International Journal of Pure and Applied Mathematics
Volume 104 No. 2 2015, 181-191
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v104i2.3
ijpam.eu

OSCILLATION OF SECOND ORDER NEUTRAL DIFFERENTIAL EQUATIONS WITH MIXED NEUTRAL TERM

R. Arul[1 §], V.S. Shobha[2]
[1,2]Department of Mathematics
Kandaswami Kandar's College
Velur – 638 182, Namakkal Dt.
Tamil Nadu, INDIA

Abstract: This paper deals with the following second order neutral differential equation of the form

$$(r(t)z'(t))' + q(t)x(\sigma(t)) = 0, \quad t \geq t_0 \geq 0$$

where $z(t) = x(t) + a(t)x(t-\tau) + b(t)x(t+\delta)$, and $\int_{t_0}^{\infty} \frac{1}{r(t)} dt = \infty$. We obtain some new oscillation criteria which extend some known results. Examples are presented to illustrate the main results.

AMS Subject Classification: 34K11
Key Words: second order, neutral differential equation, mixed neutral term, oscillation

1. Introduction

In this paper, we study the oscillatory behavior of solution of the second order neutral differential equation of the form

$$(r(t)z'(t))' + q(t)x(\sigma(t)) = 0, \quad t \geq t_0 \geq 0 \tag{1.1}$$

where $z(t) = x(t) + a(t)x(t-\tau) + b(t)x(t+\delta)$. In what follows we assume that:

Received: April 22, 2015

[§]Correspondence author

(A_1) $a, b, q \in C([t_0, \infty), \mathbb{R})$, $0 \leq a(t) \leq a < \infty$, $0 \leq b(t) \leq b < \infty$, and $q(t) > 0$;

(A_2) $r \in C([t_0, \infty), \mathbb{R})$, $r(t) > 0$, and $\int_{t_0}^{\infty} \frac{1}{r(t)} dt = \infty$;

(A_3) τ, δ are nonnegative constants, $\sigma \in C([t_0, \infty), \mathbb{R})$, $\sigma'(t) > 0$, $\lim_{t \to \infty} \sigma(t) = \infty$, and $\sigma(t \pm \delta) = \sigma(t) \pm \delta$.

By a solution of equation (1.1), we mean a continuous function defined on an interval $[t_x, \infty)$ such that $r(t)z'(t)$ is continuously differentiable and x satisfies equation (1.1) for all $t \in [t_x, \infty)$. We consider only solutions satisfying condition $\sup\{|x(t)| : t \geq T \geq t_x\} > 0$, and tacitly assume that equation (1.1) possess such solutions. As usual, a solution of equation (1.1) is called oscillatory if it is neither eventually positive nor eventually negative; otherwise we call it nonosicllatory.

From the literature, it is known that several classes of neutral type differential equations are often encountered in applied problems in natural sciences and engineering, see for example [3] and [11]. Recently, a great deal interest in studying the oscillatory properties of neutral type functional differential equations, see for example [1, 2, 5, 6, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] and the references cited therein.

In [9], the authors established oscillation criteria for the equation (1.1) when $r(t) \equiv 1$, $b(t) = 0$, $0 \leq a(t) \leq 1$ and $q(t) \geq 0$. In [18], the author, by employing Riccati technique and averaging function method, established some general oscillation criteria for the equation (1.1) when $b(t) \equiv 0$ and $\sigma(t) = t - \sigma$.

In [20], the authors studied the oscillatory behavior of equation (1.1) when $b(t) \equiv 0$. From the review of literature it is known that very few results are available in the literature when the neutral term contain both delay and advanced argument, see for example [4, 7, 8] . This and the technique used in [16] motivated the investigation and present some general new oscillation criteria for equation (1.1) by employing a class of function Y the operator T, and the Riccati technique.

Following [16] , we say that a function $\phi = \phi(t, s, \ell)$ belongs to the function class Y, denoted by $\phi \in Y$ if $\phi \in C(E, \mathbb{R})$, where $E = \{(t, s, \ell) : t_0 \leq \ell \leq s \leq t < \infty\}$, which satisfies $\phi(t, t, \ell) = 0$, $\phi(t, \ell, \ell) = 0$ and $\phi(t, s, \ell) > 0$ for $\ell < s < t$, and has the partial derivative $\frac{\partial \phi}{\partial s}$ on E such that $\frac{\partial \phi}{\partial s}$ is locally integrable with respect to s in E. By choosing the special function ϕ, it is possible to derive several oscillation criteria for a wide range of differential equations.

