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FUZZY Γ-SEMIRINGS

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Abstract: In this paper we introduce and study the concept of fuzzy Γ-semiring and investigated some of the properties related to them. Further we studiey fuzzy ideals on Γ-semirings and established a one-one correspondence between fuzzy left(right) ideal of a Γ-semiring R and level set μ_t, $t \in [0, 1]$ left (right) ideal of R.

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1. Introduction

The concept of gamma in algebra was introduced and studied first by N. Nobusawa [7] in 1964 and further established Γ-ring. Infact, there have been a few slightly different definition on a Γ-ring. In 1995, M.K. Rao [6] introduced the notion of Γ-semiring as a generalization of Γ-ring as well as semiring and studied the concepts of Γ-semirings and its sub Γ-semirings with a left(right) unity. Later on much has been developed and this concepts by different researchers. Fuzzy sets introduced by L.A. Zadeh [9] and there after several researchers developed algebraic structures and applied it on different branches of pure and

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applied mathematics. Further on Γ-semirings the study properties of fuzzy ideals, fuzzy prime ideals, fuzzy semiprime ideals and their generalizations play an important role in their structure theory, however the properties of a fuzzy ideal in semirings and Γ-semirings are some what different from the properties of the usual ring ideals. In 1992, Jun and Lee [4] introduced the notion of fuzzy ideal in Γ-ring and studied few properties. In 2005, Dutta and Chanda [2] studied the structures of fuzzy ideals in Γ-ring via operator rings of Γ-ring. In this paper, we established a one-one correspondence between fuzzy Γ-semiring and its level set μ_t, $t \in [0,1]$. Further we investigated one-one correspondence between fuzzy left(right) ideal of a Γ-semiring R and level set μ_t, t ∈ [0,1] left(right) ideal of R.

2. Preliminaries

Definition 2.1. Let R and Γ be two additive commutative semigroups. Then R is called Γ- semiring if there exists a mapping $R \times \Gamma \times R \to R$ (image to be denoted by aαb for a, b ∈ R and $\alpha \in \Gamma$) satisfying the following conditions:

1. $a\alpha(b+c) = a\alpha b + a\alpha c$;
2. $(a+b)\alpha c = a\alpha c + b\alpha c$;
3. $a(\alpha+\beta)c = a\alpha c + a\beta c$;
4. $a\alpha(b\beta c) = (a\alpha b)\beta c, \forall\ a,b,c \in R; \alpha,\beta \in \Gamma$.

Example. Let ℕ be the set of natural numbers and Γ={1, 2, 3}. Then $\{\mathbb{N}, \max\}$ and $(\Gamma, \max)$ are commutative semigroups.

Define a mapping $\mathbb{N} \times \Gamma \times \mathbb{N} \to \mathbb{N}$ by $a\alpha b = \min\{a,\alpha,b\}, \forall\ a,b \in \mathbb{N}$ and $\alpha \in \Gamma$.

Then ℕ is a Γ -semiring.

Definition 2.2. A non-empty subset S of a Γ-semiring R is called a sub Γ-semiring of R if $(S,+)$ is a sub semigroup of $(R,+)$ and $a\gamma b \in S$, $\forall\ a,b \in S; \gamma \in \Gamma$.

Definition 2.3. A non-empty subset S of a Γ-semiring R is called a Γ-ideal of R if $(S,+)$ is a sub semigroup of $(R,+)$ and $x\gamma a \in S$ and $a\gamma x \in S$, $\forall\ a \in S\ ;\ x \in R\ ;\ \gamma \in \Gamma$.

Definition 2.4. Let R be any non-empty set. A mapping $\mu : R \to$ [0,1] is called a fuzzy subset of R.

Definition 2.5. Let μ be any fuzzy subset of a set R and let t ∈ [0,1].

The set $\mu_t = \{x \in R/\mu(x) \geq t\}$ is called a level subset of μ.

The set of all level subsets of μ is denoted by F_μ i.e., $F_\mu = \{\mu_t/t \in im\mu\}$.

Definition 2.6. Let μ and σ be any two fuzzy subsets of a set R. Then μ is said to be contained in σ, denoted by $\mu \subseteq \sigma$, if $\mu(x) \leq \sigma(x),\ \forall\, x \in R$.

If $\mu(x) = \sigma(x)$, $\forall\, x \in R$, then μ and σ are said to be equal.

Definition 2.7. The union of two fuzzy subsets μ and σ of a set R, denoted by $\mu \cup \sigma$, is a fuzzy subset of R defined by $(\mu \cup \sigma)(x) = \max\{\mu(x), \sigma(x)\}$, $\forall\, x \in R$.

