International Journal of Pure and Applied Mathematics
Volume 104 No. 2 2015, 265-284
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v104i2.10

ijpam.eu

ON THE STABILITY OF TRIBONACCI AND *k*-TRIBONACCI FUNCTIONAL EQUATIONS IN MODULAR SPACE

Roji Lather[1], Ashish[2 §], Manoj Kumar[3]

[1,3]Department of Mathematics
Maharshi Dayanand University
Rohtak, 124001, INDIA

[2]Central University of Haryana
Jant-Pali, Mahendergarh, INDIA

Abstract: The purpose of this paper is to establish the Hyers-Ulam stability of the following Tribonacci and k-Tribonacci functional equations

$$f(x) = f(x-1) + f(x-2) + f(x-3),$$
$$f(k,x) = kf(k,x-1) + f(k,x-2) + f(k,x-3)$$

in modular space.

AMS Subject Classification: 39B82, 39B52, 39B72
Key Words: Hyers-Ulam stability, modular space, Tribonacci functional equation, k-Tribonacci functional equation

1. Introduction

Stability is investigated when one is asking whether a small error of parameters in one problem causes a large deviation of its solution. Give an approximate homomorphism, is it possible to approximate it by a true homomorphism? In other words, we are looking for the situations when the homo-

Received: August 8, 2015

§Correspondence author

morphisms are stable, that is, if a mapping is almost a homomorphism, then there exists a true homomorphism near it will small error as much as possible. This problem was posed by Ulam in 1940 [34] and is called the stability of functional equations. For Banach spaces, the problem was solved by Hyers [5] in the case of approximately additive mappings. Later, Hyers result was generalized by Aoki [35] for additive mappings and by Rassias [36] for linear mappings by allowing the Cauchy difference to be unbounded. During the last few decades, a number of papers and research monographs have been published on various generalizations and applications of generalized Hyers-Ulam-Rassias stability to a number of functional equations and mappings (see [2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 15, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 33]).

In 2009, S. M. Jung [32] investigated the Hyers-Ulam stability of Fibonacci functional equation. In 2011, Alvaro H. Salas [1] investigate about the k-Fibonacci number and their associated number. After that, M. Bidkhan and M. Hosslini [16] proved the stability of k-Fibonacci functional equation. Later on, M. Bidkhan et al. [17] succeeded to prove the Hyers-Ulam stability of (k, s)-Fibonacci functional equation. Furthermore, in 2012, M. Gordji, M. Naderi and Th. M. Rassias [22] et al. proved the stability of Tribonacci functional equation in non-Archimedean space and in 2014, M. E. Gordji, Ali Divandi, M. Rostannian, C. Park and D. Y. Sin [21] also proved the stability of Tribonacci functional equation in 2-normed space.

Recently, In 2014, M. N. Parizi et al. [23] and in 2015, Iz. El.-Fassi and S. Kabbaj [12] proved the stability of Fibonacci functional equation and orthogonal quadratic functional equation in Modular space respectively. In the first section of this paper, we denote by T_n the nth Tribonacci number where

$$T_n = T_{n-1} + T_{n-2} + T_{n-3} \quad \text{for } n = 3$$

with initial conditions $T_0 = 0$, $T_1 = 1$, $T_2 = 1$. From this, we may derive a functional equation

$$f(x) = f(x-1) + f(x-2) + f(x-3) \tag{1}$$

which is called the Tribonacci functional equation if a function $f : N \times R \to X$ satisfies the above equation for all $x \in R$. We denote the roots of equation $x^3 - x^2 - x - 1 = 0$ by p, q and r where q, r are complex, $|q| = |r|$ and p is greater than one. We obtain

$$p + q + r = 1, \quad pq + qr + pr = -1, \quad pqr = 1.$$

And in the second section, we denote by $F_{k,n}$ the nth k-Tribonacci number where

$$F_{k,n} = kF_{k,n-1} + F_{k,n-2} + F_{k,n-3} \quad \text{for } n = 3$$

with initial conditions $F_{k,0} = 0$, $F_{k,1} = 1$, $F_{k,2} = 1$. From this, we may derive a functional equation

$$f(k,x) = kf(k,x-1) + f(k,x-2) + f(k,x-3) \tag{2}$$

which if called the k-Tribonacci function equation if a function $f : N \times R \to X$ satisfies the above equation for all $x \in R$, $K \in N$, characteristic equation of the k-Tribonacci sequence is $x^3 - kx^2 - x - 1 = 0$, and p, q, r denote the roots of characteristic equation where p is greater than one and $q, r \in C$ and $|q| = |r|$. We know that $p + q + r = k$, $pq + qr + pr = -1$, $pqr = 1$. For each $x \in R$, $[x]$ stands for the largest integer that does not exceed x. Finally, we prove the Hyers-Ulam stability of functional equations (3.1) and (3.2) respectively in modular space.

2. Preliminaries

In this section, we recall some definitions, basic notions and facts about Modular space. As, we know $p + q + r = 1$, $pq + qr + pr = -1$ and $pqr = 1$.

Now it follows from that

$$\begin{aligned} &f(x) - p(f(x-1) - rf(x-2)) - rf(x-1) \\ &\quad = q[f(x-1) - (r+p)f(x-2) + prf(x-3)] \end{aligned}$$

for all $x = 0$. By mathematical induction, we verify that for all $x = 0$ and all m belonging to the set $\{0, 1, 2, \ldots\}$, we obtain,

$$\begin{aligned} &f(x) - p(f(x-1) - rf(x-2)) - rf(x-1) \\ &\quad = q^m[f(x-m) - rf(x-m-1) + prf(x-m-2) - pf(x-m-1)] \\ &f(x) - r[f(x-1) - qf(x-2)] - qf(x-1) \\ &\quad = p^m[f(x-m) - rf(x-m-1) + qrf(x-m-2) - qf(x-m-1)] \\ &f(x) - q[f(x-1) - pf(x-2)] - pf(x-1) \\ &\quad = r^m[f(x–m) - pf(x-m-1) + qpf(x-m-2) - qf(x-m-1)] \end{aligned}$$

for all $x = 0$ and all $m \in \{0, 1, 2, \ldots\}$.

And in the similar way we can define for equation .

Definition 2.1 ([12]). Let X be an arbitrary vector space.

