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ON HYPERCLOSED SETS

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Abstract: In this paper we introduce hyperclosed sets and study properties of hyper-closure and hyper-interior of a set. We also obtain characterizations of some of the well known separation axioms in topology like regular, Urysohn, T_1 etc. in terms of these sets.

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1. Introduction

In [8], Veličko introduced the concept of θ-closed sets and θ-closure for studying the important class of H-closed spaces in terms of filter bases. Jankovič [4] utilized these sets to obtain a characterization of Hausdorff spaces showing that a space is Hausdorff if and only if every compact set is θ-closed. Also various characterizations of other separation axioms R_0 and R_1 were also obtained in terms of θ-closure operator by Jankovič in [3] and [4]. In [6], necessary and sufficient conditions for a QHC set to be θ-closed in $S_{2\frac{1}{2}}$ and normal spaces have been obtained. In [2], it was proved that for an almost regular space the θ-closure of every set is θ-closed.

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In this paper a stronger form of closed sets called hyperclosed sets is introduced in terms of hyper-closure operator and it is shown that it is even stronger than the concept of θ-closed sets of Veličko. Some of the basic properties of hyper-closure and hyper-interior operators have been given and these operators have been utilized to obtain various characterizations of S_i separation axioms of Csàszàr [1]. It is proved in Section 3 below that in almost regular spaces, the concepts of hyper-closure and θ-closure coincide. It is also shown that the closure, θ-closure and hyper-closure operators coincide for regular spaces. Also analogous to the above characterization of Hausdorff spaces it is obtained that a space is Urysohn if and only if every compact set is hyperclosed.

Throughout, by a space X we shall mean a topological space and $\mathfrak{T}$ will denote the topology associated with it. Also and $cl(A)$, $int(A)$ and $X\backslash A$ will denote the closure, interior the complement of A in X respectively.

Definition 1.1. If X is a space, $A \subset X$, and $x \in X$, then

1. $ker(A) = \cap\{U : U \in \mathfrak{T} \text{ and } A \subset U\} = \{x \in X : cl\{x\} \cap A \neq \phi\}$.
2. $< x >= cl\{x\} \cap ker\{x\}$.
3. $Z(A) = \cup\{< x >: x \in A\} = \{x \in X :< x > \cap A \neq \phi\}$.
4. $x \in int_\theta(A)$, if there exists a open set U containing x such that $cl(U) \subset A$ and set A is θ-open if $A = int_\theta(A)$.
5. $x \in cl_\theta(A)$, if $cl(U) \cap A \neq \phi$ for any open set U containing x and set A is θ-closed if $A = cl_\theta(A)$.
6. R will denote the equivalence relation, xRy if and only if $y \in< x >$.
7. $G(R) = \{(x, y) \in X \times X : xRy\}$, is the graph of the equivalence relation.
8. $\Delta = \{(x, x) \in X \times X : x \in X\}$, the diagonal of $X \times X$.
9. $p_2(x, y) = y$, is the projection from $X \times X$ onto X.
10. A set A is regular closed if $A = cl(int(A))$ or equivalently if it is closure of some open set.
11. A set which can be expressed as the closure of a singleton is said to be a point closure set. The complement of a point closure set is said to be a co-point closure set.

Definition 1.2. A space X is said to be,

1. Urysohn ($T_{2\frac{1}{2}}$) space [1] if for every pair of points x and y in X there exist neighborhoods U of x and V of y such that $cl(U) \cap cl(V) = \phi$.

2. Regular space [1] if for any point x and a closed set F not containing x there exist disjoint neighborhoods containing them.

3. $S_{2\frac{1}{2}}$ space [6] if for every pair of points x and y in X, whenever $cl\{x\} \neq cl\{y\}$ then there exist neighborhoods U of x and V of y such that $cl(U) \cap cl(V) = \phi$. Every $T_{2\frac{1}{2}}$ space or regular space is $S_{2\frac{1}{2}}$.