Define the operator T by

$$T[g;\ell,t] = \int_{\ell}^{t} \phi(t,s,\ell)g(s)ds, \tag{1.2}$$

for $t \geq s \geq \ell \geq t_0$ and $g \in C'[t_0,\infty)$. The function $\psi = \psi(t,s,\ell)$ is defined by

$$\frac{\partial \psi}{\partial s}(t,s,\ell) = \psi(t,s,\ell)\phi(t,s,\ell) \tag{1.3}$$

then, it is easy to see that T is a linear operator and

$$T[g';\ell,t] = -T[g\psi;\ell,t] \quad \text{for} \quad g \in C'[t_0,\infty). \tag{1.4}$$

2. Oscillation Results

In this section, we obtain some new oscillation criteria for the equation (1.1). We begin with the following theorem.

Theorem 2.1. *If*

$$\int_{t_0}^{\infty} Q(t)dt = \infty \tag{2.1}$$

where $Q(t) = min\{q(t), q(t-\tau), q(t+\delta)\}$, *then every solution of equation* (1.1) *is oscillatory.*

Proof. Let $x(t)$ be a nonsocillatory solution of equation (1.1) for $t \geq t_0$. Then there exists a $t_1 \geq t_0$ such that $x(t) \neq 0$ for all $t \geq t_1$. Without loss of generality, we may assume that $x(t) > 0$, $x(t-\tau) > 0$, $x(t+\delta) > 0$ and $x(\sigma(t)) > 0$ for all $t \geq t_1$. From equation (1.1), we have

$$(r(t)z'(t))' = -q(t)x(\sigma(t)) < 0, \quad t \geq t_1. \tag{2.2}$$

Therefore $r(t)z'(t)$ is a decreasing function. We claim that $z'(t) > 0$ for $t \geq t_1$. Otherwise, there exists a $t_2 \geq t_1$ such that $z'(t_2) < 0$. Then

$$r(t)z'(t) \leq r(t_2)z'(t_2), \quad t \geq t_2 \tag{2.3}$$

and hence

$$z(t) \leq z(t_2) + r(t_2)z'(t_2) \int_{t_2}^{t} \frac{ds}{r(s)}. \tag{2.4}$$

Let $t \to \infty$ in (2.4), we obtain $z(t) \to -\infty$ as $t \to \infty$. This implies that $z'(t) > 0$ for all $t \geq t_1$. From the definition of $z(t)$ and equation (1.1), we have

$$\begin{aligned}(r(t)z'(t))' + q(t)x(\sigma(t)) + a(r(t-\tau)z'(t-\tau))'\\ + aq(t-\tau)x(\sigma(t-\tau)) + b(r(t+\delta)z'(t+\delta))'\\ + bq(t+\delta)x(\sigma(t+\delta)) = 0,\end{aligned} \tag{2.5}$$

and thus

$$\begin{aligned}(r(t)z'(t))' + a(r(t-\tau)z'(t-\tau))' + b(r(t+\delta)z'(t+\delta))'\\ + Q(t)z(\sigma(t)) \leq 0, \quad t \geq t_1.\end{aligned} \tag{2.6}$$

Integrating (2.6) from t_1 to t, we obtain

$$\begin{aligned}\int_{t_1}^{t} Q(s)z(\sigma(s))ds \;\; \leq \;\; & r(t_1)z'(t_1) - r(t)z'(t) + r(t_1-\tau)z'(t_1-\tau)\\ & - r(t-\tau)z'(t-\tau) + r(t_1+\delta)z'(t+\delta)\\ & - r(t+\delta)z'(t+\delta).\end{aligned} \tag{2.7}$$

Since $z'(t) > 0$ for $t \geq t_1$. We can find a constant $c > 0$ such that $z(t) \geq c > 0$ for all $t \geq t_1$. Then from (2.7), we have

$$\int_{t_1}^{\infty} Q(t)dt < \infty, \tag{2.8}$$

which contradiction (2.1). This completes the proof. □

Theorem 2.2. *Assume that $\sigma(t) = t - k$ such that $k > \tau$ and k is a positive constant. If*

$$\liminf_{t\to\infty} \int_{t-k+\tau}^{t} R(s-k)Q(s)ds > \frac{1+a+b}{e} \tag{2.9}$$