(a) A functional $\rho : X \to [0, \infty]$ is called a modular if for arbitrary $x, y \in X$,

(i) $\rho(x) = 0$ if and only if $x = 0$,
(ii) $\rho(\alpha x) = \rho(x)$ for every scalar α with $|\alpha| = 1$,
(iii) $\rho(\alpha x + \beta y) = \rho(x) + \rho(y)$ if and only if $\alpha + \beta = 1$ and $\alpha, \beta = 0$,

(b) if (iii) is replaced by

(iii) $\rho(\alpha x + \beta y) = \alpha\rho(x) + \beta\rho(y)$ if and only if $\alpha + \beta = 1$ and $\alpha, \beta = 0$,

then we say that ρ is a convex modular.

(c) A modular ρ defines a corresponding modular space, i.e., the vector space X_ρ given by

$$X_\rho = \{x \in X : \rho(\lambda x) \to 0 \text{ as } \lambda \to 0\}$$

Definition 2.2 ([14]). Let ρ be a convex modular, the modular space X_ρ can be equipped with a norm called the Luxemburg norm, defined by

$$\|x\|_\rho = \inf \left\{ \lambda > 0 : \rho \frac{x}{\lambda} \leq 1 \right\}$$

A function modular is said to be satisfy the Δ_2-condition if there exit $k > 0$ such that $\rho(2x) \leq k\rho(x)$ for all $x \in X_\rho$.

Example 2.3 ([23]). Let $(X, \|\cdot\|)$ be a norm space, then $\|\cdot\|$ is a convex modular on X. But converse is not true.

In general the modular ρ does not behave as a norm or as a distance because it is not sub-additive. But one can associate to a modular the F-norm (see [4]).

Definition 2.4 ([12]). Let $\{x_n\}$ and x be in X_ρ. Then

(i) we say $\{x_n\}$ is a ρ-convegent to x and write $x_n \rho x$ if and only if $\rho(x_n - x) \to 0$ as $n \to 0$,

(ii) the sequence $\{x_n\}$, with $x_n \to X_\rho$, is called ρ-Cauchy if $\rho(x_n - x_m) \to 0$ as m, $n \to \infty$,

(iii) a subset S of X_ρ is called ρ-complete if and only if any ρ-Cauchy sequence is ρ-convergent to an element of S.

The modular ρ has the Fatou property if and only if any $\rho(x) \leq \lim_{n\to\infty} \inf \rho(x_n)$ whenever the sequence $\{x_n\}$ is ρ-convergent to x. For further details and proofs, we refer the reader to [14].

Remark 2.5 ([12]). If $x \in X_\rho$ then $\rho(ax)$ is a nondecreasing function of $a \geq 0$. Suppose that $0ab$, then property (iii) of Definition 2.1 with $y = 0$ shows that

$$\rho(ax) = \rho\left(\frac{a}{b}bx\right) \leq \rho(bx).$$

Moreover, if ρ is convex modular on X and $|\alpha| \leq 1$ then, $\rho(\alpha x) \leq |\alpha|\rho(x)$ and also $\rho(x) \leq \frac{1}{2}\rho(2x) \leq \frac{k}{2}\rho(x)$ if ρ satisfy the Δ_2-condition for all $x \in X$.

3. Main Results

3.1. Stability of Tribonacci Functional Equation in Modular Space

In the following theorem, we prove the Hyers-Ulam stability of the Tribonacci functional equation .

Theorem 3.1. *Let (X, ρ) be a Banach Modular space. If a function $f : R \to X$ satisfies the inequality*

$$\rho(f(x) - f(x-1) - f(x-2) - f(x-2)) = \epsilon \tag{3}$$

for all $x \in R$, and for some $\epsilon > 0$, then there exist a Tribonacci function $H : N \times R \to X$ such that

$$\rho(f(x)-H(x)) \leq \frac{2(1+|q|)+|q|^2}{||q^2(r-p)+r^2(p-q)+p^2(q-r)} \times \frac{\epsilon}{1-|q|^2}\,. \tag{4}$$

Proof. It follows from that

$$\begin{aligned}&\rho(f(x) - p\{f(x-1) - rf(x-2)\} - rf(x-1)\\ &\quad - q[f(x-1) - (r+p)f(x-2) + prf(x-3)]) \leq \epsilon\end{aligned}$$

If we replace x by $x - \alpha$ in the last inequality, then we get

$$\begin{aligned}&\rho(f(x-\alpha) - p\{f(x-\alpha-1) - rf(x-\alpha-2)\} - rf(x-\alpha-1)\\ &\quad - qpf(x-\alpha-1) - (r+p)f(x-\alpha-2) + prf(x-\alpha-3)) \leq \epsilon\end{aligned}$$

for all $x \in R$.

Now multiplying both sides by q^{α},

$$\begin{aligned}
&\rho(q^{\alpha}\{f(x-\alpha)-p[f(x-\alpha-1)-rf(x-\alpha-2)]-rf(x-\alpha-1)\} \\
&-q^{\alpha+1}\{f(x-\alpha-1)-(r+p)f(x-\alpha-2)+prf(x-\alpha-3)\}) \\
&\quad \leq |q^{\alpha}|\rho(\{f(x-\alpha)-p[f(x-\alpha-1)-rf(x-\alpha-2)]-rf(x-\alpha-1)\} \\
&\qquad -q^{\alpha+1}\{f(x-\alpha-1)-(r+p)f(x-\alpha-2)+prf(x-\alpha-3)\}) \\
&\quad \leq |q^{\alpha}|\epsilon \qquad (5)
\end{aligned}$$

for all $x \in R$ and $\alpha \in N$. Furthermore, we have

$$\begin{aligned}
&\rho(\{f(x) - p[f(x-1) - rf(x-2)] - rf(x-1)\} \\
&- q^{m}[f(x-m) - (r+p)f(x-m-1) + prf(x-m-2)]) \\
&\leq \rho\bigg(\sum_{\alpha=0}^{n-1}(q^{\alpha}[f(x-\alpha)-p\{f(x-\alpha-1)-rf(x-\alpha-2)\}-rf(x-\alpha-1)] \\
&\quad - q^{\alpha+1}[f(x-\alpha-1) - (r+p)f(x-\alpha-2) + prf(x-\alpha-3)])\bigg) \\
&\leq \sum_{\alpha=0}^{m-1}|q|^{\alpha}(\rho([f(x-\alpha)-p\{f(x-\alpha-1)-rf(x-\alpha-2)\}-rf(x-\alpha-1)] \\
&\quad - q[f(x-\alpha-1) - (r+p)f(x-\alpha-2) + prf(x-\alpha-3)])) \\
&\leq \sum_{\alpha=0}^{m-1}|q|^{\alpha} \\
&\leq \frac{\epsilon}{1-|q|} \qquad (6)
\end{aligned}$$

for all $x \in R$, $m \in N$.

