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A STUDY ON (λ, μ) ANTI-FUZZY NORMAL SUBRING OF A RING

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Abstract: In this paper, we made an attempt to study the algebraic nature of (λ, μ)-anti-fuzzy normal subring of a ring.

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Key Words: fuzzy set, (λ, μ) fuzzy subring, (λ, μ)-anti-fuzzy subring, (λ, μ)-anti-fuzzy normal subring, pseudo (λ, μ)-anti-fuzzy

1. Introduction

After the introduction of fuzzy sets by L.A. Zadeh [20], several researchers explored on the generalization of the notion of fuzzy set. Azriel Rosenfeld [5] defined a fuzzy group. Asok Kumer Ray [4] defined a product of fuzzy subgroups and Fuzzy subgroups and Anti-fuzzy subgroups have introduced R. Biswas [6]. The concepts of (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals was introduced by Bingxue Yao [18]. In this paper, we introduce the some theorems in (λ, μ)-anti-fuzzy normal subring of a ring. Throughout this article, we will always assume that $0 \leq \lambda < \mu \leq 1$.

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2. Preliminaries

Definition 1. Let X be a non-empty set.A fuzzy subset A of X is a function $A : X \to [0,1]$.

Definition 2. Let R be a subring.A fuzzy subset A of R is said to be a (λ,μ)-fuzzy subring of R if it satisfies the following conditions:
(i) $A(x+y) \vee \lambda \geq \min\{A(x), A(y)\} \wedge \mu$, (ii)$A(-x) \vee \lambda \geq A(x) \wedge \mu$,
(iii)$A(xy) \vee \lambda \geq \min\{A(x), A(y)\} \wedge \mu$,for all x and y in R.

Definition 3. Let R be a subring. A fuzzy subset A of R is said to be an (λ,μ)-anti-fuzzy subring of R if it satisfies the following conditions:
(i) $A(x+y) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$,
(ii) $A(-x) \wedge \mu \leq A(x) \vee \lambda$,
(iii) $A(xy) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$, for all x and y in R.

Definition 4. Let R be a subring. An anti-fuzzy subring A of R is said to be an (λ,μ)anti-fuzzy normal subring of R if $A(xy) = A(yx) \vee \lambda \wedge \mu$,for all x and y in R.

Definition 5. Let A and B be fuzzy subsets of sets G and H, respectively.The (λ,μ)anti-product of A and B,denoted by $A\times B$,is defined as $A\times B = \{\langle (x,y), A\times B(x,y) \wedge \mu\rangle \,/$ for all x in G and y in $H\}$,where $A\times B(x,y)\wedge\mu = \max\{A(x), B(y)\} \vee \lambda$.

Definition 6. Let $(R,+,.)$ and $(R^{'},+,.)$ be any two subrings. Let $f : R \to R^{'}$ be any function and A be an anti-fuzzy subring in R, V be an anti-fuzzy subring in $f(R) = R^{'}$, defined by $V(y) = \inf\limits_{x\in f^{-1}(y)} A(x)$, for all x in R and y in $R^{'}$. Then A is called a preimage of V under f and is denoted by $f^{-1}(V)$.

Definition 7. A (λ,μ)anti-fuzzy subset A of a set X is said to be normalized if there exists an element x in X such that $A(x) \vee \lambda \wedge \mu = 1$.

Definition 8. Let A be an anti-fuzzy subring of a subring $(R,+,.)$ and a in R. Then the pseudo (λ,μ) anti-fuzzy coset $(aA)^p$ is defined by $((aA)^p)(x)\wedge\mu = p(a)A(x) \vee \lambda$,for every x in R and for some p in P.

3. Properties of (λ,μ) Anti-fuzzy Normal Subring of a Ring

Theorem 9. *Let $(R,+,.)$ be a subring. If A and B are two (λ,μ)-anti -fuzzy normal subrings of R,then their union $A\cup B$ is an anti-fuzzy normal subring of R.*

Proof. Let x and $y \in R$. Let $A = \{\langle x, A(x)\rangle / x \in R\}$ and $B = \{\langle x, B(x)\rangle / x \in R\}$ be (λ, μ)-anti-fuzzy normal subrings of a ring R. Let $C = A \cup B$ and $C = \{\langle x, C(x)\rangle / x \in R\}$. Then, Clearly C is an (λ, μ)-anti-fuzzy subring of a ring R, since A and B are two (λ, μ)-anti-fuzzy subrings of a ring R. And, $C(xy) = \max\{A(xy), B(xy)\} = \max\{A(yx) \vee \lambda \wedge \mu, B(yx) \vee \lambda \wedge \mu\}$ $= \max\{A(yx), B(yx)\} \vee \lambda \wedge \mu = C(yx) \vee \lambda \wedge \mu$, for all x and y in R. Therefore, $C(xy) = C(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence $A \cup B$ is an (λ, μ)-anti-fuzzy normal subring of a ring R. □

