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A MATHEMATICAL MODEL FOR THE SECRETION OF VASOPRESSIN USING FUZZY TRUNCATED NORMAL DISTRIBUTION

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Abstract: The present article is devoted to investigate the increase of the hormone vasopressin due to the administration of adenosine tri-phosphate and phenylephrine in the explants. A mathematical model using fuzzy truncated normal distribution was developed and used this model to calculate the mean values of vasopressin release in the truncated time intervals. The result shows that a synergistic effect of adenosine tri-phosphate and phenylephrine on vasopressin.

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Key Words: fuzzy truncated normal distribution, vasopressin, adenosine tri-phosphate, phenylephrine

1. Introduction

Truncated distributions occur in many practical situations, particularly in several industrial settings. The truncated normal distribution is an important distribution in the world of probability and statistics. It appears quite naturally when the normal distribution itself arises. Most of the available literatures tend

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to focus on the mean and variance of the truncated normal distribution. This is partially provoked from the statistical community. They were paying interest in obtaining unbiased mean and variance estimators for data that is screened or threshold. Truncated distributions can also be used to model intensity statistics in the study of atomic heterogeneity. A mathematically defensible way to protect the main features of the normal distribution while avoiding extreme values involves the truncated normal distribution, in which the range of definition is made finite at one or both ends of the interval. It is the purpose of this paper to describe the truncation process, to consider how certain basic statistical properties of the new distribution can be determined.

Informally, the truncated normal probability density function is defined in two steps. We choose a general normal probability density function by specifying parameters μ and σ^2, and then a truncation range (a; b). The probability density function associated with the general normal distribution is modified by setting values outside the range to zero, and uniformly scaling the values in the range so that the total integral is 1.

The truncation range can be classified into four classes.

1. nontruncated case: $-\alpha = a$; $b = +\alpha$;
2. lower truncated case: $-\alpha < a$; $b = +\alpha$;
3. upper truncated case: $-\alpha = a$; $b < +\alpha$;
4. doubly truncated case: $-\alpha < a$; $b < +\alpha$.

Truncated sample of normal distribution arise, in practice, with various types of experimental data in which recorded measurements available over only a partial range of the variable. The Maximum likelihood estimation of singly truncated and doubly truncated normal distribution was considered by Cohen [2], [11]. Numerical solution to the estimators of the mean and variance for singly truncated samples were computed with an auxiliary function which is tabulated in Cohen [3]. The properties and applications of the fuzzy probabilities have been studied by many authors, including Zadeh [4], [7], Yager, R.R. et al [5], [8], J.J. Buckley, E. Eslami [14], [15].

Diverse effect of norepinephrine on vasopressin release was reported by C. Sladek, C. Yagil [10] and the effect of actinomycin and cycloheximide on stimulation of vasopressin was studied by Z. Song, H.E. Sidorowicz, C.D. Sladek [12]. In this paper, we investigate the increase of the hormone vasopressin (VP) due to the administration of adenosine tri-phosphate(ATP) and phenylephrine (PE) in the explants using fuzzy truncated normal distribution.

2. Notation

μ	Mean value of normal distribution;
σ	Standard deviation of normal distribution;
φ	pdf of normal distribution;
ϕ	cdf of normal distribution;
a	lower truncation interval;
b	upper truncation interval
$\overline{\mu}[\alpha]$	alpha cut of mean;
$\overline{\sigma}[\alpha]$	alpha cut of standard deviation;
$E[\alpha]$	mean value of fuzzy truncation normal distribution.

3. Truncated Normal Distribution

Suppose A denotes the closed interval [a, b] and let Y = μ+ σX, where a, b $\in$ R, a <b and $X \sim N(0,1)$. Then, we say that the random variable Y ((a, b); μ, σ) has a doubly truncated normal distribution in A if:

$$Y((a,b);\mu,\sigma) = Y(\mu,\sigma)/(a<Y(\mu,\sigma)<b),$$

in symbol

$$Y((a,b);\mu,\sigma) \sim TN((a,b);\mu,\sigma),$$

if its probability density function is of the following form

$$f(y(\mu,\sigma)/y(\mu,\sigma)\in A) = \frac{\frac{1}{\sigma}\varphi\left(\frac{y-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right)-\phi\left(\frac{a-\mu}{\sigma}\right)}, \quad a \le y \le b.$$

Here φ (.) and ϕ(.) are the probability density function and cumulative density function of the standard normal distribution respectively