Let $x \in R$ be fixed, than implies that $\{q^{m}[f(x-m) - p(f(x-m-1) - rf(x-m-2)) - rf(x-m-1))]$ is a cauchy sequence ($|q| < 1$). So by the completeness of X, we may define a function $H_1 : R \to X$ such that

$$H_1(x) = \lim_{m\to\infty} q^{m}[f(x-m)-(p+r)f(x-m-1) + prf(x-m-2)] \quad \text{for all } x \in R.$$

Applying the definition of H_1, we introduce the Tribonacci function

$H_1(x-1)+H_1(x-2)+H_1(x-3)$

$$\begin{aligned}
&= q^{-1} \lim_{m\to\infty} q^{m+1}[f(x-(m+1))-(p+r)f(x-(m+1)-1)+prf(x-(m+1)-2)] \\
&\quad +q^{-2} \lim_{m\to\infty} q^{m+2}[f(x-(m+2))-(p+r)f(x-(m+2)-1)+prf(x-(m+2)-2)] \\
&\quad +q^{-3} \lim_{m\to\infty} q^{m+3}[f(x-(m+3))-(p+r)f(x-(m+3)-1)+prf(x-(m+3)-2)] \\
&= q^{-1}H_1(x)+q^{-2}H_1(x)+q^{-3}H_1(x) \\
&= H_1(x) \quad \text{for all } x \in R.
\end{aligned}$$

Hence H_1 is a Tribonacci function.

If $m \to \infty$, then from , we obtain

$$\rho(f(x) - (p + r)f(x - 1) + prf(x - 2) - H_1) = \frac{1}{1 - |q|}\epsilon\,. \tag{7}$$

for all $x \in R$. Furthermore, it follows from that

$$\begin{aligned}
&\rho([f(x) - q\{f(x - 1) - pf(x - 2)\} - pf(x - 1)] \\
&\quad - r[f(x - 1) - pf(x - 2) + pqf(x - 3) - qf(x - 2)]) = \epsilon
\end{aligned}$$

for all $x \in R$. Now, we replace x by $x - \alpha$ in above inequality, we have

$$\begin{aligned}
&\rho([f(x-\alpha)-q(f(x-\alpha-1)-pf(x-\alpha-2))-pf(x-\alpha-1)] \\
&\quad -r[f(x-\alpha-1)-pf(x-\alpha-2)+pqf(x-\alpha-3)-qf(x-\alpha-2)]) = \epsilon
\end{aligned}$$

and now multiplying by r^α on both sides.

$$\begin{aligned}
&\rho(r^\alpha[f(x-\alpha)-q\{f(x-\alpha-1)-pf(x-\alpha-2)\}-pf(x-\alpha-1)] \\
&\quad -r^{\alpha+1}[f(x-\alpha-1)-pf(x-\alpha-2)+pqf(x-\alpha-3)-qf(x-\alpha-2)]) \\
&\\
&\le |r^\alpha|(\rho([f(x-\alpha)-q\{f(x-\alpha-1)-pf(x-\alpha-2)\}-pf(x-\alpha-1)] \\
&\quad -r^{\alpha+1}[f(x-\alpha-1)-pf(x-\alpha-2)+pqf(x-\alpha-3)-qf(x-\alpha-2)]) \\
&\le |r^\alpha|\epsilon
\end{aligned} \tag{8}$$

for all $x \in R$, $\alpha \in Z$. Now, we have

$$\begin{aligned}
&\rho([f(x)-q\{f(x-1)-pf(x-2)\}-pf(x-1)] \\
&\qquad -r^m[f(x-m)-(q+p)f(x-m-1)+pq(f(x-m-2)]) \\
&\le \rho\Bigg(\sum_{k=1}^{m} (r^\alpha[f(x-\alpha)-q\{f(x-\alpha-1)-pf(x-\alpha-2)\}-pf(x-\alpha-1)]
\end{aligned}$$

$$
\begin{aligned}
&\quad -r^{\alpha+1}[f(x-\alpha-1)-(p+q)f(x-\alpha-2)+pqf(x-\alpha-3)]) \Big) \\
&\le \sum_{k=1}^{m} |r|^{\alpha}(\rho([f(x-\alpha)-q\{f(x-\alpha-1)-pf(x-\alpha-2)\}-pf(x-\alpha-1)] \\
&\quad -r[f(x-\alpha-1)-(p+q)f(x-\alpha-2)+pqf(x-\alpha-3)])) \\
&\le \sum_{k=1}^{m} |r|^{\alpha} \\
&\le \frac{\epsilon}{1-|r|} \qquad (9)
\end{aligned}
$$

for all $x \in R$ and $m \in N$. We have

$$\{r^m[f(x-m)-(q+p)f(x-m-1)+pqf(x-m-2)]\}$$

is a cauchy sequence ($|r| < 1$) for all $x \in R$. Hence, we can define a function $H_2 : R \to X$ by

$$H_2(x) = \lim_{m\to\infty} r^m[f(x-m)-(q+p)f(x-m-1)+pqf(x-m-2)]$$

for all $x \in R$. Using the above definition of H_2, we have

$$
\begin{aligned}
&H_2(x-1)+H_2(x-2)+H_2(x-3) \\
&= r^{-1}\lim_{m\to\infty} r^{m+1}[f(x)-(m+1))-(q+p)f(x-(m+1)-1)+pqf(x-(m+1)-2)] \\
&\quad +r^{-2}\lim_{m\to\infty} r^{m+2}[f(x-(m+2))-(q+p)f(x-(m+2)-1)+pqf(x-(m+2)-2)] \\
&\quad +r^{-3}\lim_{m\to\infty} r^{m+3}[f(x-(m+3))-(q+p)f(x-(m+3)-1)+pqf(x-(m+3)-2)] \\
&= r^{-1}H_2(x)+r^{-2}H_2(x)+r^{-3}H_2(x) \\
&= H_2(x) \quad \text{for all } x \in R.
\end{aligned}
$$

So, we can say that H_2 is also a Tribonacci function. If m tends to ∞, then from , we have

$$
\begin{aligned}
\rho(f(x)-(q+p)f(x-1)+qpf(x-2)-H_2(x)) &= \frac{1}{1-|r|}\epsilon \\
&= \frac{1}{1-|q|}\epsilon. \qquad (10)
\end{aligned}
$$

for all $x \in R$. Finally, analogous to , we obtain

$$\rho([f(x)-r\{f(x-1)-qf(x-2)\}-qf(x-1)]$$

$$- p[f(x-1) - rf(x-2) + qrf(x-3) - qf(x-2)]) \leq \epsilon$$

for all $x \in R$.