Definition 2.8. The intersection of two fuzzy subsets μ and σ of a set R, denoted by $\mu \cap \sigma$, is a fuzzy subset of R defined by $(\mu \cap \sigma)(x) = \min\{\mu(x), \sigma(x)\}$, $\forall\, x \in R$.

Definition 2.9. The union and intersection of any family $\{\ \mu_i\ /\ i \in I\ \}$ of fuzzy subsets of a set R are defined by

$$(\bigcup_{i\in I} \mu_i)(x) = \sup_{i\in I} \mu_i(x), \quad \forall\, x \in R,$$

$$(\bigcap_{i\in I} \mu_i)(x) = \inf_{i\in I} \mu_i(x), \quad \forall\, x \in R.$$

Definition 2.10. Let μ be a fuzzy subset of a Γ - Semiring R. Then μ is called a fuzzy Γ-Semiring if:

1. $\mu(x + y) \geq min\{\mu(x), \mu(y)\}$;
2. $\mu(x\gamma y) \geq min\{\mu(x), \mu(y)\}; \forall\, x, y \in R;\ \gamma \in \Gamma$.

Example 1. Let R be the set of natural numbers with zero and let $\Gamma = \{0, 1\}$. Define the mapping $R \times \Gamma \times R \to R$ by $a\alpha b$ usual product of $a, \alpha, b,\ \forall\, a, b \in R;\ \alpha \in \Gamma$.
Then R is a Γ-semiring.

Define $\mu : R \to [0, 1]$ by

$$\mu(x) = \begin{cases} 0.7 & \text{if } x = 0 \\ 0.6 & \text{if } x \textit{ is even} \\ 0.4 & \text{if } x \textit{ is odd} \end{cases}$$

Then μ is a fuzzy Γ-semiring.

Example 2. Let R be the set of negative integers and Γ be the set of negative even integers. Then R, Γ are additive commutative semigroups.

Define the mapping $R\times\Gamma\times R\rightarrow R$ by $a\alpha b$ usual product of $a,\alpha,b,\ \forall\ a,b\in$ R; $\alpha\in\Gamma$. Then R is a Γ-semiring.

Define μ :$R\rightarrow$ $[0,1]$ by

$$\mu(x)=\begin{cases} 0.4 & \text{if } x=-1 \\ 0.6 & \text{if } x=-2 \\ 0.7 & \text{if } x<-2 \end{cases}$$

Then μ is a fuzzy Γ-semiring.

Example 3. Consider the additively abelian groups

$$Z_8=\{0,1,2,3,4,5,6,7\} \text{ and } \Gamma = \{0,2,4,6\}.$$

Define $Z_8\times\Gamma\times Z_8\rightarrow Z_8$ by aαb usual product of a,α,b, $\forall\ a,b\in$ R; $\alpha\in\Gamma$.
Then Z_8 is a Γ-semiring.
Define $\mu : Z_8 \rightarrow [0,1]$ by

$$\mu(x)=\begin{cases} 0.8 & \text{if } x=0 \\ 0.5 & \text{if } x\neq 0 \end{cases}$$

Then μ is a fuzzy Γ-semiring.

Definition 2.11. Let μ be a fuzzy subset of a Γ-semiring R. Then μ is called a fuzzy left (right) ideal of R if:

1. $\mu(x+y)\ \geq\ \min\ \{\mu(x),\mu(y)\}$;

2. $\mu(x\gamma y)\ \geq\ \mu(y)\ (\mu(x))\ \forall\ x,y\in R\ ;\ \gamma\ \in\ \Gamma$. μ is fuzzy Γ-ideal if it is both fuzzy left and fuzzy right ideal.

Definition 2.12. Let S is a subset of a Γ-semiring R. The characteristic function of S taking values in [0,1] is a fuzzy set given by

$$\chi_S(x)=\begin{cases} 1 & \text{if } x\in S \\ 0 & \text{if } x\notin S \end{cases}$$

Then χ_S is a fuzzy characteristic function of S in [0,1].

3. Fuzzy Γ-Semiring

Theorem 3.1. Let R be a Γ-semiring. A fuzzy subset μ of R is a fuzzy Γ-semiring iff its level set $\mu_t,\ t \in [0,1]$ is a sub Γ-semiring of R.

