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RARELY s^*g-CONTINUOUS FUNCTIONS

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Abstract: The notion of rare continuity was introduced by Popa. In the present
paper, we introduce a new class of functions, called rarely s^*g-continuous functions and investigate some of its fundamental properties.

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Key Words: rarely s^*g-continuous function, $I.s^*g$-continuous function, weakly s^*g-continuous, S^*GO-compact, rare set

1. Introduction

In 1961, Levine introduced the concept weak continuity [7] in topological spaces. As a generalization of this concept, Popa [12] introduced the notion of rare continuity in the year 1979 and it has been further investigated by Long and Herrington [9] in 1982. Jafari [5, 6] studied some more properties of them. Moreover, Popa [10, 11] studied the notions of Weakly continuous multifunctions and rarely continuous multifunctions.

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Meanwhile, Levine [8] introduced the concept of generalized closed sets of a topological space as a generalization of closed sets to the larger family and a class of topological spaces called $T_{\frac{1}{2}}$-spaces. Dunham [3], Dunham and Levine [4] and Caldas [1] further studied some properties of generalized closed sets and $T_{\frac{1}{2}}$-spaces.

Caldas and Jafari [2] introduced the concept of rare g-continuity in topological spaces as a generalization of rare continuity and weak continuity. They
investigated several properties of rarely g-continuous functions. The notion of $I.g$-continuity is also introduced which is weaker than g-continuity and stronger than rare g-continuity. Further they showed that when the codomain of a function is regular, then the notions of rare g-continuity and $I.g$-continuity are equivalent.

In the present paper, we introduce a new class of functions called rarely s^*g-continuous functions and investigate some of its fundamental properties.

2. Preliminaries

Throughout this paper, X and Y are topological spaces. The closure of a set A is the intersection of all closed sets that contain A and is denoted by $cl(A)$ and the interior of a set A is the union of all open sets contained in A and is denoted by $int(A)$. Now we shall require some known definitions.

Definition 2.1. A set A of a topological space (X, τ) is called

(a) semi open if there exists an open set U such that $U \subseteq A \subseteq cl(U)$.

Equivalently, a set A is semi open if $A \subseteq cl[int(A)]$.

(b) semi closed if $X - A$ is semi open.

Equivalently, a set A of a topological space (X, τ) is called semi closed if there exists a closed set F such that $int(F) \subseteq A \subseteq F$.

(c) generalized closed (g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(d) generalized open (g-open) if $X - A$ is g-closed.

(e) semi star generalized closed (s^*g-closed) if $cl(A) \subseteq U$ whenever $A \subseteq$U and U is semi open in X.

(f) semi star generalized open (s^*g-open) if $X - A$ is s^*g-closed.

(g) rare set if $\text{Int}(A) = \phi$.

(h) nowhere dense set if $Int[Cl(A)] = \phi$ where $Cl(A)$ is codense.

The family of all open {resp. g-open, s^*g-open} sets will be denoted by $O(X)$ {resp. $GO(X)$, $S^*GO(X)$} and the family of all open {resp. g-open, s^*g-open} sets containing the point $x \in X$ will be denoted by $O(X, x)$ {resp. $GO(X, x)$, $S^*GO(X, x)$}.

Definition 2.2. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be

(a) continuous if $f^{-1}(V)$ is open (closed) in X for every open (closed) set V in Y,

(b) closed if $f(U)$ is closed in Y for every closed set U in X,

(c) g-continuous if $f^{-1}(U)$ is g-closed for each closed set U in Y,

(d) s^*g-continuous if $f^{-1}(U)$ is s^*g-closed for each closed set U in Y,

(e) weakly continuous if for each $x \in X$ and each open set G containing $f(x)$, there exists $U \in O(X, x)$ such that $f(U) \subseteq Cl(G)$.

(f) weakly g-continuous if for each $x \in X$ and each open set G containing $f(x)$, there exists $U \in GO(X, x)$ such that $f(U) \subseteq Cl(G)$.

(g) rarely continuous if for each $x \in X$ and each $G \in O(Y; f(x))$, there exist a rare set R_G with $G \cap cl(R_G) = \phi$; and $U \in O(X, x)$ such that $f(U) \subseteq G \cup R_G$.

(h) rarely g-continuous if for each $x \in X$ and each $G \in O(Y, f(x))$, there exist a rare set R_G with $G \cap cl(R_G) = \phi$ and $U \in GO(X, x)$ such that $f(U) \subseteq G \cup R_G$.