Now we replace x by $x + \alpha$ in above inequality, that we have

$$\rho(f(x+\alpha) - r\{f(x+\alpha-1) - qf(x+\alpha-2)\} - qf(x+\alpha-1)$$
$$- p[f(x+\alpha-1) - (r+q)f(x-\alpha-2) + qrf(x+\alpha-3)]) \leq \epsilon$$

and

$$\begin{aligned} &\rho(p^{-\alpha}[f(x+\alpha) - r\{f(x+\alpha-1) - qf(x+\alpha-2)\} - qf(x+\alpha-1)] \\ &- p^{-\alpha+1}[f(x+\alpha-1) - (r+q)f(x-\alpha-2)) + qrf(x+\alpha-3)]) \\ &\quad \leq |\alpha^{-1}|^k \epsilon \end{aligned} \tag{11}$$

for all $x \in R$ and $\alpha \in Z$. Applying , we obtain that

$$\rho(p^{-m}[f(x+m) - r\{f(x+m-1) - qf(x+m-2)\}$$
$$- qf(x+m-1)] - [f(x) - (r+q)f(x-1) + rqf(x-2)])$$

$$\begin{aligned} &\leq \sum_{\alpha=1}^{m} \rho(p^{-\alpha}[f(x+\alpha) - r\{f(x+\alpha-1) - qf(x+\alpha-2)\} - qf(x+\alpha-1)] \\ &\quad - p^{-\alpha+1}[f(x+\alpha-1) - (r+q)f(x+\alpha-2) + qrf(x+\alpha-3)]) \\ &\leq \sum_{\alpha=1}^{m} p^{-\alpha}(\rho([f(x+\alpha) - r\{f(x+\alpha-1) - qf(x+\alpha-2)\} - qf(x+\alpha-1)] \\ &\quad - p[f(x+\alpha-1) - (r+q)f(x+\alpha-2) + qrf(x+\alpha-3)])) \\ &\leq \sum_{\alpha=1}^{m} p^{-\alpha} \epsilon \end{aligned} \tag{12}$$

for all $x \in R$, $m \in N$. We obviously have

$$\{p^{-m}[f(x+m) - (r+q)f(x+m-1) + qrf(x+m-2)]\}$$

is a cauchy sequence by definition of completeness for a fixed $x \in R$. Hence, we may define a function $H_3 : R \to X$ by

$$H_3(x) = \lim_{m \to \infty} p^{-m}[f(x+m) - (r+q)f(x+m+1) + qrf(x+m-2)]$$

for all $x \in R$. In view of above definition of H_3, we obtain

$$\begin{aligned}
&H_3(x-1)+H_3(x-2)+H_3(x-3)\\
&=p^{-1}\lim_{m\to\infty}p^{-(m-1)}[f(x+m-1)-(r+q)f(x+(m-1)-1)+qrf(x+(m-1)-2)]\\
&\quad+p^{-2}\lim_{m\to\infty}p^{-(m-2)}[f(x+m-2)-(r+q)f(x+(m-2)-1)+qrf(x+(m-2)-2)]\\
&\quad+p^{-3}\lim_{m\to\infty}p^{-(m-3)}[f(x+m-3)-(r+q)f(x+(m-3)-1)+qrf(x+(m-3)-2)]\\
&=p^{-1}H_3(x)+p^{-2}H_3(x)+p^{-3}H_3(x)\\
&=H_3(x)\quad \text{for all } x\in R.
\end{aligned}$$

Hence, we can say that H_3 is also a Tribonacci function. If we suppose, m tends to infinity in then we have

$$\rho(H_3(x)-f(x)+(r+q)f(x-1)-qrf(x-2))=\frac{\alpha^{-1}}{1-|\alpha^{-1}|}\epsilon \tag{13}$$

for all $x \in R$. From , and , we observe that

$$\begin{aligned}
&\rho\left(f(x)-\left[\frac{q^2(r-p)H_1(x)+r^2(p-q)H_2(x)-p^2(q-r)H_3(x)}{q^2(r-p)+r^2(p-q)+p^2(q-r)}\right]\right)\\
&=\frac{1}{|q^2(r-p)+r^2(p-q)+p^2(q-r)|}\rho(\{q^2(r-p)+r^2(p-q)+p^2(q-r)\}\\
&\qquad\times\ f(x)-q^2(r-p)H_1(x)-r^2(p-q)H_2(x)+p^2(q-r)H_3(x))
\end{aligned}$$

Now, we assume that

$$\begin{aligned}
&\frac{1}{|q^2(r-p)+r^2(p-q)+p^2(q-r)|}=\frac{1}{|A|} \qquad (14)\\
&\le\frac{1}{|A|}\rho([q^2(r-p)f(x)-q^2(r^2-p^2)f(x-1)+q^2(r-p)prf(x-2)-q^2(r-p)H_1(x)]\\
&\quad+[r^2(p-q)f(x)-r^2(p^2-q^2)f(x-1)+r^2(p-q)qpf(x-2)-r^2(p-q)H_2(x)]\\
&\quad+[p^2(q-r)f(x)-p^2(q^2-r^2)f(x-1)+p^2(q-r)qrf(x-2)-p^2(q-r)H_3(x)])\\
&\le\frac{1}{|A|}\left[\frac{1}{1-|q|}+\frac{1}{1-|q|}+\frac{|q^2|}{1-|q^2|}\right]\epsilon\\
&=\frac{1}{|A|}\left[\frac{2}{1-|q|}+\frac{|q^2|}{1-|q^2|}\right]\epsilon\\
&=\frac{1}{|A|}\left[\frac{2(1+|q|)+|q|^2}{1-|q^2|}\right]\epsilon
\end{aligned}$$

Putting the value of $|A|$ from we get the required result.

