersection graph $\mathbb{G}(n, m, p)$ minus 1, that is

$$W_i = W_{n,r} - 1. \tag{9}$$

In case $X_i = 0$. For $j \in \Gamma_{n,r}$ such that $j \cap i = \emptyset$ and $j \neq i$, then

$$W_i = W_{n,r} + \sum_{j \in \Gamma_{n,r}, j \cap i = \emptyset} E_{ij} X_j^{(i)} \tag{10}$$

that is the number of isolated complete graphs in $\mathbb{G}(n, m, p) - i$ equals to the sum of the number of isolated complete graphs in $\mathbb{G}(n, m, p)$ and the number of isolated complete graphs in $\mathbb{G}(n, m, p) - i$ which are connected to i .

We know that

$$|W_{n,r} - W_i| = (W_{n,r} - W_i)^+ + (W_{n,r} - W_i)^-$$

where $(W_{n,r} - W_i)^+ = \max\{W_{n,r} - W_i, 0\}$ and $(W_{n,r} - W_i)^- = -\min\{W_{n,r} - W_i, 0\}$. Since $-\min\{W_{n,r} - W_i, 0\} = \max\{W_i - W_{n,r}, 0\} = (W_i - W_{n,r})^+$, we have

$$\mathbb{E}|W_{n,r} - W_i| = \mathbb{E}(W_{n,r} - W_i)^+ + \mathbb{E}(W_i - W_{n,r})^+.$$

Form (9) and (10), we have

$$\begin{aligned} (W_{n,r} - W_i)^+ &\leq X_i \\ (W_i - W_{n,r})^+ &\leq \sum_{j \in \Gamma_{n,r}, j \cap i = \emptyset} E_{ij} X_j^{(i)}. \end{aligned}$$

We note that,

$$\begin{aligned} \mathbb{E}(X_i) &= \frac{\mathbb{E}(W_{n,r})}{\binom{n}{k}} \\ &= \frac{\binom{n}{r} \sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-r)s}}{\binom{n}{r}} \\ &= \sum_{s=0}^m \binom{m}{s} p^{s+2r-r} (1-p)^{m-s} (1-p)^{(n-r)s} \\ &= p^{2r-2} \left[1 - p + p(1-p)^{(n-r)} \right]^m. \end{aligned} \tag{11}$$

and

$$\begin{aligned} \sum_{j \in \Gamma_{n,r}, j \cap i = \emptyset} \mathbb{E}[E_{ij} X_j^{(i)}] &= \sum_{j \in D, j \cap i = \emptyset} P(E_{ij} = 1, X_j^{(i)} = 1) \\ &= \binom{n-r}{r} (1 - q^{r^2}) \sum_{s=0}^m \binom{m}{s} p^{s+2r-2} (1-p)^{m-s} (1-p)^{(n-2r)s} \\ &\leq \frac{n^r}{r!} p^{2r-2} r^2 \left[1 - p + p(1-p)^{(n-2r)} \right]^m. \end{aligned} \tag{12}$$

From (11), (12) and use the fact that $1 - p \leq \frac{1}{e^p}$, we have

$$\begin{aligned}
 \mathbb{E}(|W - W_i|) &= \mathbb{E}(W - W_i)^+ + \mathbb{E}(W_i - W)^+ \\
 &\leq \mathbb{E}(X_i) + \sum_{j \in \Gamma_{n,r}, j \cap i = \emptyset} \mathbb{E}(E_{ij} X_j^{(i)}) \\
 &\leq p^{2r-2} \left[1 - p + p(1 - p)^{(n-r)} \right]^m + \frac{n^r}{r!} p^{2r-2} r^2 \left[1 - p + p(1 - p)^{(n-2r)} \right]^m \\
 &\leq \left[1 - p + p(1 - p)^{(n-2r)} \right]^m \left(1 + \frac{n^r}{r!} r^2 \right) p^{2r-2} \\
 &\leq \frac{\left(1 + \frac{n^r}{r!} r^2 \right) p^{2r-2}}{e^{mp(1-(1-p)^{n-2r})}} \tag{13}
 \end{aligned}$$