Proof. Let R be a Γ-semiring and let μ be a fuzzy subset of R.
Suppose μ is fuzzy Γ-semiring.
Claim. $\mu_t = \{x \in R/\mu(x) \geq t\}$ is a sub Γ- semiring of R.
Let $x, y \in \mu_t$ and $\gamma \in \Gamma$.
$\Rightarrow \mu(x) \ \geq t,\ \mu(y) \geq t$
$\Rightarrow \min\ \{\mu(x),\ \mu(y)\} \ \geq\ t$
$\Rightarrow \mu(x+y) \ \geq\ t$ and $\mu(x\gamma y) \ \geq\ t$
$\Rightarrow x+y \in\ \mu_t,\ x\gamma y \in \mu_t.$
Then μ_t is sub Γ-semiring of R.

Conversely suppose μ_t is a sub Γ-semiring of R.
Claim: μ is fuzzy Γ-semiring of R.
Let $x, y \in R$ and $\gamma \in \Gamma$.
Let $\mu(x) = a;\ \mu(y) = b$.
Let t=min$\{a, b\}$.
Then $\mu(x) = a \geq t;\ \mu(y) = b \geq t.$
$\Rightarrow x,\ y \in\ \mu_t.$
$\Rightarrow x+y\ \in\ \mu_t$ and $x\gamma y \in \mu_t.$
$\Rightarrow \mu(x+y)\ \geq\ t$ and $\mu(x\gamma y) \geq t.$
$\Rightarrow \mu(x+y)\ \geq\ \min\ \{\mu(x), \mu(y)\}$ and $\mu(x\gamma y) \geq\ \min\ \{\mu(x), \mu(y)\}$
Then μ is a fuzzy Γ-semiring of R. □

Theorem 3.2. Let R be a Γ-semiring. Let μ and σ be fuzzy Γ-semirings of R. Then $\mu \cap \sigma$ is a fuzzy Γ-semiring of R.

Proof. Let R be a Γ-semiring and let μ, σ be two fuzzyΓ-semirings of R.
Let $x, y \in R$; $\gamma \in \Gamma$.

1. $(\mu \cap \sigma)(x+y) = \min\{\mu(x+y), \sigma(x+y)\}$
$\geq \min\{\ \min\{\mu(x), \mu(y)\}, \min\{\sigma(x), \sigma(y)\}\ \}$
$= \min\{\ \min\{\mu(x), \sigma(x)\}, \min\{\mu(y), \sigma(y)\}\ \}$
$= \min\ \{(\mu \cap \sigma)(x), (\mu \cap \sigma)(y)\}$

2. $(\mu \cap \sigma)(x\gamma y) = \min\{\mu(x\gamma y), \sigma(x\gamma y)\}$
$\geq \min\{\ \min\{\mu(x), \mu(y)\}, \min\{\sigma(x), \sigma(y)\}\ \}$

$= \min\{ \min\{\mu(x), \sigma(x)\}, \min\{\mu(y), \sigma(y)\} \}$
$= \min \{(\mu \cap \sigma)(x), (\mu \cap \sigma)(y)\}$.

Thus $\mu \cap \sigma$ is a fuzzzy Γ-semiring of R. □

In general union of two fuzzy Γ-semirings may not be a fuzzy Γ-semiring.

Example 3.3. consider the additive abelian groups $Z_4 = \{0, 1, 2, 3\}$ and $\Gamma = \{0, 2\}$.

Define $Z_4 \times \Gamma \times Z_4 \to Z_4$ by aαb usual product of a,α,b, $\forall\ a, b \in$ R; $\alpha \in \Gamma$.

Then Z_4 is a Γ-semiring.

Define $\mu : Z_4 \to [0, 1]$ and $\sigma : Z_4 \to [0, 1]$ by

$$\mu(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.6 & \text{if } x = 1 \\ 0.4 & \text{otherwise} \end{cases}$$

and

$$\sigma(x) = \begin{cases} 0.6 & \text{if } x = 0 \\ 0.5 & \text{if } x = 2 \\ 0.2 & \text{otherwise} \end{cases}$$

Then μ and σ are fuzzy Γ-semirings of Z_4.

But $\mu \cup \sigma$ is not a fuzzy Γ-semiring.

Inperticular we have the following theorem.

Theorem 3.4. Let R be a Γ-semiring. Let μ, σ be two fuzzy Γ-semirings of R. Then $\mu \cup \sigma$ is a fuzzy Γ-semiring of R if $\mu \subseteq \sigma$ or $\sigma \subseteq \mu$.

Proof. Let R be a Γ semiring.

Let μ, σ be two fuzzy Γ-semirings on R.

Suppose $\mu \subseteq \sigma$.