4. S_2 space [1] (R_1 in sense of [4] [5]) if for every pair of points x and y in X, whenever $cl\{x\} \neq cl\{y\}$ then there exist disjoint neighborhoods containing them. Every $S_{2\frac{1}{2}}$ space is S_2.

5. S_1 space [1] (R_0 in sense of [4] [5]) if for every pair of points x and y, whenever x has a neighborhood not containing y, then y has a neighborhood not containing x. Every S_2 space is S_1.

6. Almost regular [5] if for any point x and a regular closed set F not containing x there exist disjoint neighborhoods containing them. Every regular space is almost regular.

The following results will be used in the next sections.

Lemma 1.1. *For a space X, $A \subset X$, and $x \in X$, then*

(a) $y \in\ < x >$ *if and only if* $< x > \ = \ < y >$*.[4]*

(b) For each $x, y \in X$*, either* $< x > \ = \ < y >$ *or* $< x > \cap < y > = \phi$*.[4]*

(c) For each $x, y \in X$, $< x > \ = \ < y >$ *if and only if* $cl\{x\} = cl\{y\}$ *if and only if* $ker\{x\} = ker\{y\}$*. [4]*

(d) For each open set $U \subset X$*, if* $x \in U$ *then* $< x > \subset U$*. [4]*

(e) For each closed set $F \subset X$*, if* $x \in F$ *then* $< x > \subset F$*. [4]*

(f) For each open set $U \subset X$, $cl(U) = cl_\theta(U)$*.[4]*

(g) $< x > = p_2((\{x\} \times X) \cap G(R))$*. [3]*

(h) For each open set $U \subset X$, $Z(U) = U$*. [3]*

(i) For each closed set $F \subset X$, $Z(F) = F$*. [3]*

Lemma 1.2. *(a) X is S_1 if and only if $G(R) = ker(\Delta)$.[3]*

(b) X is S_2 if and only if $G(R) = cl(\Delta)$.[3]

(c) X is regular if and only if for every subset $A \subset X$, $cl(A) = cl_\theta(A)$.[4]

(d) X is T_0 if and only if for each $x \in X$, $cl\{x\}\backslash\{x\}$ is union of closed sets. [4]

(e) X is almost regular if and only if for any point x and a regular closed set F not containing x there exist neighborhoods U of x and V of F such that $cl(U) \cap cl(V) = \phi$. [5]

(f) X is S_1 if and only if every open set in X contains the closure of each of its points.[1]

2. Hyperclosed Sets

Firstly we give some basic properties of hyper-closure (Theorem 2.1 below), hyper-interior (Theorem 2.2 below) and hyperclosed sets (Theorem 2.3 below). The proofs are consequences of definitions and are omitted.

Definition 2.1. [4] For any subset $A \subset X$, $K(A) = \cap\{cl_\theta(cl_\theta(U)) : U \in T \text{ and } A \subset U\}$. We will call this set hyper-closure of the set A and will denote this by $cl_H(A)$ from now on.

Definition 2.2. A subset $A \subset X$ is said to be hyperclosed if $A = cl_H(A)$ and the complement of a hyperclosed set is said to be hyperopen.

Theorem 2.1. *For subsets A, B of a space X, the following statements hold:*

(a) If $A \subset B$ then $cl_H(A) \subset cl_H(B)$.

(b) $cl_H(A \cup B) = cl_H(A) \cup cl_H(B)$.

(c) $cl_H(A \cap B) \subset cl_H(A) \cap cl_H(B)$.

(d) $cl_H(A)$ is not a hyperclosed set.

(e) $cl_H(A) = \{x \in X$: For each open set U containing x and each open set V containing A, $cl(U) \cap cl(V) \neq \phi\}$.

(f) For each open set $U \subset X$, $cl_H(U) = cl_\theta(cl(U))$.

Definition 2.3. A point $x \in X$ is said to be hyper-interior point of A if there exists an open set U containing x such that $cl(U) \subset int(F)$ for some closed set F contained in A. The set of all hyper-interior points of A is denoted by $int_H(A)$.