3. Rarely s^*g-Continuous Functions

Definition 3.1. A function $f : X \to Y$ is called rarely s^*g-continuous if for each $x \in X$ and each $G \in O(Y; f(x))$, there exist a rare set R_G with $G \cap cl(R_G) = \phi$ and $U \in S^*GO(X, x)$ such that $f(U) \subseteq G \cup R_G$.

Example 3.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b, c\}\}$, $\sigma = \{\phi, Y, \{b\}, \{a, c\}\}$ and $f : (X, \tau) \to (Y, \sigma)$ be a identity function. Then f is a rarely s^*g-continuous

Theorem 3.3. The following statements are equivalent for a function $f : X \to Y$:

(1) The function f is rarely s^*g-continuous at $x \in X$.

(2) For each set $G \in O(Y, f(x))$, there exists $U \in S^*GO(X, x)$ such that $Int[f(U) \cap (Y \setminus G)] = \phi$.

(3) For each set $G \in O(Y, f(x))$, there exists $U \in S^*GO(X, x)$ such that $int[f(U)] \subseteq cl(G)$.

Proof. (1) $\Rightarrow$ (2) : Let $G \in O(Y, f(x))$. By $f(x) \in G \subseteq int[cl(G)]$ and the fact that $int[cl(G)] \in O(Y, f(x))$, there exist a rare set R_G with $int[cl(G)] \cap cl(R_G) = \phi$ and a s^*g-open set $U \subseteq X$ containing x such that $f(U) \subseteq int[cl(G)] \cup R_G$. Hence, $int[f(U) \cap (Y \setminus G)] = int[f(U)] \cap int(Y \setminus G) \subseteq int[cl(G) \cup R_G] \cap [Y \setminus cl(G)] \subseteq [cl(G) \cup int(R_G)] \cap [Y \setminus cl(G)] = \phi$.

(2) $\Rightarrow$ (3): It is straightforward.

(3) $\Rightarrow$ (1): Let $G \in O(Y, f(x))$. Then by (3), there exists $U \in S^*GO(X, x)$ such that $int[f(U)] \subseteq cl(G)$. Hence $f(U) = \{f(U) \setminus int[f(U)]\} \cup int[f(U)] \subseteq \{f(U) \setminus int[f(U)]\} \cup cl(G) = \{f(U) \setminus int[f(U)]\} \cup G \cup [cl(G) \setminus G] = \{f(U) \setminus int[f(U)]\} \cap (Y \setminus G) \cup G \cup [cl(G) \setminus G]$.

Set $R^* = \{f(U) \setminus int[f(U)]\} \cap (Y \setminus G)$ and $R^{**} = [cl(G) \setminus G]$. Then R^* and R^{**} are rare sets. Moreover, $R_G = R^* \cup R^{**}$ is a rare set such that $cl(R_G) \cap G = \phi$ and $f(U) \subseteq G \cup R_G$. This shows that f is rarely s^*g-continuous. □

We define the following notion which is a new generalization of s^*g-continuity.

Definition 3.4. A function $f : X \to Y$ is $I.s^*g$-continuous at $x \in X$ if for each set $G \in O(Y, f(x))$, there exists $U \in S^*GO(X, x)$ such that $int[f(U)] \subseteq G$. If f has this property at each point $x \in X$, then we say that f is $I.s^*g$-continuous on X.

Theorem 3.5. Let Y be a regular space. Then the function $f : X \to Y$ is $I.s^*g$-continuous on X if and only if f is rarely s^*g-continuous on X.

Proof. We prove only the sufficient condition since the necessity condition is evident. Let f be rarely s^*g-continuous on X and $x \in X$. Suppose that

$f(x) \in G$, where G is an open set in Y. By the regularity of Y, there exists an open set $G_1 \in O(Y, f(x))$ such that $cl(G_1) \subseteq G$. Since f is rarely s^*g-continuous, then there exists $U \in S^*GO(X, x)$ such that $int[f(U)] \subseteq cl(G_1)$ (Theorem 3.3). This implies that $int[f(U)] \subseteq G$ and therefore f is $I.s^*g$-continuous on X. □

We say that a function $f : X \to Y$ is $r.s^*g$-open if the image of a s^*g-open set is open.

Theorem 3.6. If $f : X \to Y$ is an $r.s^*g$-open rarely s^*g-continuous function, then f is weakly s^*g-continuous.