where $R(t) = \int_{t_0}^{t} \frac{1}{r(s)}ds$, then every solution of equation (1.1) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1) for $t \geq t_0$. Then proceeding as in the proof of Theorem 2.1, we obtain (2.6). Let

$$w(t) = r(t)z'(t) + ar(t-\tau)z'(t-\tau) + br(t+\delta)z'(t+\delta)$$

then from (2.6), we obtain

$$w'(t) + Q(t)z(t-k) \leq 0, \quad t \geq t \geq t_1. \tag{2.10}$$

Since $r(t)z'(t)$ is decreasing, we have

$$r(s)z'(s) \geq r(t)z'(t), \quad t \geq s.$$

Dividing the last inequality by $r(s)$ and then integrating from t_1 to t, we obtain

$$z(t) \geq R(t)r(t)z'(t), \quad t \geq t_1. \tag{2.11}$$

From the definition $w(t)$, we have

$$w(t) \leq (1+a+b)r(t-\tau)z'(t-\tau). \tag{2.12}$$

Combining (2.10), (2.11) and (2.12), we obtain

$$w'(t) + \frac{R(t-k)}{1+a+b}w(t+\tau-k) \leq 0. \tag{2.13}$$

By Theorem 2.3.1 in [10], the condition (2.13) implies that equation (2.9) has no eventually positive solution. This contradiction completes the proof. □

Theorem 2.3. *Assume that* $\sigma(t) \leq t-\tau$, *and there exist functions* $\phi \in Y$ *and* $k \in C'([t_0,\infty),\mathbb{R}^+)$ *such that*

$$\lim_{t\to\infty}\sup T\left[k(s)Q(s) - \frac{(1+a+b)\left(\psi + \frac{k'(s)}{k(s)}\right)^2}{4\sigma'(s)} r(\sigma(s))k(s);\ell,t\right] > 0 \tag{2.14}$$

where $Q(t)$ *is defined as in Theorem 2.1, the operator* T *defined by* (1.2), *and* $\psi = \psi(t,s,\ell)$ *is defined by* (1.3). *Then every solution of equation* (1.1) *is oscillatory.*

Proof. Let $x(t)$ be a nonoscillatory solution of equation (1.1). Then there exists a $t_1 \geq t_0$ such that $x(t) \neq 0$ for all $t \geq t_1$. Without loss of generality, we may assume that $x(t) > 0,\ x(t-\tau) > 0,\ x(t+\delta) > 0$ and $x(\sigma(t)) > 0$ for all $t \geq t_1$. Define

$$w(t) = k(t)\frac{r(t)z'(t)}{z(\sigma(t))}, \quad t \geq t_1. \tag{2.15}$$

Then $w(t) > 0$, and

$$\begin{aligned} w'(t) &= k(t)\frac{(r(t)z'(t))'}{z(\sigma(t))} + k'(t)\frac{(r(t)z'(t))}{z(\sigma(t))} \\ &\quad - k(t)\frac{(r(t)z'(t))}{z^2(\sigma(t))}z'(\sigma(t))\sigma'(t). \end{aligned} \tag{2.16}$$

From (2.2) and the fact that $z'(t) > 0$, we have

$$\frac{z'(\sigma(t))}{z'(t)} \geq \frac{r(t)}{r(\sigma(t))}. \tag{2.17}$$

Using (2.17) and (2.15) in (2.16), we obtain

$$w'(t) \leq k(t)\frac{(r(t)z'(t))'}{z(\sigma(t))} + \frac{k'(t)}{k(t)}w(t) - \frac{\sigma'(t)}{r(\sigma(t))k(t)}w^2(t). \tag{2.18}$$

Next, define

$$u(t) = k(t)\frac{(r(t-\tau)z'(t-\tau))}{z(\sigma(t))}, \quad t \geq t_1. \tag{2.19}$$

Then $u(t) > 0$, and

$$\begin{aligned} u'(t) &= k'(t)\frac{(r(t-\tau)z'(t-\tau))}{z(\sigma(t))} + k(t)\frac{(r(t-\tau)z'(t-\tau))'}{z(\sigma(t))} \\ &\quad - k(t)\frac{(r(t-\tau)z'(t-\tau))}{z^2(\sigma(t))}z'(\sigma(t))\sigma'(t). \end{aligned} \tag{2.20}$$

By (2.2) and the fact $z'(t) > 0$, noting that $\sigma(t) \leq t - \tau$, we have

$$\frac{z'(\sigma(t))}{z'(t-\tau)} \geq \frac{r(t-\tau)}{r(\sigma(t))}. \tag{2.21}$$