Hence,

$$H(x) = \frac{q^2(r-p)H_1(x) + r^2(p-q)H_2(x) - p^2(q-r)H_3(x)}{q^2(r-p) + r^2(p-q) + p^2(q-r)}$$

for all $x \in R$. It is not difficult to show that H is a Tribonacci function satisfying .

3.2. Stability of k-Tribonacci Functional Equation in Modular Space

Throughout the following theorem, we prove the Hyers-Ulam stability of the k-Tribonacci functional equation .

Theorem 3.2. *Let (X, ρ) be a Banach modular space. If a function $f : R \to X$ satisfies the inequality*

$$\rho(f(k,x) - kf(k,x-1) - f(k,x-2) - f(k,x-2)) = \epsilon \tag{15}$$

for all $x \in R$, $k \in N$ and for some $\epsilon > 0$, then there exist a k-Tribonacci function $H : N \times R \to X$ such that

$$\begin{aligned} &\rho(f(k,x) - H(k,x)) \\ &\quad = \frac{2(1+|q|) + |q|^2}{||q^2(r-p) + r^2(p-q) + p^2(q-r)} \times \frac{\epsilon}{1-|q|^2}. \end{aligned} \tag{16}$$

Proof. Since, $p + q + r = k$, $pq + qr + pr = -1$ and $pqr = 1$. So from , we obtain

$$\begin{aligned} &\rho(f(k,x) - (p+q+r)f(k,x-1) + (pq+qr+pr)f(k,x-2) \\ &\quad - pqrf(k,x-3)) = \epsilon. \end{aligned} \tag{17}$$

for all $x \in R$, $k \in N$. Now it follows from that

$$\begin{aligned} &f(k,x) - p(f(k,x-1) - rf(k,x-2)) - rf(k,x-1) \\ &\quad - q[f(k,x-1) - (r+p)f(k,x-2) + prf(k,x-3)] = \epsilon \end{aligned} \tag{18}$$

for all $k \in N$, $x \geq 0$.

If we replace x by $x - \alpha$ in inequality then we get

$$\begin{aligned} &\rho(f(k,x-\alpha) - p[f(k,x-\alpha-1) - rf(k,x-\alpha-2)] \\ &\quad - rf(k,x-\alpha-1) - qpf(K,x-\alpha-1) - (r+p)f(k,x-\alpha-2) \end{aligned}$$

$$+\,prf(k,x-\alpha-3)) = \epsilon$$

for all $x \in R$, $k \in N$.

Now multiplying both sides by q^{α},

$$\begin{aligned}
&\rho(q^{\alpha}[f(k,x-\alpha)-p\{f(k,x-\alpha-1)-rf(k,x-\alpha-2)\}-rf(k,x-\alpha-1)] \\
&-q^{\alpha+1}[f(k,x-\alpha-1)-(r+p)f(k,x-\alpha-2)+prf(k,x-\alpha-3)]) \\
&\quad \le |q^{\alpha}|\rho([f(k,x-\alpha)-p\{f(k,x-\alpha-1)-rf(k,x-\alpha-2)\}-rf(k,x-\alpha-1)] \\
&-q[f(k,x-\alpha-1)-(r+p)f(k,x-\alpha-2)+prf(k,x-\alpha-3)]) \\
&\quad \le |q^{\alpha}|\epsilon
\end{aligned} \tag{19}$$

for all $x \in R$, $k \in N$. Furthermore, we have

$$\begin{aligned}
&\rho(f(k,x)-p\{f(k,x-1)-rf(k,x-2)\}-rf(k,x-1) \\
&-q^{m}[f(k,x-m)-(r+p)f(k,x-m-1)+prf(k,x-m-2)])
\end{aligned}$$

$$\begin{aligned}
&\le \rho\Bigg(\sum_{\alpha=0}^{m-1} q^{\alpha}[f(k,x-\alpha)-p\{f(k,x-\alpha-1)-rf(k,x-\alpha-2)\}-rf(k,x-\alpha-1)] \\
&\quad -q^{\alpha+1}[f(k,x-\alpha-1)-(r+p)f(k,x-\alpha-2)+prf(k,x-\alpha-3)]\Bigg) \\
&\le \sum_{\alpha=0}^{m-1} |q|^{\alpha}\rho([f(k,x-\alpha)-p\{f(k,x-\alpha-1)-rf(k,x-\alpha-2)\}-rf(k,x-\alpha-1)] \\
&\quad -q[f(k,x-\alpha-1)-(r+p)f(k,x-\alpha-2)+prf(k,x-\alpha-3)]) \\
&\le \sum_{\alpha=0}^{m-1} |q|^{\alpha}\epsilon \\
&\le \frac{\epsilon}{1-|q|}
\end{aligned} \tag{20}$$

for all $x \in R$, $m \in N$, $k \in N$.

Let $x \in R$ be fixed, than implies that $\{q^{m}[f(k,x-m)-p(f(k,x-m-1)-rf(k,x-m-2))-rf(k,x-m-1))]$ is a cauchy sequence ($|q|<1$). So by the completeness of X, we may define a function $H_1 : R \to X$ such that

$$\begin{aligned}
H_1(k,x) = \lim_{m\to\infty} q^{m}[&f(k,x-m)-(p+r)f(k,x-m-1) \\
&+prf(k,x-m-2)] \quad \text{for all } x \in R,\ k \in N.
\end{aligned}$$

Applying the definition of H_1, we introduce the k-Tribonacci function

$$\begin{aligned}
&kH_1(k,x-1) + H_1(k,x-2) + H_1(k,x-3)\\
&= kq^{-1} \lim_{m\to\infty} q^{m+1}[f(k,x-(m+1))\\
&\qquad -(p+r)f(k,x-(m+1)-1) + prf(k,x-(m+1)-2)]\\
&\quad + q^{-2} \lim_{m\to\infty} q^{m+2}[f(k,x-(m+2))\\
&\qquad -(p+r)f(k,x-(m+2)-1) + prf(k,x-(m+2)-2)]\\
&\quad + q^{-3} \lim_{m\to\infty} q^{m+3}[f(k,x-(m+3))\\
&\qquad -(p+r)f(k,x-(m+3)-1) + prf(k,x-(m+3)-2)]
\end{aligned}$$

$$\begin{aligned}
&= kq^{-1}H_1(k,x) + q^{-2}H_1(k,x) + q^{-3}H_1(k,x)\\
&= H_1(k,x) \quad \text{for all } x \in R,\ k \in N.
\end{aligned}$$

Hence H_1 is a k-Tribonacci function.