Theorem 10. *Let R be a subring. The union of a family of (λ, μ)anti−fuzzy normal subrings of R is an (λ, μ)anti−fuzzy normal subring of R.*

Proof. Let $\{A_i\}_{i \in I}$ be a family of (λ, μ)-anti-fuzzy normal subrings of a ring R and let $A = \bigcup_{i \in I} A_i$. Then for x and y in R. Clearly the union of a family of (λ, μ)-anti -fuzzy subrings of a ring R is an (λ, μ)-anti-fuzzy subring of a ring R. $A(xy) = \sup_{i \in I} A_i(xy) = \sup_{i \in I} A_i(yx) \vee \lambda \wedge \mu = A(yx) \vee \lambda \wedge \mu$, for all x and y in R. Therefore, $A(xy) = A(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence the union of a family of (λ, μ)-anti-fuzzy normal subrings of a ring R is an (λ, μ)-anti-fuzzy normal subring of a ring R. □

Theorem 11. *Let A and B be (λ, μ)-anti-fuzzy subring of the rings G and H, respectively. If A and B are (λ, μ)-anti-fuzzy normal subrings, then $A \times B$ is an (λ, μ)-anti-fuzzy normal subring of $G \times H$.*

Proof. Let A and B be (λ, μ)-anti-fuzzy normal subrings of the rings G and H respectively. Clearly $A \times B$ is an (λ, μ)-anti-fuzzy subring of $G \times H$. Let x_1 and x_2 be in G, y_1 and y_2 be in H. Then (x_1, y_1) and (x_2, y_2) are in $G \times H$.Now,$A \times B[(x_1, y_1)(x_2, y_2)] = A \times B(x_1x_2, y_1y_2) = \max\{A(x_1x_2), B(y_1y_2)\} = \max\{A(x_2x_1) \vee \lambda \wedge \mu, B(y_2y_1) \vee \lambda \wedge \mu\} = \max\{A(x_2x_1), B(y_2y_1)\} \vee \lambda \wedge \mu = A \times B(x_2x_1, y_2y_1) \vee \lambda \wedge \mu = A \times B[(x_2, y_2)(x_1, y_1)] \vee \lambda \wedge \mu$. Therefore, $A \times B[(x_1, y_1)(x_2, y_2)] = A \times B[(x_2, y_2)(x_1, y_1)] \vee \lambda \wedge \mu$. Hence $A \times B$ is an (λ, μ)-anti-fuzzy normal subring of $G \times H$. □

Theorem 12. *Let A be a fuzzy subset in a subring R and V be the strongest (λ, μ)-anti-fuzzy relation on R. Then A is an (λ, μ)- anti-fuzzy normal subring of R if and only if V is an (λ, μ)- anti-fuzzy normal subring of $R \times R$.*

Proof. Suppose that A is a (λ, μ)-anti-fuzzy normal subring of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$. Clearly V is a (λ, μ)-anti-fuzzy subring of $R \times R$. We have, $V(xy) = V[(x_1, x_2)(y_1, y_2)] = V((x_1y_1, x_2y_2)) =$

$A((x_1y_1)) \wedge A((x_2y_2)) = \{A((y_1x_1)) \vee \lambda \wedge \mu\} \wedge \{A((y_2x_2)) \vee \lambda \wedge \mu\} = \{A((y_1x_1)) \wedge A((y_2x_2))\} \vee \lambda \wedge \mu = V((y_1x_1, y_2x_2)) \vee \lambda \wedge \mu = V[(y_1, y_2)(x_1, x_2)] \vee \lambda \wedge \mu = V(yx) \vee \lambda \wedge \mu$. Therefore, $V(xy) = V(yx) \vee \lambda \wedge \mu$, for all x and y in $R \times R$.This proves that V is a (λ, μ)-anti-fuzzy normal subring of $R \times R$. Conversely, assume that V is a (λ, μ)-anti-fuzzy normal subring of $R \times R$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R \times R$, we have $A(x_1y_1) \wedge A(x_2y_2) = V((x_1y_1, x_2y_2)) = V[(x_1, x_2)(y_1, y_2)] = V(xy) = V(yx) \vee \lambda \wedge \mu = V[(y_1, y_2)(x_1, x_2)] \vee \lambda \wedge \mu = V((y_1x_1, y_2x_2)) \vee \lambda \wedge \mu = \{A(y_1x_1) \vee \lambda \wedge \mu\} \wedge \{A(y_2x_2) \vee \lambda \wedge \mu\} = \{A(y_1x_1) \wedge A(y_2x_2)\} \vee \lambda \wedge \mu$. We get, $A((x_1y_1)) = A(y_1x_1) \vee \lambda \wedge \mu$, for all x_1 and y_1 in R. Hence A is a (λ, μ)-anti-fuzzy normal subring of R. □