Using (2.21) and (2.19) in (2.20), we obtain

$$u'(t) \leq k(t)\frac{(r(t-\tau)z'(t-\tau))'}{z(\sigma(t))} + \frac{k'(t)}{k(t)}u(t) - \frac{\sigma'(t)}{r(\sigma(t))k(t)}u^2(t). \tag{2.22}$$

Similarly, define

$$v(t) = k(t)\frac{(r(t+\delta)z'(t+\delta))}{z(\sigma(t))}, \quad t \geq t_1. \tag{2.23}$$

Then $v(t) > 0$, and

$$\begin{aligned} v'(t) &= k'(t)\frac{(r(t+\delta)z'(t+\delta))}{z(\sigma(t))} + k(t)\frac{(r(t+\delta)z'(t+\delta))'}{z(\sigma(t))} \\ &\quad - k(t)\frac{(r(t+\delta)z'(t+\delta))}{z^2(\sigma(t))}z'(\sigma(t))\sigma'(t). \end{aligned} \tag{2.24}$$

From (2.2) and the fact $z'(t) > 0$, noting that $\sigma(t) \le t - \tau \le t + \delta$, we have

$$\frac{z'(\sigma(t))}{z'(t+\delta)} \ge \frac{r(t+\delta)}{r(\sigma(t))}. \tag{2.25}$$

From (2.23), (2.24) and (2.25), we have

$$v'(t) \le k(t)\frac{(r(t+\delta)z'(t+\delta))'}{z(\sigma(t))} + \frac{k'(t)}{k(t)}v(t) - \frac{\sigma'(t)}{r(\sigma(t))k(t)}v^2(t). \tag{2.26}$$

Combining (2.18), (2.22) and (2.26), we obtain

$$\begin{aligned} w'(t) &+ au'(t) + bv'(t) \\ &\le k(t)\frac{(r(t)z'(t))'}{z(\sigma(t))} + ak(t)\frac{(r(t-\tau)z'(t-\tau))'}{z(\sigma(t))} \\ &\quad + bk(t)\frac{(r(t+\delta)z'(t+\delta))'}{z(\sigma(t))} + \frac{k'(t)}{k(t)}w(t) \\ &\quad - \frac{\sigma'(t)}{r(\sigma(t))k(t)}w^2(t) + a\frac{k'(t)}{k(t)}u(t) \\ &\quad - a\frac{\sigma'(t)}{r(\sigma(t))k(t)}u^2(t) + b\frac{k'(t)}{k(t)}v(t) \\ &\quad - b\frac{\sigma'(t)}{r(\sigma(t))k(t)}v^2(t). \end{aligned} \tag{2.27}$$

From (2.6) and (2.27) we obtain

$$\begin{aligned} w'(t) &+ au'(t) + bv'(t) \\ &\le -k(t)Q(t) + \frac{k'(t)}{k(t)}w(t) - \frac{\sigma'(t)}{r(\sigma(t))k(t)}w^2(t) \\ &\quad + a\frac{k'(t)}{k(t)}u(t) - a\frac{\sigma'(t)}{r(\sigma(t))k(t)}u^2(t) + b\frac{k'(t)}{k(t)}v(t) \\ &\quad - b\frac{\sigma'(t)}{r(\sigma(t))k(t)}v^2(t). \end{aligned} \tag{2.28}$$

Applying the operator T to (2.28), we obtain

$$\begin{aligned} T[w'(s) &+ au'(s) + bv'(s); \ell, t] \\ &\le T[-k(s)Q(s) + \frac{k'(s)}{k(s)}w(s) - \frac{\sigma'(s)}{r(\sigma(s))k(s)}w^2(s) \\ &\quad + a\frac{k'(s)}{k(s)}u(s) - a\frac{\sigma'(s)}{r(\sigma(s))k(s)}u^2(s) \end{aligned}$$

$$+ b\frac{k'(s)}{k(s)}v(s) - b\frac{\sigma'(s)}{r(\sigma(s))k(s)}v^2(s); \ell, t].$$

By (1.4), and the last inequality, we have

$$\begin{aligned} &T[k(s)Q(s); \ell, t] \\ &\quad \le T[\left(\psi + \frac{k'(s)}{k(s)}\right) w(s) - \frac{\sigma'(s)}{r(\sigma(s))k(s)}w^2(s) \\ &\qquad + a\left(\psi + \frac{k'(s)}{k(s)}\right) u(s) - a\frac{\sigma'(s)}{r(\sigma(s))k(s)}u^2(s) \\ &\qquad + b\left(\psi + \frac{k'(s)}{k(s)}\right) v(s) - b\frac{\sigma'(s)}{r(\sigma(s))k(s)}v^2(s); \ell, t]. \end{aligned} \tag{2.29}$$