The mean value of truncated normal distribution is

$$E(y/y\in A) = \mu - \sigma\left[\frac{\varphi\left(\frac{b-\mu}{\sigma}\right)-\varphi\left(\frac{a-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right)-\phi\left(\frac{a-\mu}{\sigma}\right)}\right].$$

4. Fuzzy Truncated Normal Distribution

Let $\bar{A}$ be a fuzzy event in [a, b], with $\mu\bar{A} : [a, b] \to [0, 1]$. Then, the random variable $Y((a, b); \mu, \sigma)$ has a fuzzy truncated normal distribution in symbol:

$$Y((a, b); \mu, \sigma) \sim FTN((a, b); \mu, \sigma),$$

i.e. if it has the following probability density function

$$f(y(\mu,\sigma)/y(\mu,\sigma) \in \bar{A}) = \frac{\left[\frac{\frac{1}{\sigma}\varphi\left(\frac{y-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right)-\phi\left(\frac{a-\mu}{\sigma}\right)}\right]\mu_{\bar{A}}(y)}{\int\limits_{a}^{b}\left[\frac{\frac{1}{\sigma}\varphi\left(\frac{y-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right)-\phi\left(\frac{a-\mu}{\sigma}\right)}\right]\mu_{\bar{A}}(y)dy}, \quad a \leq y \leq b.$$

Here φ (.) and ϕ(.) are the probability density function and cumulative density function of the standard normal distribution respectively

The mean value of truncated normal distribution is

$$E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}) = \mu - \sigma \left[\frac{\varphi\left(\frac{b-\mu}{\sigma}\right) - \varphi\left(\frac{a-\mu}{\sigma}\right)}{\phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)}\right].$$

5. Application

Vasopressin is a hormone that is secreted by the posterior pituitary gland. In the body, vasopressin act on the kidneys and blood vessels. Vasopressin helps prevent the loss of water from the body by reducing urine output and serving the kidneys reabsorb water in the body. Vasopressin also raises blood pressure by constricting (narrowing) blood vessels. It acts as a neurotransmitter in the brain to control circadian rhythm, thermoregulation, and adrenocorticotrophic hormone (ACTH) release. The therapeutic use of vasopressin has become important in the critical care environment in the management of cranial diabetes insipidus, bleeding abnormalities, oesophageal variceal hemorrhage, asystolic cardiac arrest, and septic shock. The A1 catecholamine neurons of the caudal ventrolateral medulla transmit hemodynamic information to the vasopressin neurons in the hypothalamus. Simultaneous exposure of explants to 100 μM adenosine tri-phosphate (ATP) and 100 μM phenylephrine(PE) resulted in synergistic responses that were delayed, larger and sustained relative to the responses observed with ATP or PE individually. Table 5.1 shows the % basal of vasopressin release due to the administration of ATP and PE.

Time (mins.)	120	135	150	165	180	195	210	225	240
VP release % basal	100	1 00	150	200	180	240	270	230	225

Table 5.1: Vasopressin release due to the administration of ATP and PE

We truncate the time intervals into two cases such that from 120 min. to 180 min. is taken as the first case and 180 min. to 240 min. is taken as the second case and the mean value of truncated normal distribution are calculated accordingly.

5.1. Case I: 120 min. to 180 min.

In the interval 120 min. to 180 min., the mean and standard deviations of vasopressin release are $\mu = 146$ and $\sigma = 6.8$. The Fuzzy triangular numbers for the scale and location parameters are taken as

$$\overline{\mu} = [145, 146, 147], \quad \overline{\sigma} = [6.3, 6.8, 7.3].$$

The alpha cut of the mean and Standard deviations are respectively given as

$$\overline{\mu}[\alpha] = [145 + \alpha, 147 - \alpha], \quad \overline{\sigma}[\alpha] = [6.3 + 0.5\alpha, 7.3 - 0.5\alpha].$$

Here a and b specify the truncation interval, i.e. $a = 120$ and $b = 180$.

