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A POINTWISE BINOMIAL APPROXIMATION FOR BERNOULLI RANDOM SUMMANDS

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Abstract: The aim of this paper is to give a pointwise bound for the point metric between the distribution of random sums of independent Bernoulli random variables and a binomial distribution. Two examples have been given to illustrate the result obtained.

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1. Introduction

Let $X_1, X_2, ...$ be a sequence of independent Bernoulli random variables, each with probability $p_i = P(X_i = 1) = 1 - P(X_i = 0)$, and let N be a non-negative integer-valued random variable and independent of the X_i's. Let S_N be random sums of N independent Bernoulli random variables, that is, $S_N = \sum_{i=1}^{N} X_i$. For $N = n \in \mathbb{N}$ is fixed, there have been some research related to the binomial approximation for the sum S_n, which can be found in [1], [2] and [3]. Especially, [3] gave a pointwise bound for the point metric between the distribution of S_n and an appropriate binomial distribution, which is similar to the aim of this study. Let $\widehat{n} = E(N)$ and $\widehat{p} = 1 - \widehat{q} = \frac{\lambda}{\widehat{n}}$, where $\widehat{n} \in \mathbb{N}$

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and $\lambda = E(\lambda_N) = E\left(\sum_{i=1}^N p_i\right)$. Let $\mathrm{s}_N(x)$ be the probability function of S_N and $\mathrm{B}_{\widehat{n},\widehat{p}}(x)$ the binomial probability function with parameters $\widehat{n}$ and $\widehat{p}$, where $x \in \{0, ..., \widehat{n}\}$. In this paper, we are interested to give a pointwise bound on the point metric $|\mathrm{s}_N(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)|$ for $x \in \{0, ..., \widehat{n}\}$, which is in Section 2. In Section 3, two examples have been given to illustrate the desired result. Conclusion of this study is presented in the last section.

2. Result

The following theorem presents a pointwise bound for $|\mathrm{s}_N(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)|$.

Theorem 2.1. *For $\widehat{n} \in \mathbb{N}$ and $x \in \{1, ..., \widehat{n}\}$, then*

$$|\mathrm{s}_N(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)| \leq \min\left\{\frac{1-e^{-\lambda}}{\lambda}, \frac{1}{x}\right\}\left(\frac{\lambda^2}{\widehat{n}} + E|\lambda_N - \lambda|\right) + \min\left\{E\left(\frac{1-e^{-\lambda_N}}{\lambda_N}\sum_{i=1}^N p_i^2\right), \frac{1}{x}E\left(\sum_{i=1}^N p_i^2\right)\right\}, \quad (2.1)$$

where $\mathrm{s}_N(0) = E(\prod_{i=1}^N q_i)$.

Proof. Let $\mathrm{P}_\lambda(x)$ be the Poisson probability function with mean λ. It follows the fact that

$$|\mathrm{s}_N(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)| \leq |\mathrm{s}_N(x) - \mathrm{P}_\lambda(x)| + |\mathrm{P}_\lambda(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)|. \quad (2.2)$$

Teerapabolarn [4] showed that

$$|\mathrm{s}_N(x) - \mathrm{P}_\lambda(x)| \leq \min\left\{\frac{1-e^{-\lambda}}{\lambda}, \frac{1}{x}\right\} E|\lambda_N - \lambda| + \min\left\{E\left(\frac{1-e^{-\lambda_N}}{\lambda_N}\sum_{i=1}^N p_i^2\right), \frac{1}{x}E\left(\sum_{i=1}^N p_i^2\right)\right\} \quad (2.3)$$

and Kun and Teerapabolarn [5] showed that

$$|\mathrm{P}_\lambda(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)| \leq \min\left\{\frac{1-e^{-\lambda}}{\lambda}, \frac{1}{x}\right\}\widehat{n}\widehat{p}^2 = \min\left\{\frac{1-e^{-\lambda}}{\lambda}, \frac{1}{x}\right\}\frac{\lambda^2}{\widehat{n}}. \quad (2.4)$$

Hence, the inequality (2.1) is obtained by taking the bounds in (2.3) and (2.4) into (2.2). □

If X_i's are identically distributed, then the following corollary is an immediately consequence of the Theorem 2.1

Corollary 2.1. *For $\widehat{n} \in \mathbb{N}$ and $x \in \{1, ..., \widehat{n}\}$, if $p_1 = p_2 = \cdots = p$, then we have the following:*

$$|\mathrm{S}_N(x) - \mathrm{B}_{\widehat{n},\widehat{p}}(x)| \leq \min\left\{\frac{1-e^{-\widehat{n}p}}{\widehat{n}p}, \frac{1}{x}\right\}\left(\widehat{n}p^2 + E|N-\widehat{n}|p\right) + \min\left\{E\left(1-e^{-Np}\right)p, \frac{\widehat{n}p^2}{x}\right\}, \tag{2.5}$$

where $\mathrm{S}_N(0) = E(q^N)$.

3. Examples

Two examples are given to illustrate the result in the case of X_i's are identically distributed.

Example 3.1. For n ($n \in \mathbb{N}$) is fixed, let N be a positive integer-valued random variable with probability function

$$P(N = k) = \begin{cases} \frac{1}{2} & , k = 2n, \\ \frac{1}{2} & , k = 4n, \\ 0 & , \text{otherwise}. \end{cases}$$

Therefore $\widehat{n} = 3n$, $E|N - \widehat{n}| = n$ and $E\left(1 - e^{-Np}\right) = 1 - \frac{e^{-2np}+e^{-4np}}{2}$. Let $p_1 = p_2 = \cdots = p$, then we have

$$|\mathrm{S}_N(x) - \mathrm{B}_{3n,p}(x)| \leq \min\left\{\frac{1-e^{-3np}}{3np}, \frac{1}{x}\right\}\left(3np^2 + np\right) + \min\left\{\left(1 - \frac{e^{-2np}+e^{-4np}}{2}\right)p, \frac{3np^2}{x}\right\},$$

$x \in \{1, ..., 3n\}$.

Example 3.2. Let N be a positive integer-valued random variable with probability function

$$P(N = n) = \frac{1}{20}, \ n = 1, 2, ..., 21,$$

then we have $\widehat{n} = 11$ and $E|N - \widehat{n}| \le \sqrt{Var(N)} = \sqrt{\frac{891}{20}} = 6.67$. Therefore, if $p_1 = p_2 = \cdots = p$, then we obtain

$$|\mathrm{S}_N(x) - \mathrm{B}_{11,p}(x)| \le \min\left\{\frac{1 - e^{-11p}}{11p}, \frac{1}{x}\right\}\left(11p^2 + 6.67p\right) + \min\left\{p, \frac{11p^2}{x}\right\},$$

$x \in \{1, ..., 11\}$.

4. Conclusion

In this study, a pointwise bound for the point metric between the distribution of random sums of independent Bernoulli random variables and an appropriate binomial distribution was obtained. It is pointed out that the binomial probability function with parameters $\widehat{n}$ and $\widehat{p}$ can be used as an estimate of the probability function of random sums of independent Bernoulli random variables when $\widehat{p}$ is small.

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