If $m \to \infty$, then from we obtain

$$\rho(f(k,x) - (p+r)f(k,x-1) + prf(k,x-2) - H_1(k,x)) \tag{21}$$

$$\leq \frac{1}{1-|q|}\epsilon \tag{22}$$

for all $x \in R$, $k \in N$. Furthermore, it follows from that

$$\begin{aligned}
&\rho(f(k,x)-q(f(k,x-1)-pf(k,x-2))-pf(k,x-1)\\
&\quad -r[f(k,x-1)-pf(k,x-2) + pqf(k,x-3)-qf(k,x-2)]) = \epsilon
\end{aligned}$$

for all $x \in R$, $k \in N$. Now, we replace x by $x-\alpha$ in above inequality, we have

$$\begin{aligned}
&\rho(f(k,x-\alpha)-q(f(k,x-\alpha-1)-pf(k,x-\alpha-2))-pf(k,x-\alpha-1)\\
&\quad -r[f(k,x-\alpha-1)-pf(k,x-\alpha-2) + pqf(k,x-\alpha-3)-qf(k,x-\alpha-2)]) = \epsilon
\end{aligned}$$

and now multiplying by r^α on both sides.

$$\begin{aligned}
&\rho(r^\alpha[f(k,x-\alpha)-q(f(k,x-\alpha-1)-pf(k,x-\alpha-2))-pf(k,x-\alpha-1)]\\
&\quad -r^{\alpha+1}[f(K,x-\alpha-1)-pf(K,x-\alpha-2) + pqf(k,x-\alpha-3)-qf(k,x-\alpha-2)])\\
&\leq |r^\alpha|\rho([f(k,x-\alpha)-q(f(k,x-\alpha-1)-pf(k,x-\alpha-2))-pf(k,x-\alpha-1)]\\
&\quad -r[f(K,x-\alpha-1)-pf(K,x-\alpha-2) + pqf(k,x-\alpha-3)-qf(k,x-\alpha-2)])\\
&\leq |r^\alpha|\epsilon
\end{aligned} \tag{23}$$

for all $x \in R$, $\alpha \in Z$. Now, we have

$$
\begin{aligned}
&\rho(f(k,x)-q\{f(k,x-1)-pf(k,x-2)\}-pf(k,x-1)\\
&\quad -r^m[f(k,x-m)-(q+p)f(k,x-m-1)+pq(f(k,x-m-2)])\\
&\le \rho\Bigg(\sum_{\alpha=0}^{m-1} r^\alpha[f(k,x-\alpha)-q\{f(k,x-\alpha-1)-pf(k,x-\alpha-2)\}-pf(k,x-\alpha-1)]\\
&\quad -r^{\alpha+1}[f(k,x-\alpha-1)-(p+q)f(k,x-\alpha-2)+pqf(k,x-\alpha-3)]\Bigg)\\
&\le \sum_{\alpha=0}^{m-1} |r|^\alpha \rho([f(k,x-\alpha)-q\{f(k,x-\alpha-1)-pf(k,x-\alpha-2)\}-pf(k,x-\alpha-1)]\\
&\quad -r[f(k,x-\alpha-1)-(p+q)f(k,x-\alpha-2)+pqf(k,x-\alpha-3)])\\
&\le \sum_{\alpha=0}^{m-1} |r|^\alpha \epsilon\\
&\le \frac{\epsilon}{1-|r|}
\end{aligned}
\tag{24}
$$

for all $x \in R$ and $m \in N$. We have

$$\{r^m[f(k,x-m)-(q+p)f(k,x-m-1)+pqf(k,x-m-2)]\}$$

is a cauchy sequence ($|r| < 1$) for all $x \in R$. Hence, we can define a function $H_2 : R \to X$ by

$$
\begin{aligned}
&H_2(k,x)\\
&= \lim_{m\to\infty} r^m[f(k,x-m)-(q+p)f(k,x-m-1)+pqf(k,x-m-2)]
\end{aligned}
$$

for all $x \in R$. Using the above definition of H_2, we have

$$
\begin{aligned}
&kH_2(k,x-1)+H_2(k,x-2)+H_2(k,x-3)\\
&= kr^{-1} \lim_{m\to\infty} r^{m+1}[f(k,x-(m+1))\\
&\qquad -(q+p)f(k,x-(m+1)-1)+pqf(k,x-(m+1)-2)]\\
&\quad +r^{-2} \lim_{m\to\infty} r^{m+2}[f(k,x-(m+2))\\
&\qquad -(q+p)f(k,x-(m+2)-1)+pqf(k,x-(m+2)-2)]\\
&\quad +r^{-3} \lim_{m\to\infty} r^{m+3}[f(k,x-(m+3))
\end{aligned}
$$

$$-(q+p)f(k,x-(m+3)-1)+pqf(k,x-(m+3)-2)]$$
$$= kr^{-1}H_2(k,x)+r^{-2}H_2(k,x)+r^{-3}H_2(k,x)$$
$$= H_2(k,x) \quad \text{for all } x \in R.$$

So, we can say that H_2 is also a k-Tribonacci function. If m tends to ∞, then from , we have

$$\rho(f(k,x)-(q+p)f(k,x-1)+qpf(k,x-2)-H_2(k,x)) \\ \leq \frac{1}{1-|r|}\epsilon \leq \frac{1}{1-|q|}\epsilon. \tag{25}$$

for all $x \in R$. Finally, analogous to , we obtain

$$\rho(f(k,x)-r\{f(k,x-1)-qf(k,x-2)\}-qf(k,x-1) \\ -p[f(k,x-1)-rf(k,x-2)+qrf(k,x-3)-qf(k,x-2)]) = \epsilon$$

for all $x \in R$. Now we replace x by $x+\alpha$ in above inequality, that we have

$$\rho(f(k,x+\alpha)-r\{f(k,x+\alpha-1)-qf(k,x+\alpha-2)\}-qf(k,x+\alpha-1) \\ -p[f(k,x+\alpha-1)-(r+q)f(k,x-\alpha-2)+qrf(k,x+\alpha-3)]) \leq \epsilon$$