Theorem 13. *A is a (λ, μ) anti-fuzzy subring of a ring $(R, +, .)$ if and only if $A(x - y) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$ and $A(xy) \wedge \mu \leq \max\{A(x), A(y)\} \vee \lambda$, for all x and y in R.*

Proof. Let $(R, +, .)$ and $(R^|, +, .)$ be any two subrings and $f : R \to R^|$ be a homomorphism. Then, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where A is an (λ, μ)-anti-fuzzy normal subring of a ring R. We have to prove that V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^|$. Now, for $f(x), f(y) in R^|$, clearly V is an (λ, μ)-anti-fuzzy subring of a ring $R^|$, since A is an (λ, μ)-anti-fuzzy subring of a ring R. Now, $V(f(x)f(y)) = V(f(xy)) \leq A(xy) = A(yx) \vee \lambda \wedge \mu \geq V(f(yx)) \vee \lambda \wedge \mu = V(f(y)f(x)) \vee \lambda \wedge \mu$, which implies that $V(f(x)f(y)) = V(f(y)f(x)) \vee \lambda \wedge \mu$, for all $f(x)$ and $f(y)$ in $R^|$. Hence V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^|$. □

Theorem 14. *Let $(R, +, .)$ and $(R^|, +, .)$ be any two subrings. The homomorphic preimage of an (λ, μ)-anti-fuzzy normal subring of $R^|$ is an (λ, μ)-anti-fuzzy normal subring of R.*

Proof. Let $(R, +, .)$ and $(R^|, +, .)$ be any two subrings and $f : R \to R^|$ be a homomorphism. Then, $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$, for all x and y in R. Let $V = f(A)$, where V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^|$. We have to prove that A is an (λ, μ)-anti-fuzzy normal subring of a ring R. Let x and y in R. Then, clearly A is an (λ, μ)-anti-fuzzy subring of a ring R, since V is an (λ, μ)-anti-fuzzy subring of a ring $R^|$. Now, $A(xy) = V(f(xy)) = V(f(x)f(y)) = V(f(y)f(x)) \vee \lambda \wedge \mu = V(f(yx)) \vee \lambda \wedge \mu = A(yx) \vee \lambda \wedge \mu$,which implies that $A(xy) = A(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence A is an (λ, μ)-anti-fuzzy normal subring of a ring R. □

Theorem 15. *Let $(R, +, .)$ and $(R^{|}, +, .)$ be any two subrings. The anti-homomorphic image of an (λ, μ)-anti-fuzzy normal subring of R is an (λ, μ)-anti-fuzzy normal subring of $R^{|}$.*

Proof. Let $(R, +, .)$ and $(R^{|}, +, .)$ be any two subrings and $f : R \to R^{|}$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all x and y in R. Let $V = f(A)$, where A is an (λ, μ)- anti-fuzzy normal subring of a ring R. We have to prove that V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^{|}$. Now, for $f(x)$ and $f(y)$ in $R^{|}$, clearly V is an (λ, μ)-anti-fuzzy subring of a ring $R^{|}$, since A is an (λ, μ)-anti-fuzzy subring of a ring R. Now, $V(f(x)f(y)) = V(f(yx)) \vee \lambda \wedge \mu \leq A(yx) \vee \lambda \wedge \mu = A(xy) \vee \lambda \wedge \mu \geq V(f(xy)) \vee \lambda \wedge \mu = V(f(y)f(x)) \vee \lambda \wedge \mu$, which implies that $V(f(x)f(y)) = V(f(y)f(x)) \vee \lambda \wedge \mu$, for all $f(x)$ and $f(y)$ in $R^{|}$. Hence V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^{|}$. □