Proof. Suppose that $x \in X$ and $G \in O(Y, f(x))$. Since f is rarely s^*g-continuous, there exist a rare set R_G with $cl(R_G) \cap U = \phi$ and $U \in S^*GO(X, x)$ such that $f(U) \subseteq G \cup R_G$. This means that $\{f(U) \cap (Y \setminus cl(G)\} \subseteq R_G$. Since the function f is $r.s^*g$-open, then $f(U) \cap [Y \setminus cl(G)]$ is open. But the rare set R_G has no interior points. Then $f(U) \cap [Y \setminus cl(G)] = \phi$. This implies that $f(U) \subseteq cl(G)$ and thus f is weakly s^*g-continuous. □

Theorem 3.7. If $f : X \to Y$ is rarely s^*g-continuous function, then the graph function $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for every $x \in X$ is rarely s^*g-continuous.

Proof. Suppose that $x \in X$ and W is any open set containing $g(x)$. It follows that there exist open sets U and V in X and Y, respectively, such that $(x, f(x)) \in U \times V \subseteq W$. Since f is rarely s^*g-continuous, there exists $G \in S^*GO(X, x)$ such that $int[f(G)] \subseteq cl(V)$. Let $E = U \cap G$. It follows that $E \in S^*GO(X, x)$ and we have $int[g(E)] \subseteq int(U \times f(G)) \subseteq U \times cl(V) \subseteq cl(W)$. Therefore, g is rarely s^*g-continuous. □

Definition 3.8. Let $A = \{G_i\}$ be a class of subsets of X. By rarely union sets of A we mean $\{Gi \cup R_{G_i}\}$, where each R_{G_i} is a rare set such that each of $\{G_i \cap cl(R_{G_i})\}$ is empty.

Recall that, a subset B of X is said to be rarely almost compact relative to X if every open cover of B by open sets of X, there exists a finite subfamily whose rarely union sets cover B.

A topological space X is said to be rarely almost compact if the set X is rarely almost compact relative to X. A topological space X is called S^*GO-

compact if every cover of X by s^*g-open sets has a finite subcover.

Theorem 3.9. Let $f : X \to Y$ be rarely s^*g-continuous and K a S^*GO-compact set relative to X. Then $f(K)$ is rarely almost compact subset relative to Y.

Proof. Suppose that Ω is an open cover of $f(K)$. Let B be the set of all $V \in \Omega$ such that $V \cap f(K) \neq \phi$. Then B is a open cover of $f(K)$. Hence for each $k \in K$, there is some $V_k \in B$ such that $f(k) \in V_k$. Since f is rarely s^*g-continuous there exist a rare set R_{V_k} with $V_k \cap cl(R_{V_k}) = \phi$ and a s^*g-open set U_k containing k such that $f(U_k) \subseteq V_k \cup R_{V_k}$. Hence there is a finite subfamily $\{U_k\}_{k\in\Delta}$ which covers K, where Δ is a finite subset of K. The subfamily $\{V_k \cup R_{V_k}\}_{k\in\Delta}$ also covers $f(K)$. □

Recall that a space X is called T_S-space if every s^*g-closed set in X is closed in X.

Theorem 3.10. Let $f : X \to Y$ be rarely s^*g-continuous and X a T_S-space. Then f is rarely continuous.

A space X is called a door space if every subset of X is either open or closed. W. Dunham [[3] Corollary 3.7] proved the following result:

Lemma 3.11. A door space is a $T_{\frac{1}{2}}$-space.

Theorem 3.12. Let $f : X \to Y$ be a rarely s^*g-continuous and X be a door space. Then f is rarely continuous.

Proof. It is an immediate consequence of Lemma 3.11 and Theorem 3.10. □

Lemma 3.13. (Long and Herrington [9]). If $g : Y \to Z$ is continuous and one-to-one, then g preserves rare sets.

Theorem 3.14. If $f : X \to Y$ is rarely s^*g-continuous and $g : Y \to Z$ is continuous and one-to-one, then $gof : X \to Z$ is rarely s^*g-continuous.

Proof. Suppose that $x \in X$ and $(gof)(x) \in V$, where V is an open set in

V. By hypothesis, g is continuous, therefore there exists an open set $G \subseteq Y$ containing $f(x)$ such that $g(G) \subseteq V$. Since f is rarely s^*g-continuous, there exist a rare set R_G with $G \cap cl(R_G) = \phi$ and a s^*g-open set U containing x such that $f(U) \subseteq G \cup R_G$. It follows from Lemma 3.13 that $g(R_G)$ is a rare set in Z. Since R_G is a subset of $Y \setminus G$ and g is injective, we have $cl[g(R_G)] \cap V = \phi$. This implies that $(gof)(U) \subseteq V[g(R_G)$. Hence the result. □

Recall that a function $f : X \to Y$ is called pre-s^*g-open if $f(U)$ is s^*g-open in Y for every s^*g-open set U of X.