and

$$\rho(p^{-\alpha}[f(k,x+\alpha)-r(f(k,x+\alpha-1)-qf(k,x+\alpha-2))-qf(k,x+\alpha-1)] \\ -p^{-\alpha+1}[f(k,x+\alpha-1)-(r+q)f(k,x-\alpha-2))+qrf(k,x+\alpha-3)]) \\ = |\alpha^{-1}|^k\epsilon \tag{26}$$

for all $x \in R$ and $\alpha \in Z$. Applying , we obtain that

$$\rho(p^{-m}[f(k,x+m)-r(f(k,x+m-1)-qf(k,x+m-2)) \\ -qf(k,x+m-1)]-[f(k,x)-(r+q)f(k,x-1)+rqf(k,x-2)]) \\ \leq \sum_{\alpha=1}^{m}\rho(p^{-\alpha}[f(k,x+\alpha)-r(f(k,x+\alpha-1)-qf(k,x+\alpha-2))-qf(k,x+\alpha-1)] \\ -p^{-\alpha+1}[f(k,x+\alpha-1)-(r+q)f(k,x+\alpha-2)+qrf(k,x+\alpha-3)]) \\ \leq \sum_{\alpha=1}^{m}p^{-\alpha}\rho([f(k,x+\alpha)-r(f(k,x+\alpha-1)-qf(k,x+\alpha-2))-qf(k,x+\alpha-1)] \\ -p^{-\alpha+1}[f(k,x+\alpha-1)-(r+q)f(k,x+\alpha-2)+qrf(k,x+\alpha-3)])$$

$$\leq \sum_{\alpha=1}^{m} p^{-\alpha}\epsilon \tag{27}$$

for all $x \in R$, $m \in N$. By using we see that

$$\{p^{-m}[f(k,x+m) - (r+q)f(k,x+m-1) + qrf(k,x+m-2)]\}$$

is a cauchy sequence by definition of completeness for a fixed $x \in R$. Hence, we may define a function $H_3 : R \to X$ by

$$H_3(k,x) = \lim_{m\to\infty} p^{-m}[f(k,x+m) - (r+q)f(k,x+m+1) + qrf(k,x+m-2)]$$

for all $x \in R$. In view of above definition of H_3, we obtain

$$\begin{aligned}
&kH_3(k,x-1)+H_3(k,x-2)+H_3(k,x-3)\\
&= kp^{-1}\lim_{m\to\infty} p^{-(m-1)}[f(k,x+m-1)\\
&\qquad -(r+q)f(k,x+(m-1)-1)+qrf(k,x+(m-1)-2)]\\
&\quad +p^{-2}\lim_{m\to\infty} p^{-(m-2)}[f(k,x+m-2)\\
&\qquad -(r+q)f(k,x+(m-2)-1)+qrf(k,x+(m-2)-2)]\\
&\quad +p^{-3}\lim_{m\to\infty} p^{-(m-3)}[f(k,x+m-3)\\
&\qquad -(r+q)f(k,x+(m-3)-1)+qrf(k,x+(m-3)-2)]\\
&= kp^{-1}H_3(k,x)+p^{-2}H_3(k,x)+p^{-3}H_3(k,x)\\
&= H_3(k,x) \quad \text{for all } x \in R,\ k \in N.
\end{aligned}$$

Hence, we can say that H_3 is also a k-Tribonacci function. If we suppose, m tends to infinity in then we have

$$\begin{aligned}
&\rho(H_3(k,x) - f(k,x) + (r+q)f(k,x-1) - qrf(k,x-2))\\
&\quad \leq \frac{\alpha^{-1}}{1-|\alpha^{-1}|}\epsilon
\end{aligned} \tag{28}$$

for all $x \in R$. From (22), (25) and (28), we observe that

$$\begin{aligned}
&\rho\left(f(k,x) - \left[\frac{q^2(r-p)H_1(k,x) + r^2(p-q)H_2(k,x) - p^2(q-r)H_3(k,x)}{q^2(r-p) + r^2(p-q) + p^2(q-r)}\right]\right)\\
&= \frac{1}{|q^2(r-p) + r^2(p-q) + p^2(q-r)|}\\
&\qquad \times \rho((q^2(r-p) + r^2(p-q) + p^2(q-r)f(k,x) - q^2(r-p)H_1(k,x)
\end{aligned}$$

$$- r^2(p-q)H_2(k,x) + p^2(q-r)H_3(k,x))$$

For convince, we assume that

$$\frac{1}{|q^2(r-p)+r^2(p-q)+p^2(q-r)|} = \frac{1}{|A|} \tag{29}$$

$$\begin{aligned}
&\leq \frac{1}{|A|}\rho[(q^2(r-p)\,f(k,x)-q^2(r^2-p^2)\,f(k,x-1)\\
&\qquad +q^2(r-p)pr\,f(k,x-2)-q^2(r-p)H_1(k,x))
\end{aligned}$$

$$\begin{aligned}
&+(r^2(p-q)\,f(k,x)-r^2(p^2-q^2)\,f(k,x-1)+r^2(p-q)qp\,f(k,x-2)\\
&\qquad -r^2(p-q)H_2(k,x))\\
&+(p^2(q-r)f(k,x)-p^2(q^2-r^2)f(k,x-1)+p^2(q-r)qr\,f(k,x-2)\\
&\qquad -p^2(q-r)H_3(k,x)|)]\\
&\leq \frac{1}{|A|}\left[\frac{1}{1-|q|}+\frac{1}{1-|q|}+\frac{|q^2|}{1-|q^2|}\right]\epsilon\\
&= \frac{1}{|A|}\left[\frac{2}{1-|q|}+\frac{|q^2|}{1-|q^2|}\right]\epsilon\\
&= \frac{1}{|A|}\left[\frac{2(1+|q|)+|q|^2}{1-|q^2|}\right]\epsilon
\end{aligned}$$

Putting the value of $|A|$ from we get the required result.

Hence,

$$H(k,x) = \frac{q^2(r-p)H_1(k,x) + r^2(p-q)H_2(k,x) - p^2(q-r)H_3(k,x)}{q^2(r-p) + r^2(p-q) + p^2(q-r)}$$

for all $x \in R$. It is easy to show that H is a k-Tribonacci function satisfying .