Theorem 16. *Let $(R, +, .)$ and $(R^{|}, +, .)$ be any two subrings. The anti-homomorphic preimage of an (λ, μ)-anti-fuzzy normal subring of $R^{|}$ is an (λ, μ)-anti-fuzzy normal subring of R.*

Proof. Let $(R, +, .)$ and $(R^{|}, +, .)$ be any two subrings and $f : R \to R^{|}$ be an anti-homomorphism. Then, $f(x + y) = f(y) + f(x)$ and $f(xy) = f(y)f(x)$, for all$x$ and y in R. Let $V = f(A)$, where V is an (λ, μ)-anti-fuzzy normal subring of a ring $R^{|}$. We have to prove that A is an (λ, μ)-anti-fuzzy normal subring of a ring R. Let x and y in R, then clearly A is an (λ, μ)-anti-fuzzy subring of a ring R, since V is an (λ, μ)-anti-fuzzy subring of a ring $R^{|}$. Now, $A(xy) = V(f(xy)) = V(f(y)f(x)) \vee \lambda \wedge \mu = V(f(x)f(y)) \vee \lambda \wedge \mu = V(f(yx)) \vee \lambda \wedge \mu = A(yx) \vee \lambda \wedge \mu$, which implies that $A(xy) = A(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence A is an (λ, μ)-anti-fuzzy normal subring of a ring R. □

Theorem 17. *A (λ, μ)-anti-fuzzy normal subring A of a ring R is normalized if and only if $A(e) \vee \lambda \wedge \mu = 1$, where e is the identity element of R.*

Proof. If A is normalized, then there exists x in R such that $A(x) = 1$, but by properties of a (λ, μ)-anti-fuzzy normal subring A of R,$A(x) \vee \lambda \wedge \mu \leq A(e) \vee \lambda \wedge \mu$, for every x in R. Since $A(x) \vee \lambda \wedge \mu = 1$ and $A(x) \vee \lambda \wedge \mu \leq A(e) \vee \lambda \wedge \mu, 1 \leq A(e) \vee \lambda \wedge \mu$. But $1 \geq A(e) \vee \lambda \wedge \mu$.Hence $A(e) \vee \lambda \wedge \mu = 1$.Conversely,if $A(e) \vee \lambda \wedge \mu = 1$,then by the definition of normalized (λ, μ)-anti-fuzzy subset,A is normalized. □

Theorem 18. *Let A and B be (λ,μ)-anti-fuzzy subrings of the rings R and H, respectively. Suppose that e and $e^{|}$ are the identity element of R and H, respectively. If $A \times B$ is a (λ,μ)-anti -fuzzy normal subrings of $R \times H$, then at least one of the following two statements must hold.*
(i) $B(e^{|}) \geq A(x) \vee \lambda \wedge \mu$, for all x in R,
(ii) $A(e) \geq B(y) \vee \lambda \wedge \mu$, for all y in H.

Proof. It is trivial. □

Theorem 19. *Let A be a (λ,μ)-anti-fuzzy normal subring of a ring R, then the pseudo (λ,μ)-anti-fuzzy coset $(aA)^p$ is a (λ,μ)-anti-fuzzy normal subring of the ring R,for a in R.*

Proof. Let A be a (λ,μ)-anti -fuzzy normal subring of a ring R. For every x and y in R, we have, clearly $(aA)^p$ is a (λ,μ)-anti-fuzzy subring of the ring R and $((aA)^p)(xy) = p(a)A(xy) = p(a)A(yx) \vee \lambda \wedge \mu = ((aA)^p)(yx) \vee \lambda \wedge \mu$. Therefore, $((aA)^p)(xy) = ((aA)^p)(yx) \vee \lambda \wedge \mu$,for x and y in R. Hence $(aA)^p$ is a (λ,μ)-anti-fuzzy normal subring of the ring R. □

Theorem 20. *Let A and B be (λ,μ)-anti-fuzzy subsets of the rings R and H, respectively and $A \times B$ is a (λ,μ)-anti -fuzzy normal subring of $R \times H$. Then the following are true:*

1. *if $A(x) \vee \lambda \wedge \mu \leq B(e^{|})$, then A is a (λ,μ)-anti -fuzzy normal subring of R.*
2. *if $B(x) \vee \lambda \wedge \mu \leq A(e)$, then B is a (λ,μ)-anti -fuzzy normal subring of H.*
3. *either A is a (λ,μ)-anti -fuzzy normal subring of R or B is a (λ,μ)-anti-fuzzy normal subring of H.*