Theorem 2.2. *For subsets A, B of a space X, the following statements hold:*

(a) *If $A \subset B$ then $int_H(A) \subset int_H(B)$.*

(b) $int_H(A) \cup int_H(B) \subset int_H(A \cup B)$.

(c) $int_H(A \cap B) = int_H(A) \cap int_H(B)$.

(d) $X \backslash int_H(A) = cl_H(X \backslash A)$.

(e) $X \backslash cl_H(A) = int_H(X \backslash A)$.

(f) $int_H(A) = \cup\{U \subset X : U \in \mathfrak{T}, cl(U) \subset int(F), F$ *closed and* $F \subset A\}$ *i.e.* $int_H(A)$ *is union of all those open subsets X whose closures are contained in the interior of some closed set contained in A.*

(g) $int_H(A) = \cup\{int_\theta(int_\theta(F)) : F$ *closed and* $F \subset A\}$.

(h) *A is hyperopen if and only if $A = int_H(A)$.*

(i) *$int_H(A)$ is not a hyperopen set.*

(j) *$int_H(A) \subset int_\theta(A) \subset int(A)$, so every hyperopen set is θ-open and hence open.*

Theorem 2.3. *For any space X,*

(a) *ϕ and X are hyperclosed.*

(b) *Arbitrary intersection and finite unions of hyperclosed sets are hyperclosed.*

(c) *For each subset A, $cl_H(A)$ is closed set.*

(d) *For each subset A, $cl(A) \subset cl_\theta(A) \subset cl_\theta(cl_\theta(A)) \subset cl_H(A)$, so every hyperclosed set is θ-closed and hence closed.*

Example 1.1 below shows that reverse inclusion of Theorem 2.1(c) and Theorem 2.3(d) does not hold. It also validates Theorem 2.1(d) and shows that Theorem 2.1(f) does not hold for the sets which are not open.

Example 2.1. [7, this is an improvisation of Example 81] If S is the set of points in the interior of the unit square, we define, $X = S \cup \{(0,0), (1/2, 0), (1, 0)\}$. Points in S will be given the Euclidean local basis neighborhoods, while

$$U_p(0,0) = \{(0,0)\} \cup \{(x,y) \mid 0 < x < \frac{1}{4}, 0 < y < \frac{1}{p}\},$$

$$U_q(\frac{1}{2},0) = \{(\frac{1}{2},0)\} \cup \{(x,y) \mid \frac{1}{4} < x < \frac{3}{4}, 0 < y < \frac{1}{q}\},$$

$$U_r(1,0) = \{(1,0)\} \cup \{(x,y) \mid \frac{1}{2} < x < 1, 0 < y < \frac{1}{r}\},$$

where p, q, r are integers, are the local bases for $(0,0)$, $(\frac{1}{2},0)$ and $(1,0)$ respectively.

Let, $A = \{(0,0)\}$ be a singleton set. So we have the following:

1. As, $cl_\theta(cl(A)) = cl_\theta(A) = A$ for the non-open set A. But, $(\frac{1}{2},0) \in cl_H(A)$ and so we have that, $cl_\theta(cl(A)) \neq cl_H(A)$.
2. Now, $(1,0) \in cl_\theta(cl_H(A)) \subset cl_H(cl_H(A))$ and $(1,0) \notin cl_H(A)$. Therefore, $cl_H(A)$ is not a θ-closed even if $cl_\theta(A)$ is θ-closed set . Also $cl_H(A)$ need not be a hyperclosed set.

Again if we let $A = \{(0,0)\}$ and $B = \{(1,0)\}$ we have that $(\frac{1}{2},0) \in cl_H(A) \cap cl_H(B)$ but $(\frac{1}{2},0) \notin cl_H(A \cap B)$ and so $cl_H(A \cap B) \neq cl_H(A) \cap cl_H(B)$.

We shall also need the following.

Theorem 2.4. *For a space X and $x, y \in X$, the following statements hold:*

(a) $x \in cl_H\{y\}$ *if and only if* $y \in cl_H\{x\}$.

(b) $cl_H\{x\} = p_2((\{x\} \times X) \cap cl_\theta(\Delta))$.