The mean values of truncated normal distribution for the interval 120 min. to 180 min. are calculated by using

$$E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}) = \{E_1[\alpha], \ E_2[\alpha]\}$$

Here

$$E_1[\alpha] = \text{Minimum of } E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A})$$
$$E_2[\alpha] = \text{Maximum of } E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}).$$

5.2. Case II: 180 min. to 240 min.

In the interval 180 min. to 240 min., the mean and standard deviations of vasopressin release are $\mu = 229$ and $\sigma = 5.7$. The Fuzzy triangular numbers for the scale and location parameters are taken as

$$\overline{\mu} = [228, 229, 230], \quad \overline{\sigma} = [5.2, 5.7, 6.2].$$

α	$\overline{E}_1[\alpha]$	$\overline{E}_2[\alpha]$
0	138.7	139.7
0.1	138.75	139.65
0.2	138.8	139.6
0.3	138.85	139.55
0.4	138.9	139.5
0.5	138.95	139.45
0.6	139	139.4
0.7	139.05	139.35
0.8	139.1	139.3
0.9	139.15	139.25
1	139.2	139.2

Table 5.2: α-cut for fuzzy truncated normal distribution

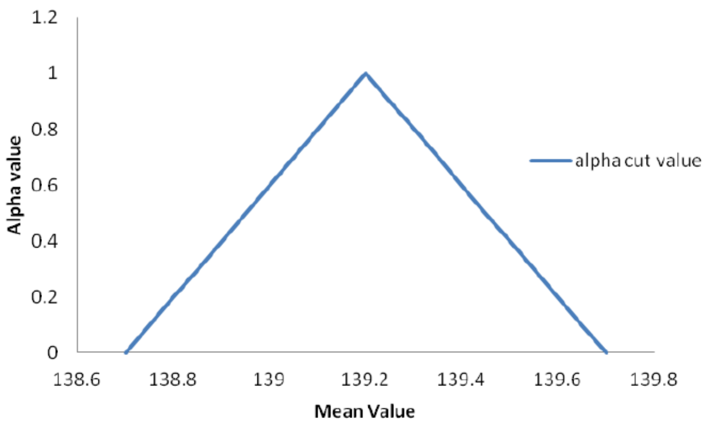

Figure 5.1: Membership function for the interval 120 to 180

The alpha cut of the mean and Standard deviations are respectively given as

$$\overline{\mu}[\alpha] = [228 + \alpha, 230 - \alpha], \quad \overline{\sigma}[\alpha] = [5.2 + 0.5\alpha, 6.2 - 0.5\alpha].$$

Here a and b specify the truncation interval, i.e. $a = 180$ and $b = 240$.

The mean values of truncated normal distribution for the interval 180 min. to 240 min. are calculated by using

$$E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}) = \{E_1[\alpha], \ E_2[\alpha]\},$$

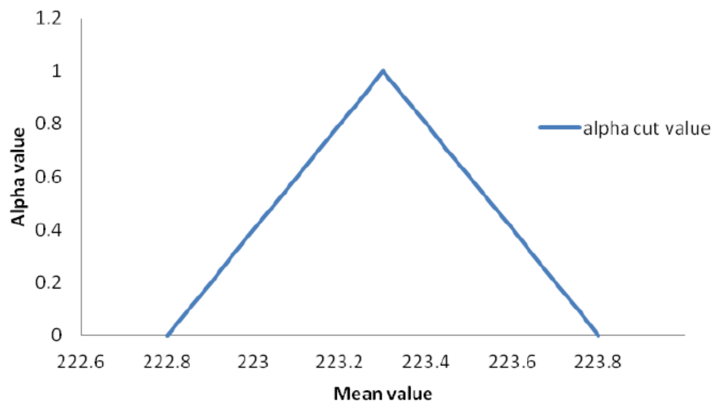

Figure 5.2: Membership function for the interval 180 to 240

where

$$E_1[\alpha] = \text{Minimum of } E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}),$$

$$E_2[\alpha] = \text{Maximum of } E(y_{\mu,\sigma}/y_{\mu,\sigma} \in \bar{A}).$$

α	$\overline{E}_1[\alpha]$	$\overline{E}_2[\alpha]$
0	222.80	223.80
0.1	222.85	223.75
0.2	222.90	223.70
0.3	222.95	223.65
0.4	223.00	223.60
0.5	223.05	223.55
0.6	223.10	223.50
0.7	223.15	223.45
0.8	223.20	223.40
0.9	223.25	223.35
1	223.30	223.30

Table 5.3: α-cut for fuzzy truncated normal distribution

6. Conclusion

Here we proposed two cases to study, the mean value of vasopressin releasing level due to the administration of ATP and PE by using fuzzy truncated normal distribution. The membership function of the mean values showed that, after the administration of ATP and PE the releasing level of vasopressin is increased in both the two cases. These results suggest that the release of cotransmitters may be responsible for maintaining a sustained increase in plasma vasopressin in response to blood pressure.

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