References

[1] A. H. Sales, About k-Fibonacci numbers and their associated numbers, *Int. J. of Math Forum*, **6** (50) (2011), 2473-2479.

[2] Ashish, R. Chugh, Hyers-Ulam-Rassias stability of orthogonality cubic and quadratic functional equations, *Int. J. of Pure and Applied Mathematics*, **81** (1) (2012), 9-20.

[3] Ashish, R. Chugh, Manoj Kumar, Hyers-Ulam-Rassias stability of functional equations in multi-Banch space, *Int. J. of Pure and Applied Mathematics*, **86** (4) (2013), 621-631.

[4] C. Park, S. Y. Jang, R. Saadati, Fuzzy approximate of homomorphisms, *J. Comp. Anal. Appl.*, **14** (5) (2012), 833-841.

[5] D. H. Hyers, On the stability if linear functional equation, *Proc. Natl. Acad. Sci. USA.*, **27** (1941), 221-224.

[6] D. H. Hyers, G. Isac, Th. M Rassias, *Stability of Functional Equations in Several Variables*, Birkhauser, Boston (1998).

[7] D. H. Hyers, Th. M. Rassias, Approximate homomorphisms, *Aequationes Math.*, **44** (1992), 125-153.

[8] G. L. Forti, Hyers-Ulam stability of functional equations in several variables, *Aequationes Math.*, **50** (1995), 143-190.

[9] G. Z. Eskandani, On the Hyers-Ulam-Rassias stability of an additive functional equation in quasi-Banach spaces, *J. Math. Anal. Appl.*, **345** (2008), 405-409.

[10] G. Z. Eskandani, Th. M proper JCQ*triples, *Mediterr. J. Math.*, **10** (2013), 1391-1400.

[11] G. Z. Zamani, H. Vaezi, Fuzzy approximation of an additive functional equation, *J. Funct. Spaces Appl.*, **9** (2) (2011), 205-215.

[12] Iz. El-Fassi, S. Kabbaj, Hyers-Ulam-Rassias stability of orthogonal quadratic functional equations in modular spaces, ., **26** (1) (2015), 61-75.

[13] J. M. Rassias, On approximately. Rassias: Hyers-Ulam-Rassias stability of derivations in of approximately linear mappings by linear mappings, *J. Functional Anal. USA*, **46** (1982), 126-130.

[14] J. Musielak, Orlicz spaces and modular spaces, *Lecture Notes in Math.*, Springer-Verlag, Berlin, (1034) (1983).

[15] K. Jun, Y. Lee, On the Hyers-Ulam-Rassias stability of a Pexiderized quadratic inequality, *Math. Inequal. Appl.*, **4** (2001), 93-118.

[16] M. Bidkham, M. Hosseini, Hyers-Ulam stability of k-Fibonacci functional equation, *Int. J. Nonlinear Anal. Appl.*, **2** (2011), 42-49.

[17] M. Bidkham, M. Hosseini, C. Park, M. Eshaghi Gordji, Nearly (k, s)-Fibonacci functional equations in β-normed spaces, *Aequationes Math.*, **83** (2012), 131-141.

[18] M. Kumar, Ashish, R. Chugh, The stability of cubic functional equations in 2-Banach space, *Applied Mathematical Sciences*, **7** (60) (2013), 2967-2977.

[19] M. Kumar, Renu Chugh, Ashish, Hyers-Ulam-Rassias stability of quadratic functional equations in 2-Banach spaces, *Int. J. of Comp. Appl.*, **63** (8) (2013), 1-4.

[20] M. Kumar, Ashish, Stability of functional equations in multi-Banach space via fixed point approach, *Int. J. of Comp. Appl.*, **44** (7) (2012), 35-40.

[21] M. Eshaghi Gordji, A. Divandari, C. Park, D. Y. Shin, Hyers-Ulam stability of a Tribonacci functional equation in 2-normed space, *J. Comp. Anal. and Appl.*, **6** (3) (2014), 503-508.

[22] M. Gordji, M. Naderi, Th. M. Rassias: Solution and stability of tribonacci functional equation in non-archinedean Banach spaces, *?*, (2011), 67-74.

[23] M. N. Parizi, M. E. Gordji, Hyers-Ulam stability of functional equations in modular functional spaces, J. of Math. and Comp. Sci., **10** (2014), 1-6.

[24] M. S. Moslehian, Th. M. Rassias: Stability of functional equation in Non-Archimedean spaces, *Applicable Analysis and Discrete Mathe.*, **1** (2007), 325-334.

[25] P. Gavruta, A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings, *J. Math. Anal. Appl.*, **184** (1994), 431-436.

[26] P. Gavruta, L. Gavruta, A new method for the generalized Hyers-Ulam-Rassias stability, *Int. J. Nonlinear Anal. Appl.*, **1** (2010), 11-18.

[27] R. Ger and P. Semrl, The stability of exponential equation, *Proc. Amer. Math. Soc.*, **124** (1996), 779-787.

[28] R. Chugh, Ashish, On the stability of functional equations in Random Normed spaces, *J. Comp. Anal. Appl.*, **45** (11) (2012), 25-34.

[29] R. Chugh, Ashish, The stability of pexiderized cauchy functional equation, *Int. J. of Pure and Applied Mathematics*, **77** (5) (2012), 649-666.

[30] R. Chugh and Ashish, On the stability of generalized Cauchy linear functional equation, Int. J. of Mathematical Analysis, **6**, No.29 (2012), 1403-1413.

[31] S. Czerwik, *Functional Equations and Inequalities in Servel Variables*, World Scientific, New Jersey, London, Singapore, Hong Kong (2002).

[32] S. Jung, Hyers-Ulam stability of Fibonacci functional equation, *Bull. Iranian Math. Soc.*, **35** (2009), 217-227.

[33] S. Jung, Hyers-Ulam-Rassias stability of functional equations in nonlinear analysis, *Springer Optimization and Its Applications*, **48**, Springer, New York (2011).

[34] S. M. Ulam, *Problems in Modern Mathematics*, Chapter VI, Science ed., Wiley, New York (1940).

[35] T. Aoki, On the stability of linear transformation in Banach spaces, *J. Math. Soc. Jpn.*, **2** (1950), 64-66.

[36] Th. M. Rassias, On the stability ofthe linear mapping in Banach spaces, *Proc. Amer. Math. Soc.*, **72** (1978), 297-300.