International Journal of Pure and Applied Mathematics
Volume 105 No. 3 2015, 543-547
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: http://dx.doi.org/10.12732/ijpam.v105i3.20.1

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A LUENBERGER OBSERVER FOR A QUASI-STATIC DISTURBANCE ESTIMATION IN LINEAR TIME INVARIANT SYSTEMS

Paolo Mercorelli

Institute of Product and Process Innovation
Leuphana University of Lueneburg
Volgershall 1, D-21339 Lueneburg, GERMANY

Abstract: This paper deals with a Luenberger Observer structure which is devoted to the identification of state variables of a Linear Time Invariant (LTI) system. In particular, a disturbance acting as an unknown input is estimated under the hypothesis of quasi-stationarity. Without loosing the generality, a system of the second order is taken into consideration and a constructive proposition is proven.

AMS Subject Classification: 49K15, 93B10, 93B07
Key Words: problems involving ordinary differential equations, canonical structure, observability

1. Introduction and Motivation

The paper proposes a technique to identify disturbances in which the main idea is to use an integration between a state observer and a disturbance observer to estimate the non-measurable state and the disturbance. In the field of the control of actuators, observers are widely used. For instance, in [1] the author implemented a switching extended Kalman Filter with a switching dynamics

Received: November 19, 2015

and in [2] a sliding mode observer is proposed. In [3] a cascade Kalman Filter is proposed as an observer to estimate the state in sensorless control for an actuator. In this sense, observers are widely used in many industrial applications to save sensors and also to filter data.

2. Problem Formulation and its Solution

Let's consider the following disturbed linear time invariant (LTI) system represented in its controllability canonical form which arranges the coefficients of the transfer function denominator across one row of an **A** matrix as follows:

$$\underbrace{\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_{n-1}(t) \\ \dot{x}_n(t) \end{bmatrix}}_{\dot{\mathbf{X}}(t)} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}}_{\mathbf{X(t)}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\mathbf{B}} u(t) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\mathbf{D}} d(t), \quad (1)$$

where $u(t)$ and $d(t)$ are the input and the disturbance acting on the system, $\mathbf{X}(t)$ is the state vector of the system with $n \in \mathbb{N}$ which represents the order of the system. Let's consider the following extended Luenberger Observer for the linear system defined above in which a quasi-static condition for a disturbance is considered. The quasi-static condition is characterised by $\dot{d}(t) \approx 0$ and the Luenberger Observer can be written as follows:

$$\begin{bmatrix} \dot{\hat{\mathbf{X}}}(t) \\ \dot{\hat{d}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{0} & 0 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{X}}(t) \\ \hat{d}(t) \end{bmatrix} + \mathbf{B}u(t) + \mathbf{LC}\hat{\mathbf{X}}(t), \quad (2)$$

where

$$y(t) = \underbrace{\begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}}_{\mathbf{C}} \underbrace{\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix}}_{\hat{\mathbf{x}}(t)}, \tag{3}$$

and

$$\mathbf{L} = \begin{bmatrix} a_{l1} \\ a_{l2} \\ \vdots \\ a_{l(n-1)} \\ a_{ln} \end{bmatrix}, \tag{4}$$

it is always possible to show that there exists always a matrix $\mathbf{L}$ which characterises the Luenberger Observer to estimate the whole state and the quasi-static disturbance acting on the system. For sake of brevity, just the case with $n = 2$ is proven. If the following system of the second order is considered:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -a_2 x_1(t) - a_1 x_2(t) + u_{in}(t) + d(t), \end{cases} \tag{5}$$

let take into consideration the Luenberger Observer as follows:

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + a_{l1}(x_1(t) - \hat{x}_1(t)) \\ \dot{\hat{x}}_2(t) = -a_2\hat{x}_1(t) - a_1\hat{x}_2(t) + a_{l2}(x_1(t) - \hat{x}_1(t)) + u_{in}(t) + \hat{d}(t) \\ \dot{\hat{d}}(t) = a_{l3}(x_1(t) - \hat{x}_1(t)), \end{cases} \tag{6}$$

where $\hat{\mathbf{x}}(t) = [x_1(t) \ \ x_2(t) \ \ \hat{d}_v(t)]^T$ is the observed state vector which consists of the two state variables and the observed disturbance variable $\hat{d}(t)$. Considering

$$\rho(t) = [\rho_1(t) \ \ \rho_2(t) \ \ \rho_3(t)]^T, \tag{7}$$

the error vector is defined as follows:

$$\begin{cases} \rho_1(t) = x_1(t) - \hat{x}_1(t) \\ \rho_2(t) = x_2(t) - \hat{x}_2(t) \\ \rho_3(t) = d(t) - \hat{d}(t). \end{cases} \tag{8}$$

Proposition 1. If the gain parameters a_{l1}, a_{l2} and $a_{l3} \in \mathbb{R}$ with $a_{l1} > -a_1$, $a_{l2} > a_{l1}a_1 - a_2$, $a_{l3} > 0$ and $(a_1 + a_{l1})(a_{l1}a_1 + a_{l2} + a_2) > a_{l3}$, then $\lim_{t\to+\infty} \rho(t) = 0$.

Proof. Considering the above defined error

$$\rho(t) = [\rho_1(t) \;\; \rho_2(t) \;\; \rho_3(t)]^T, \tag{9}$$

considering (8) and combining relations (5) and (6), then:

$$\begin{cases} \dot{\rho}_1(t) = \rho_2(t) - a_{l1}\rho_1(t) \\ \dot{\rho}_2(t) = -(a_{l2} + a_2)\rho_1(t) - a_1\rho_2(t) + \rho_3(t) \\ \dot{\hat{\rho}}_3(t) = -a_{l3}\rho_1(t) + \dot{d}(t). \end{cases} \tag{10}$$

The observation error can be expressed as follows:

$$\dot{\rho}(t) = \mathbf{A}_e\rho(t) + \mathbf{B}_e\dot{d}_v(t), \tag{11}$$

where

$$\mathbf{A}_e = \begin{bmatrix} -a_{l1} & 1 & 0 \\ -(a_{l2} + a_2) & -a_1 & 1 \\ -a_{l3} & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{B}_e = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \tag{12}$$

The characteristic equation of matrix $\mathbf{A}_e$ is:

$$det(\lambda I - \mathbf{A}_e) = \begin{bmatrix} \lambda + a_{l1} & -1 & 0 \\ (a_{l2} + a_2) & \lambda + a_1 & -1 \\ a_3 & 0 & \lambda \end{bmatrix} = 0. \tag{13}$$

Then

$$\lambda^3 + (a_{l1} + a_1)\lambda^2 + (a_{l1}a_1 + a_{l2} + a_2)\lambda + a_{3l} = 0. \tag{14}$$

It is straightforward to show, using for instance the Routh-Hurwitz test, that if the following conditions are guaranteed, then a Hurwitz matrix is obtained:

$$a_1 + a_{l1} > 0, \tag{15}$$

$$a_{l1}a_1 + a_{l2} + a_2 > 0, \tag{16}$$

$$a_{l3} > 0, \tag{17}$$

and

$$(a_1 + a_{l1})(a_{l1}a_1 + a_{l2} + a_2) > a_{l3}, \tag{18}$$

thus from (15) it follows that $a_{l1} > a_1$ and from (16) it follows that $a_{l2} > -a_{l1}a_1 - a_2$ and choosing parameter a_{l3} in an appropriate way, it is always possible to satisfy (18), then matrix $\mathbf{A}_e$ is a Hurwitz matrix and the convergence of the estimation of the observer is shown. □

3. Conclusion

This paper deals with a Luenberger Observer used for estimation of the state of a LTI system and in particular, for a quasi-static disturbance acting as an unknown input. A general structure such an observer is shown and, without loosing the generality of the result, a proposition concerning a system of the second order is proven.

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