(c) $cl_\theta(\Delta) = \cup_{x \in X}(\{x\} \times cl_H\{x\})$.

(d) $cl_H\{x\} = cl_H(< x >)$.

Proof. (a) is obvious and (b) has been proved in [4, Lemma 4.4]

(c) Let $P = \cup_{x \in X}(\{x\} \times cl_H\{x\})$ and let $(x,y) \in P \backslash cl_\theta(\Delta)$. This implies there exists neighborhoods U containing x and V containing y such that $cl(U \times V) \cap \Delta = \phi$. Therefore, $(cl(U) \times cl(V)) \cap \Delta = \phi$ and so, $cl(U) \cap cl(V) = \phi$. Thus $y \notin cl_H\{x\}$ and so $(x,y) \notin P$ and the result follows. The reverse inclusion is obtained by reversing the above steps.

(d) $cl_H\{x\} \subset cl_H(< x >)$ is obvious. Let $y \in cl_H(< x >) \backslash cl_H\{x\}$. This implies there exists neighborhoods U containing x and V containing y such that, $cl(U) \cap cl(V) = \phi$. But since U is an open set containing x, by Lemma 1.1(d) we have that $< x > \subset U$. Thus U is a neighborhood of $< x >$ which implies $y \notin cl_H(< x >)$ and the result follows.

□

3. Applications of Hyperclosed Sets

Theorem 3.1. *A space X is almost regular if and only if every regular closed set is hyperclosed.*

Proof. The proof follows from Lemma 1.2(e). □

Theorem 3.2. *In an almost regular space X, for each set $A \subset X$, $cl_\theta(A) = cl_H(A)$.*

Proof. As, $cl_H(A) = \cap\{cl_\theta(cl_\theta(U)) : U \in \mathfrak{T}$ and $A \subset U\}$. Since closure of an open set is regular closed, by Theorem 3.1 and Theorem 2.3(d) we have that, $cl_H(A) = \cap\{cl(U) : U \in T$ and $A \subset U\} = \{x \in X :$ For each open set U containing x and each open set V containing A, $U \cap V \neq \phi\}$. Let $x \in cl_H(A) \backslash cl_\theta(A)$ which implies for any $y \in A$, there exist neighborhoods U_y of y and V_y of x such that $U_y \cap V_y = \phi$. Since $A \subset \cup_{y \in A} U_y$. Let $P = \cup_{y \in A} U_y$. Then P is a neighborhood of A such that $P \cap V_y = \phi$ which implies $x \notin cl_H(A)$ and the result follows. □

Remark 3.1. As a consequence of Theorem 3.2 we have that for almost regular spaces, the following hold:

(a) every θ-closed set is hyperclosed.

(b) for every subset A, $cl_H(A)$ is hyperclosed set.

We now see that hyper-closures serve to characterize regularity.

Theorem 3.3. *A space X is regular if and only if for every $A \subset X$, $cl(A) = cl_H(A)$.*

Proof. Necessity follows from Lemma 1.2(c) and Theorem 3.2 and sufficiency follows from Lemma 1.2(c) and Theorem 2.3(d). □

We now introduce hyper-kernel analogous to kernel in [4] and use it in Theorem 3.4 and Theorem 3.6 below to characterize $S_{2\frac{1}{2}}$ spaces.

Definition 3.1. For subset $A \subset X$ hyper-kernel is defined as $ker_H(A) = \{x \in X : cl_H\{x\} \cap A \neq \phi\}$.

Remark 3.2. For a space X some of the properties of hyper-kernel are as follows:

(a) For each set $A \subset X$, $A \subset Z(A) \subset ker(A) \subset ker_H(A) \subset cl_H(A)$.

(b) For each compact set $A \subset X$, $ker_H(A) = cl_H(A)$.

The following Example shows that Remark 3.2(b) does not hold for the sets which are not compact.