Now, from (2.29), we obtain

$$\begin{aligned} T[k(s)Q(s); \ell, t] \ \le \ & T[(\frac{(\psi + \frac{k'(s)}{k(s)})^2}{4} + a\frac{(\psi + \frac{k'(s)}{k(s)})^2}{4} \\ & + b\frac{(\psi + \frac{k'(s)}{k(s)})^2}{4})\frac{r(\sigma(s))k(s)}{\sigma'(s)}; \ell, t] \end{aligned}$$

or

$$T\left[k(s)Q(s) - \frac{(1+a+b)\left(\psi + \frac{k'(s)}{k(s)}\right)^2}{4\sigma'(s)}r(\sigma(s))k(s); \ell, t\right] \le 0.$$

Taking the sup limit in the last inequality, we obtain

$$\lim_{t\to\infty} \sup T\left[k(s)Q(s) - \frac{(1+a+b)\left(\psi + \frac{k'(s)}{k(s)}\right)^2}{4\sigma'(s)}r(\sigma(s))k(s); \ell, t\right] \le 0$$

which contradicts (2.14). This completes the proof. □

Remark 2.1. With different choices of functions k and ϕ, Theorem 2.2 can be stated with different conditions for oscillations of equation (1.1).

For example, if we take $\phi(t, s, \ell) = (t - s)^\alpha (s - \ell)^\beta$ for $\alpha > \frac{1}{2}$, $\beta > \frac{1}{2}$, then

$$\psi(t, s, \ell) = \frac{\beta t - (\alpha + \beta)s + \alpha\ell}{(t - s)(s - \ell)}.$$

From Theorem 2.2, we obtain the following oscillation criteria for equation (1.1).

Corollary 2.1. *Assume that $\sigma(t) \leq t-\tau$, and there exists a function $k \in C'([t_0,\infty),\mathbb{R}^+)$ such that*

$$\lim_{t\to\infty} \sup \int_{t_0}^{t} [(t-s)^{\alpha}(s-t_0)^{\beta}k(s)Q(s) - \frac{(1+a+b)\left(\psi+\frac{k'(s)}{k(s)}\right)^2}{4\sigma'(s)} r(\sigma(s))k(s)]ds > 0$$

where $\alpha > \frac{1}{2}$, $\beta > \frac{1}{2}$ and $\psi = \frac{\beta t-(a+b)s+\alpha t_0}{(t-s)(s-t_0)}$. Then every solution of equation (1.1) *is oscillatory.*

3. Examples

In this section, we provide three examples to illustrate the main results.

Example 3.1. Consider the following neutral differential equation

$$\big(t(x(t)+3x(t-\pi)+2x(t+\pi))'\big)' + 4x(t-\pi) = 0, \quad t \geq t_0 \tag{3.1}$$

where $r(t)=t$, $a(t)=3$, $b(t)=2$,$q(t)=4$, $\tau=\delta=\pi$, and $\sigma(t)=t-\pi$. Hence all the conditions of Theorem 2.1 are satisfied and hence every solution of equation (3.1) is oscillatory.

Example 3.2. Consider the neutral differential equation

$$\left(\frac{1}{t+3}(x(t)+4x(t-2)+3x(t+1))'\right)' + \frac{50}{t^2}x(t-3) = 0, \quad t \geq 1 \tag{3.2}$$

where $r(t)=\frac{1}{t+3}$, $a(t)=4$, $b(t)=3$ $q(t)=\frac{50}{t^2}$, $\tau=2$, $\delta=1$ and $k=3$. Since $R(t)=\frac{(t+3)^2}{2}-8$, we have

$$\begin{aligned}
&\lim_{t\to\infty}\inf \int_{t-k+\tau}^{t} R(s-\tau)Q(s)ds \\
&\qquad = \lim_{t\to\infty}\inf \int_{t-1}^{t} \left(\frac{(s+3)^2}{2}-8\right)\frac{50}{(s+1)^2}ds \\
&\qquad = 25 - \lim_{t\to\infty}\inf \left(\frac{1}{t+1}-\frac{1}{t}\right) \\
&\qquad = 25 > \frac{8}{e}.
\end{aligned}$$

Hence by Theorem 2.2, every solution of equation (3.2) is oscillatory.