Theorem 3.15. Let $f : X \to Y$ be a pre-s^*g-open surjection and $g : Y \to Z$ a function such that $gof : X \to Z$ is rarely s^*g-continuous. Then g is rarely s^*g-continuous.

Proof. Let $y \in Y$ and $x \in X$ such that $f(x) = y$. Let $G \in O(Z, (gof)(x))$. Since gof is rarely s^*g-continuous, there exist a rare set R_G with $G \cap cl(R_G)$ $= \phi$ and $U \in S^*GO(X, x)$ such that $(gof)(U) \subseteq G \cup R_G$. But $f(U)$ (say V) is a s^*g-open set containing $f(x)$. Therefore, there exist a rare set R_G with $G \cap cl(R_) = \phi$ and $V \in S^*GO(Y, y)$ such that $g(V) \subseteq G \cup R_G$, i.e., g is rarely s^*g-continuous. □

4. Upper (Lower) Rarely s^*g-Continuous Multifunctions

We provide the following definitions which will be used in the sequel. Let $F : X \to Y$ be a multifunction. The upper and lower inverses of a set $V \subseteq Y$ are denoted by $F^+(V)$ and $F^-(V)$ respectively, that is, $F^+(V) = \{x \in X/F(x) \subseteq V\}$ and $F^-(V) = \{x \in X/F(x) \cap V \neq \phi\}$.

Definition 4.1. A multifunction $F : X \to Y$ is said to be

(i) upper rarely s^*g-continuous (briefly $u.r.s^*g.c$) at $x \in X$ if for each $V \in O(Y, F(x))$, there exist a rare set R_V with $V \cap cl(R_V) = \phi$ and $U \in S^*GO(X, x)$ such that $F(U) \subseteq V \cup R_V$,

(ii) lower rarely s^*g-continuous (briefly $l.r.s^*g.c$) at $x \in X$ if for each $V \in O(Y)$ with $F(x) \cap V \neq \phi$ there exist a rare set R_V with $V \cap cl(R_V) = \phi$ and $U \in S^*GO(X, x)$ such that $F(u) \cap (V \cup R_V) \neq \phi$ for every $u \in U$,

(iii) upper/lower rarely s^*g-continuous if it is upper/lower rarely s^*g-continuous at each point of X.

Definition 4.2. A multifunction $F : X \to Y$ is said to be

(i) upper weakly s^*g-continuous at $x \in X$ if for each $V \in O(Y, F(x))$, there exist $U \in S^*GO(X, x)$ such that $F(U) \subseteq cl(V)$,

(ii) lower weakly s^*g-continuous at $x \in X$ if for each $V \in O(Y)$ with $F(x) \cap V \neq \phi$, there exists $U \in S^*GO(X, x)$ such that $F(u) \cap cl(V) \neq \phi$ for every $u \in U$,

(iii) upper/lower weakly s^*g-continuous if it is upper/lower weakly s^*g-continuous at each point of X.

Theorem 4.3. The following statements are equivalent for a multifunction $F : X \to Y$:

(i) F is $u.r.s^*g.c$ at $x \in X$,

(ii) For each $V \in O(Y, F(x))$, there exists $U \in S^*GO(X, x)$ such that $int[F(U) \cap (Y - V)] = \phi$,

(iii) For each $V \in O(Y, F(x))$, there exists $U \in S^*GO(X, x)$ such that $int[F(U)] \subseteq cl(V)$.

Proof. $(i) \Rightarrow (ii)$: Let $V \in O(Y, F(x))$. By $F(x) \subseteq V \subseteq int[cl(V)]$ and the fact that $int[cl(V)] \in O(Y)$, there exist a rare set R_V with $int[cl(V)] \cap cl(R_V) = \phi$ and a s^*g-open set $U \subseteq X$ containing x such that $F(U) \subseteq int[cl(V)] \cup R_V$. Hence $int[F(U) \cap (Y \setminus V)] = int(F(U)) \cap int(Y \setminus V) \subseteq int[cl(V) \cup R_V] \cap [Y \setminus cl(V)] \subseteq [cl(V) \cup int(R_V)] \cap [Y \setminus cl(V)] = \phi$.

$(ii) \Rightarrow (iii)$: Obvious.