Example 3.1. We take the topological space $(X, \mathfrak{T})$ as in Example 1.1. Let $A = \{(x, y) \mid 0 < x < \frac{1}{4}, 0 < y < \frac{1}{4}\}$. Since A is an open set, by Theorem 1.1(f), $cl_H(A) = cl_\theta(cl(A))$. But since, $cl(A) = \{(x, y) \mid 0 < x \leq \frac{1}{4}, 0 < y \leq \frac{1}{4}\} \cup \{(0, 0)\}$ it is easily seen that $(\frac{1}{2}, 0) \in cl_\theta(cl(A))$ but $(\frac{1}{2}, 0) \notin ker_H(A)$ and so $cl_H(A) \neq ker_H(A)$.

We now prove that hyper-kernel and hyperclosed sets characterize $S_{2\frac{1}{2}}$ spaces in terms of point closure and co-point closure sets.

Theorem 3.4. *For a space X and $x \in X$, the following statements are equivalent:*

(a) X is $S_{2\frac{1}{2}}$.

(b) For any point closure set A not containing x, there exists neighborhoods U containing x and V containing A such that, $cl(U) \cap cl(V) = \phi$.

(c) Every point closure set is hyperclosed.

(d) For any co-point closure set G containing x, there exist a neighborhood U containing x such that $cl_H(U) \subset G$.

(e) For any co-point closure set G containing x, there exist a neighborhood U containing x such that $ker_H(U) \subset G$.

Proof. $(a) \Rightarrow (b)$ Let A be a point closure set not containing x and $A = cl\{y\}$ for some $y \in X$. Thus $cl\{x\} \neq cl\{y\}$ and since X is $S_{2\frac{1}{2}}$, there exists neighborhoods U containing x and V containing y such that, $cl(U) \cap cl(V) = \phi$.

Since $S_{2\frac{1}{2}}$ is S_1 by Lemma 1.2(f), $cl\{y\} \subset V$ and so V is a neighborhood containing A and the result follows.

$(b) \Rightarrow (c)$ is easily established.

$(c) \Rightarrow (d)$ Let G be a co-point closure containing x. Thus $x \notin X \backslash G$ which is a point closure set and so there exists neighborhoods U containing x and V containing $X \backslash G$ such that, $cl(U) \cap cl(V) = \phi$. This implies $cl_\theta(cl(U)) \cap V = \phi$ and by Theorem 1.1(f) since U is an open set, $cl_H(U) \cap V = \phi$. Thus, $cl_H(U) \subset X \backslash V \subset G$ the result follows.

$(d) \Rightarrow (e)$ is obvious.

$(e) \Rightarrow (a)$ Let for any $x, y \in X$, $cl\{x\} \neq cl\{y\}$. Let $z \in cl\{x\}$ and $z \notin cl\{y\}$. Let $G = X \backslash cl\{y\}$ be a co-point closure set containing z and so there exist an open set U containing z such that $ker_H(U) \subset G$. Thus, $y \in X \backslash G \subset X \backslash ker_H(U)$ which gives that $cl_H\{y\} \cap U = \phi$. Since $z \in U$, $z \notin cl_H\{y\}$ and so there exists neighborhoods V containing z and W containing y such that, $cl(V) \cap cl(W) = \phi$. But $z \in cl\{x\}$ implies every neighborhood of z contains x. Thus V is a neighborhood containing x and W containing y such that, $cl(V) \cap cl(W) = \phi$. Hence, X is $S_{2\frac{1}{2}}$. □

It is useful to notice that S_1 spaces are characterized by the statement that every open set contains the closure of its singletons, S_2 spaces have been characterized by the statement that every open set contains the *theta*-closure of its singletons in [4] and below in Theorem 3.5, we characterize $S_{2\frac{1}{2}}$ spaces as spaces in which every open set contains the hyper-closure of its singletons. Since every Urysohn space is T_1 and for T_1 spaces $< x >= \{x\}$, hence first three characterizations of the following Theorem 3.5 are generalizations of Theorem 4.5 of [4].