Let $x, y \in R$; $\gamma \in \Gamma$.

1. $(\mu \cup \sigma)(x + y) = \max\{\mu(x + y), \sigma(x + y)\}$

$= \sigma(x + y)$
$\geq \min\{\sigma(x), \sigma(y)\}$
$= \min\{\max\{\mu(x), \sigma(x)\}, \max\{\mu(y), \sigma(y)\}\}$
$= \min\{(\mu \cup \sigma)(x), (\mu \cup \sigma)(y)\}$

2. $(\mu \cup \sigma)(x\gamma y) = \max\{\mu(x\gamma y), \sigma(x\gamma y)\}$

$= \sigma(x\gamma y)$
$\geq \min\{\sigma(x), \sigma(y)\}$

$= \min\{\max\{\mu(x), \sigma(x)\}, \max\{\mu(y), \sigma(y)\}\}$
$= \min\{(\mu \cup \sigma)(x), (\mu \cup \sigma)(y)\}$.

Thus $\mu \cup \sigma$ is a fuzzy Γ-semiring.

Similarly, if $\sigma \subseteq \mu$, we get $\mu \cup \sigma$ is a fuzzy Γ-semiring. □

Theorem 3.5. Two fuzzy Γ-semirings μ and θ of a Γ-semiring R such that card $Im\mu < \infty$ and Card $Im\theta < \infty$ are equal if and only if $Im\mu = Im\theta$ and $F_\mu = F_\theta$.

Proof. Let R be a Γ-semiring.

Let μ, θ are two fuzzy Γ-semirings of R such that card $Im\mu < \infty$ and card $Im\theta < \infty$.

Suppose μ, θ are equal.

Claim. $Im\mu = Im\theta$ and $F_\mu = F_\theta$.

Let $t \in Im\mu$.
$\Leftrightarrow \mu(x) = t$.
$\Leftrightarrow \theta(x) = t$. (because $\mu = \theta$).
$\Leftrightarrow t \in Im\theta$.
Therefore $Im\mu = Im\theta$.
F_μ=$\{\mu_t/t \in Im\mu\}$ =$\{\theta_t/t \in Im\theta\}$ =F_θ. therefore $F_\mu = F_\theta$.

Conversely suppose $Im\mu = Im\theta$ and $F_\mu = F_\theta$.

Claim. $\mu = \theta$.

Let $t \in Im\mu \Rightarrow t \in Im\theta$
Then $\mu(x) = t$ and $\theta(x) = t$, i.e. $\mu(x) = t = \theta(x), \forall\, x \in R$.
$\Rightarrow \mu = \theta$. □

Theorem 3.6. Let R be a Γ-semiring. Let μ be a fuzzy Γ-semiring on R. Define μ^* on $R \cup \{0\}$ by

$$\mu^* = \{x \in R/\mu(x) = \mu(0)\}$$

Then μ^* is a sub Γ-semiring on R.

Proof. Let μ be a fuzzy Γ-semiring on R.

Let $x, y \in \mu^* \Rightarrow \mu(x) = \mu(0)$ *and* $\mu(y) = \mu(0)$.

1. $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$
$= \min\{\mu(0), \mu(0)\}$
$= \mu(0)$.
$\Rightarrow x + y \in \mu^*$

2. $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$.
$= \min\{\mu(0), \mu(0)\}$
$= \mu(0)$.
$\Rightarrow x\gamma y \in \mu^*$.

Thus μ^* is a sub Γ-semiring on R. □

4. Fuzzy Ideals on Γ-Semiring

Theorem 4.1. Let R be a Γ-semiring. A fuzzy subset μ of R is a fuzzy left(right) ideal of a Γ-semiring R if and only if the level subset μ_t, $t \in Im\mu$ is a left(right) ideal of R.

Proof. Let R be a Γ-semiring. Let μ be a fuzzy left ideal of R.
Claim. μ_t, $t \in Im\mu$ is a left ideal of R.
Let $x, y \in \mu_t$; $\gamma \in \Gamma$.
$\Rightarrow \mu(x) \geq t$ and $\mu(y) \geq t$.

1. $\mu(x + y) \geq \min\{\mu(x), \mu(y)\} \geq$ t
$\Rightarrow x + y \in \mu_t$.
Let $x \in \mu_t; a \in R; \gamma \in \Gamma \Rightarrow \mu(x) \geq t$.

2. $\mu(a\gamma x) \geq \ \mu(x) \geq t \Rightarrow a\gamma x \in \mu_t$.