Hence, by (5), (6) and (13), we have

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C_{(\lambda,A)} \left(1 + \frac{n^r}{r!} r^2 \right) p^{2r-2}}{e^{mp(1-(1-p)^{n-2r})}}$$

where $C_{(\lambda,A)} = \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

4. Proof of Corollary 1.3.

From (13), we have

$$\begin{aligned}
 \mathbb{E}(|W - W_i|) &\leq \frac{\left(1 + \frac{n^r}{r!} r^2 \right) p^{2r-2}}{e^{mp(1-(1-p)^{n-2r})}} \\
 &= \frac{p^{2r-2}}{e^{mp(1-(1-p)^{n-2r})}} + \frac{rn^r p^{2r-2}}{(r-1)! e^{mp(1-(1-p)^{n-2r})}} \tag{14}
 \end{aligned}$$

Suppose that $q = 1 - p$ and $m = \lfloor n^\alpha \rfloor$ for some $\alpha > 0$ and $p = \frac{1}{n^\gamma}$ for any $\gamma \in \mathbb{R}^+ \setminus \{1\}$, then we obtain

$$\begin{aligned}
 \mathbb{E}(|W - W_i|) &\leq \frac{1}{n^{(2r-2)\gamma} e^{mp(1-(1-p)^{n-2r})}} + \frac{rn^r}{(r-1)! n^{(2r-2)\gamma} e^{mp(1-(1-p)^{n-2r})}} \\
 &\leq \frac{(1 + n^r)}{n^{(2r-2)\gamma} e^{mp(1-(1-p)^{n-2r})}}
 \end{aligned}$$

$$\begin{aligned}
 &\leq \frac{2}{n^{((2r-2)\gamma-r)} e^{mp(1-(1-p)^{n-2r})}} \\
 &\leq \frac{2}{n^{((2r-2)\gamma-r)} e^{n^{\alpha-\gamma}(1-(1-p)^{n-2r})}} \\
 &\leq \frac{2}{n^{((2r-2)\gamma-r)} n^{2(\alpha-\gamma)(1-(q)^{n-2r})}} \\
 &= \frac{2}{n^{2(\alpha-\gamma)(1-(q)^{n-2r})+(2r-2)\gamma-r}} \tag{15}
 \end{aligned}$$

By (5), (6) and (15), we have

$$|P(W \in A) - Poi_\lambda(A)| \leq \frac{C(\lambda, A)}{n^{2(\alpha-\gamma)(1-(q)^{n-2r})+(2r-2)\gamma-r}},$$

where $C(\lambda, A) = 2 \min \left\{ 1, \lambda, \frac{\Delta(\lambda)}{M_A+1} \right\}$.

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References

- [1] A.D. Barbour and Louis H.Y. Chen, An introduction to stein’s method, 2005, Singapore university Press and world scientific Publishing Co. Pte. Ltd.
- [2] M. Karonski, E.R.Scheinerman and K.B.Singer-Cohen. On random intersection graphs: the subgraph problem, *Combinatorics, Probability and Computing*, Vol. 8: 131-159 (1999).
- [3] Fill, J., Scheinerman, E. and Singer-Cohen, K. (2000): Random intersection graphs when $m = w(n)$: an equivalence theorem relating the evolution of the $G(n,m,p)$ and $G(n,p)$ models, *Random Structures Algorithms* 16, 156-176. *Lecture Note Ser.*, vol. 276, Cambridge University Press, Cambridge, 1999, pp. 239-298.
- [4] L.H.Y. Chen (1975), Poisson approximation for dependent trials, *Annals of probability*, 3:534-545.

- [5] P. Erdős and A. Rényi. On the evolution of random graphs, *Publ. Math. Inst. Hungar. Acad. Sci.* 5 (1960) 1761.
- [6] Y. Shang (2011), On the Isolated Vertices and Connectivity in Random Intersection Graphs, Hindawi Publishing Corporation *International Journal of Combinatorics*, Volume 2011, Article ID 872703.
- [7] A.D. Barbour, L. Holst and S. Janson, *Poisson approximation*. Oxford Studies in probability 2, Clarendon Press, Oxford, 1992.
- [8] T. Santiwipanont and K. Teerapabolarn, Two formulas of non-uniform bounds on Poisson approximation for dependent indicators, *Thai journal of mathematics* (2006).
- [9] M. Donganont, The bounds on Poisson Approximation of the number of isolated vertices in a random intersection graph, *International Journal of Pure and Applied Mathematics*, ISSN 1311-8080, Volume 87, No. 2 (2013).
- [10] M. Donganont, Poisson Approximation of the number of isolated trees in a random intersection graph, *International Journal of Pure and Applied Mathematics*, ISSN 1311-8080, Volume 88, No. 1 (2013), 113-123.