Example 3.3. Consider the neutral differential equation

$$(x(t) + 2(t-2) + x(t+1))'' + \frac{100}{(t-1)^2}x(t-3) = 0, \quad t \geq 4 \tag{3.3}$$

where $r(t) = 1,\ a(t) = 2,\ b(t) = 1,\ q(t) = \frac{100}{(t-1)^2},\ \tau = 2,\ \delta = 1$ and $\sigma(t) = t-3$. By taking $\alpha = \beta = 2$ and $k(t) = 1$, it is easy to verify that all conditions of Corollary 2.1 are satisfied and hence every solution of equation (3.3) is oscillatory.

Remark 3.1. Recent results cannot be applied to equation (3.1), (3.2) and (3.3), since the neutral term contain both delay and advanced arguments and $r(t) \neq 1$.

References

[1] R.P.Agarwal and S.R.Grace, Oscillation theorems for certain neutral functional differential equations, *Comput. Math. Appl.,* **38**(1999), 1-11.

[2] D.D.Bainov and D.P.Mishev, Oscillation Theory of Neutral Differential Equations with Delay, *Adam Hilger, Bristol,* 1991.

[3] R.D.Driver, A mixed neutral system, *Nonlinear Anal.,* **8**(1984), 155-158.

[4] J.Dzurina, J.Busa and E.A.Airyan, Oscillation criteria for second order differential equations of neutral type with mixed arguments, *Diff. Eqns.,* **38**(2002), 137-140.

[5] J.Dzurina and I.P.Stavroulakis, Oscillation criteria for second order delay differential equations, *Appl. Math. Comput.,* **140**(2003), 445-453.

[6] S.R.Grace, Oscillation theorems for nonlinear differential equations of second order, *J. Math. Anal. Appl.,* **171**(1992), 220-241.

[7] S.R.Grace, On the oscillation of mixed neutral equations, *J. Math. Anal. Appl.,* **194**(1995), 377-388.

[8] S.R.Grace, Oscillation of mixed neutral functional differential equations, *Appl. Math. Comput.,* **68**(1995), 1-13.

[9] M.K.Grammatikopouls, G.Ladas and A.Meimaridou, Oscillation of second order neutral delay differential equations, *Rad. Math.,* **1**(1985), 267-274.

[10] I.Gyori and G.Ladas, Oscillation Theory of Delay Differential Equations with Applications, *Claredon Press, Oxford,* 1991.

[11] J.Hale, Theory of Functional Differential Equations, *Springer, New York,* 1977.

[12] Z.Han, T.Li, S.Sun and W.Chen, On the oscillation of second order neutral delay differential equations, *Adv. Diff. Eqn.,* Vol. **2010**, Article ID 289340, 8 pages.

[13] B.Karpuz, J.V.Manojlovic, O.Ocalan and Y.Shoukaku, Oscillation criteria for a class of second order neutral delay differential equations, *Appl. Math. Comput.,* **210**(2009), 303-312.

[14] T.Li, Yu.V.Rogovchenko and C.Zhang, Oscillation of second order neutral differential equations, *Funkc. Ekvacioj.,* **56**(2013), 111-120.

[15] T.Li and E.Thandapani, Oscillation of solutions to second order neutral differential equations, *Elec. J. Diff. Equ.,* **2014(67)**(2014), 1-7.

[16] L.Liu and Y.Bai, New oscillation crieria for second order nonlinear neutral delay differential equations, *J. Comput. Appl. Math.,* **231**(2009), 657-663.

[17] X.Liu and X.H.Tang, Oscillation fo solutions of neutral differential equations with a superlinear neutral term, *Appl. Math. Lett.,* **20**(2007), 1016-1022.

[18] S.G.Ruan, Oscillations of second order neutral differential equations, *Canadian Math. Bull.,* **36**(1993), 485-496.

[19] Y.Sahina, On oscillation of second order neutral type delay differential equations, *Appl. MAth. Comput.,* **150**(2004), 697-706.

[20] R.Xu and F.Meng, Oscillation crieria for second order quasilinear neutral delay differential equations, *Appl. Math. Comput.,* **192**(2007), 216-222.

[21] L.Ye and Z.Xu, Oscillation crieria for second order quasilinear neutral delay differential equations, *Appl. Math. Comput.,* **207**(2009), 388-396.