$(iii) \Rightarrow (i)$: Let $V \in O(Y, F(x))$. Then, by (iii) there exists $U \in S^*GO(X, x)$ such that $int[F(U)] \subseteq cl(V)$. Thus $F(U) = [\{F(U) \setminus int[F(U)]\} \cup int[F(U)] \subseteq \{F(U) \setminus int[F(U)]\} \cup cl(V) = \{F(U) \setminus int[F(U)]\} \cup V \cup [cl(V) \setminus V] = \{(F(U) \setminus int[F(U)] \cup (Y \setminus V)\} \cup V \cup [cl(V) \setminus V]$. Put $P = \{F(U) \setminus int[F(U)]\} \cap (Y \setminus V)$ and $G = cl(V) \setminus V$, then P and G are rare sets. Moreover, $R_V = P \cup G$ is a rare set such that $cl(R_V) \cap V = \phi$ and $F(U) \subseteq V \cup R_V$. Hence F is $u.r.s^*g.c.$ □

Theorem 4.4. The following are equivalent for a multifunction $F : X \to Y$:

(i) F is $l.r.s^*g.c$ at $x \in X$,

(ii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$, there exists a rare set R_V with $V \cap cl(R_V) = \phi$ such that $x \in s^*g\text{-}int[F \setminus (V \cup R_V)]$,

(iii) For each $V \in O(Y)$ such that $F(x) \cap V \neq \phi$, there exists a rare set R_V with $cl(V) \cap R_V = \phi$ such that $x \in s^*g\text{-}int[F \setminus (cl(V) \cup R_V)]$,

(iv) For each $V \in RO(Y)$ such that $F(x) \cap V \neq \phi$, there exists a rare set R_V with $V \cap cl(R_V) = \phi$ such that $x \in s^*g\text{-}int[F \setminus (V \cup R_V)]$.

Proof. $(i) \Rightarrow)(ii)$: Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (i), there exist a rare set R_V with $V \cap cl(R_V) = \phi$ and $U \in S^*GO(X, x)$ such that $F(x) \cap (V \cup R_V) \neq \phi$ for each $u \in U$. Therefore, $u \in F - (V \cup R_V)$ for each $u \in U$ and hence $U \subseteq F - (V \cup R_V)$. Since $U \in GO(X, x)$, we obtain $x \in U \subseteq s^*g\text{-}int[F \setminus (V \cup R_V)]$.

$(ii) \Rightarrow (iii)$: Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. By (ii), there exists a rare set R_V with $V \cap cl(R_V) = \phi$ such that $x \in s^*g\text{-}int[F \setminus (V \cup R_V)]$. Hence $R_V \subseteq Y \setminus V = [Y \setminus cl(V)] \cup [cl(V) \setminus V]$ and hence $R_V \subseteq \{R_V \cap [Y \setminus cl(V)]\} \cup [cl(V) \setminus V]$. Now, put $P = R_V \cap [Y \setminus cl(V)]$. Then P is a rare set and $P \cap cl(V) = \phi$. Moreover, we have $x \in s^*g\text{-}int[F \setminus (V \cup R_V)] \subseteq s^*g\text{-}int\{F \setminus [P \cup cl(V)]\}$.

$(iii) \Rightarrow (iv)$: Let V be any regular open set of Y such that $F(x) \cap V \neq \phi$. By (iii), there exists a rare set R_V with $cl(V) \cap R_V = \phi$ such that $x \in s^*g\text{-}int\{F \setminus [cl(V) \cup R_V]\}$. Put $P = R_V \cup [cl(V) \setminus V]$, then P is a rare set and $V \cap cl(P) = \phi$. Moreover, we have $x \in s^*g\text{-}int\{F^-[cl(V) \cup R_V]\} = s^*g\text{-}int\{F^-[R \cup \{cl(V) \setminus V\} \cup V]\} = s^*g\text{-}int[F^-(P \cup V)]$.

$(iv) \Rightarrow (i)$: Let $V \in O(Y)$ such that $F(x) \cap V \neq \phi$. Then $F(x) \cap int[cl(V)] \neq \phi$ and $int[cl(V)]$ is regular open in Y. By (iv), there exists a rare set R_V with $V \cap cl(R_V) = \phi$ such that $x \in s^*g\text{-}int[F^-(V \cup R_V)]$. Therefore, there exists $U \in S^*GO(X, x)$ such that $U \subseteq F^-(V \cup R_V)$. Hence $F(u) \cap (V \cup R_V) \neq \phi$ for each $u \in U$. This shows that F is lower rarely s^*g-continuous at x. □

Corollary 4.5. The following statements are equivalent for a function $f : X \to Y$:

(i) f is rarely s^*g-continuous at $x \in X$,

(ii) For $V \in O(Y, f(x))$, there exists $U \in S^*GO(X, x)$ such that $int[f(U) \cap (Y \setminus V)] = \phi$,

(iii) For each $V \in O(Y, f(x))$, there exists $U \in S^*GO(X, x)$ such that $int[f(U)] \subseteq cl(V)$.

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