Theorem 3.5. *For a space X and $x, y \in X$, following statements are equivalent:*

- (a) X *is* $S_{2\frac{1}{2}}$.
- (b) $< x >= cl_H\{x\}$.
- (c) $cl_\theta(\Delta) = \cup_{x \in X}(\{x\} \times < x >)$.
- (d) $G(R) = cl_\theta(\Delta)$.
- (e) $< x >$ *is hyperclosed set.*
- (f) $cl_H\{x\} = cl\{x\}$.
- (g) $cl_H\{x\} = ker\{x\}$.

(h) *For each closed set $F \subset X$, if $x \in F$ then $cl_H\{x\} \subset F$.*

(i) *For each open set $U \subset X$, if $x \in U$ then $cl_H\{x\} \subset U$.*

(j) *Either $< x >=< y >$ or $cl_H\{x\} \cap cl_H\{y\} = \phi$.*

Proof. $(b) \Leftrightarrow (c)$ It follows from Theorem 2.4 (c) and (d).
$(c) \Rightarrow (d)$ Let $(x, y) \in G(R) \backslash cl_\theta(\Delta)$. Thus $y \notin cl_\theta\{x\}$ and so $y \notin < x >$ by part (c). This implies $(x, y) \notin G(R)$ and the result follows. The reverse inclusion is obtained by reversing the above steps.
$(d) \Rightarrow (b)$ follows from Theorem 2.4(b) and Lemma 1.1(g).
The rest of the proof is similar to that of [4, Theorem 3.1] □

Corollary 3.1. *For an $S_{2\frac{1}{2}}$ space X the following statements hold:*

(a) *For each $x \in X$, $cl_H\{x\}$ is a hyperclosed set.*

(b) *$\{cl_H\{x\} : x \in X\}$ forms a partition for space X.*

The proofs are straightforward and thus omitted.

Corollary 3.2. (a) *An S_2 space X is $S_{2\frac{1}{2}}$ if and only if $cl_\theta(\Delta) = cl(\Delta)$.*

(b) *An S_1 space X is $S_{2\frac{1}{2}}$ if and only if $cl_\theta(\Delta) = ker(\Delta)$.*

Proof. (a) It follows from Theorem 3.5(d) and Lemma 1.2(b)

(b) It follows from Theorem 3.5(d) and Lemma 1.2(a).

□

We now characterize $S_{2\frac{1}{2}}$ spaces in terms of unions of hyperclosed sets.

Theorem 3.6. *For a space X, then the following statements are equivalent:*

(a) *X is $S_{2\frac{1}{2}}$.*

(b) *Every open set is a union of hyperclosed sets.*

(c) *For each $A \subset X$, A is union of hyperclosed sets whenever $X \backslash A$ is union of closed sets.*

Proof. The proof follows from Theorem 3.5(i) and Corollary 3.1(a). □

We now list below some more characterizations of various separation axioms. The proofs are similar to that of Theorems in [3] [4] and hence are omitted.

Theorem 3.7. *For a space X and a compact set $A \subset X$, the following statements are equivalent:*

(a) *X is $S_{2\frac{1}{2}}$.*

(b) *$ker_H(A) = cl(A)$.*

(c) *$cl_H(A) = cl(A)$.*

(d) *$ker_H(A) = Z(A)$.*

(e) *$cl_H(A) = Z(A)$.*

(f) *For each open set $U \subset X$, $ker_H(U) = U$.*

(g) *For each closed set $F \subset X$, $ker_H(F) = F$.*

Theorem 3.8. *For a space X, the following statements are equivalent:*

(a) *X is Urysohn.*

(b) *For every distinct points $x, y \in X$, $cl_H\{x\} \cap cl_H\{y\} = \phi$.*

(c) *For each set A, $A = ker_H(A)$.*

(d) *For each compact set A, $A = ker_H(A)$.*

(e) *Every compact set is hyperclosed.*

Theorem 3.9. *A space X is S_1 if and only if for each $x \in X$, $cl_H\{x\} \backslash cl\{x\}$ is union of closed sets.*

Theorem 3.10. *A space X is T_1 if and only if for each $x \in X$, $cl_H\{x\} \backslash \{x\}$ is union of closed sets.*

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