Proof. Let $A \times B$ be a (λ,μ)-anti -fuzzy normal subring of $R \times H$ and x, y in R and $e^{|}$ in H. Then $(x, e^{|})$ and $(y, e^{|})$ are in $R \times H$. Clearly $A \times B$ is a (λ,μ)-anti -fuzzy normal subring of $R \times H$. Now, using the property that $A(x) \vee \lambda \wedge \mu \leq B(e^{|})$, for all x in R, clearly A is a (λ,μ)- anti-fuzzy normal subring of R. Now, $A(xy) = A(xy) \wedge B(e^{|}e^{|}) = A \times B(((xy),(e^{|}e^{|}))) = A \times B\left[(x,e^{|})(y,e^{|})\right] = A \times B\left[(y,e^{|})(x,e^{|})\right] \vee \lambda \wedge \mu = A \times B\left[((yx),(e^{|}e^{|}))\right] \vee \lambda \wedge \mu = A(yx) \wedge B(e^{|}e^{|}) \vee \lambda \wedge \mu = A(yx) \vee \lambda \wedge \mu$. Therefore, $A(xy) = A(yx) \vee \lambda \wedge \mu$,for all x and y in R. Hence A is a (λ,μ)-anti -fuzzy normal subring of R. Thus (i) is proved. Now, using the property that $B(x) \vee \lambda \wedge \mu \leq A(e)$, for all x in H, let x and y in H and e in R. Then (e, x) and (e, y) are in $R \times H$. Clearly B is a

(λ,μ)-anti-fuzzy normal subring of H. Now, $B(xy) = B(xy) \wedge A(ee) = A(ee) \wedge B(xy) = A \times B(((ee),(xy))) = A \times B\,[(e,x)(e,y)] = A \times B\,[(e,y)(e,x)] \vee \lambda \wedge \mu = A \times B\,[((ee),(yx))] \vee \lambda \wedge \mu = A(ee) \wedge B(yx) \vee \lambda \wedge \mu = B(yx) \vee \lambda \wedge \mu$. Therefore, $B(xy) = B(yx) \vee \lambda \wedge \mu$, for all x and y in H. Hence B is a (λ,μ)-anti-fuzzy normal subring of H. Thus (ii) is proved. (iii) is clear. □

In the following Theorem $\circ$ is the composition operation of functions.

Theorem 21. *Let A be an (λ,μ)-anti-fuzzy subring of a ring H and f is an isomorphism from a subring R onto H. If A is an (λ,μ)-anti -fuzzy normal subring of the ring H, then $A \circ f$ is an (λ,μ)- anti-fuzzy normal subring of the ring R.*

Proof. Let x and y in R and A be an (λ,μ)-anti-fuzzy normal subring of a ring H. Then we have, clearly $A \circ f$ is an (λ,μ) -anti-fuzzy subring of a ring R. Now, $(A \circ f)(xy) = A(f(xy)) = A(f(x)f(y)) = A(f(y)f(x)) \vee \lambda \wedge \mu = A(f(yx)) \vee \lambda \wedge \mu = (A \circ f)(yx) \vee \lambda \wedge \mu$, which implies that $(A \circ f)(xy) = (A \circ f)(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence $A \circ f$ is an (λ,μ)-anti-fuzzy normal subring of a ring R. □

Theorem 22. *Let A be an (λ,μ)-anti-fuzzy subring of a ring H and f is an anti-isomorphism from a subring R onto H. If A is an (λ,μ)- anti-fuzzy normal subring of the ring H, then $A \circ f$ is an (λ,μ)-anti-fuzzy normal subring of the ring R.*

Proof. Let x and y in R and A be an (λ,μ)-anti-fuzzy normal subring of a ring H. Then we have, clearly $A \circ f$ is an (λ,μ)-anti- fuzzy subring of a ring R. Now, $(A \circ f)(xy) = A(f(xy)) = A(f(y)f(x)) \vee \lambda \wedge \mu = A(f(y) \vee \lambda \wedge \mu f(x) \vee \lambda \wedge \mu) = A(f(x)f(y)) \vee \lambda \wedge \mu = A(f(yx)) \vee \lambda \wedge \mu = (A \circ f)(yx) \vee \lambda \wedge \mu$, which implies that $(A \circ f)(xy) = (A \circ f)(yx) \vee \lambda \wedge \mu$, for all x and y in R. Hence $A \circ f$ is an (λ,μ)-anti-fuzzy normal subring of a ring R. □

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