Thus μ_t, $t \in Im\mu$ is left(right) ideal of R.
Conversely suppose that μ_t is left(right) ideal of R.
Claim. μ is fuzzy left ideal.
Let $x, y \in R; \gamma \in \Gamma$. Suppose $\mu(x) = a; \mu(y) = b$, where $a, b \in [0, 1]$. Let $t = \min\{a, b\}$.
Therefore $\mu(x) \geq t$ and $\mu(y) \geq t$.
$\Rightarrow x, y \in \mu_t$.
$\Rightarrow x + y \in \mu_t$.
$\Rightarrow \mu(x + y) \geq t = \min\{\mu(x), \mu(y)\}$.
Now $x\gamma y \in \mu_t$ and $y\gamma x \in \mu_t$.
$\Rightarrow \mu(x\gamma y) \geq t$ and $\mu(y\gamma x) \geq t$.
Suppose $a < b$.
$\Rightarrow t = a$.
Therefore $\mu(y\gamma x) \geq t$= a =$\mu(x)$.
Suppose $b < a$, we get $\mu(x\gamma y) \geq \mu(y)$.

Thus μ is fuzzy left(right)ideal of R. □

Theorem 4.2. Let S be a non-empty subset of a Γ-semiring R.Then χ_S is a fuzzy left(right) ideal of R if and only if S is a left(right) ideal of R.

Proof. Suppose χ_S is a fuzzy left ideal of R.

Let $x, y \in R$; a $\in R$; $\gamma \in \Gamma$. Now, $\chi_S(x + y) \geq min\{\chi_S(x), \chi_S(y)\}$ $\Rightarrow x + y \in$S.

Also, $\chi_S(a\gamma x) \geq \chi_S(x) = 1 \Rightarrow a\gamma x \in S$.

Then S is a left ideal of R.

Conversely suppose that S is a left(right) ideal of R.

Let $x, y \in R$; $\gamma \in \Gamma$.

If $x, y \in S$, then $x + y \in S$ and $x\gamma y \in S$.

$\Rightarrow \chi_S(x + y) = 1 = min\{\chi_S(x), \chi_S(y)\}$ and $\chi_S(x\gamma y) = 1 = \chi_S(y)$.

If $x, y \notin S$, then $x + y \notin S$ and $x\gamma y \notin S$.

$\Rightarrow \chi_S(x + y) = 0 = min\{\chi_S(x), \chi_S(y)\}$ and $\chi_S(x\gamma y) = 0 = \chi_S(y)$.

If one of the x or y not in S, then then $x + y \notin S$ and $x\gamma y \notin S$.

$\Rightarrow \chi_S(x + y) = 0 = min\{\chi_S(x), \chi_S(y)\}$ and $\chi_S(x\gamma y) = 0 = \chi_S(y)$.

Then χ_S is a fuzzy left(right) ideal of R. □

Theorem 4.3. Let R be a Γ-semiring. Let μ and σ be fuzzy left(right) ideal of R. Then $\mu \cap \sigma$ is a fuzzy left(right) ideal of R.

Proof. Let R be a Γ-semiring and let μ, σ be two fuzzy left(right)ideal of R.

Let $x, y \in R$; $\gamma \in \Gamma$.

Now:

1. $(\mu \cap \sigma)(x + y) = \min\{\mu(x + y), \sigma(x + y)S\}$
$\geq \min\{\ \min\{\mu(x), \mu(y)\},\ \min\{\sigma(x), \sigma(y)\}\ \}$
$= \min\{\ \min\{\mu(x), \sigma(x)\},\ \min\{\mu(y), \sigma(y)\}\ \}$
$= \min\ \{(\mu \cap \sigma)(x), (\mu \cap \sigma)(y)\}$

2. $(\mu \cap \sigma)(x\gamma y) = \min\{\mu(x\gamma y), \sigma(x\gamma y)\}$
$\geq \min\{\mu(y), \sigma(y)\}$
$= (\mu \cap \sigma)(y)$

Thus $\mu \cap \sigma$ is a fuzzy left(right) ideal of R. □

Theorem 4.4. Let $\{\mu_i / i \in I\}$ be a family of fuzzy left(right) ideals of a Γ-semiring R, then $\bigcap_{i \in I} \mu_i$ is a fuzzy left(right) ideal of R.

Proof. Let $\{\mu_i / i \in I\}$ be a family of fuzzy left ideals of a Γ-semiring R.
Claim. $\mu = \bigcap_{i \in I} \mu_i$ fuzzy left ideal of R.
Let $x, y \in R; \gamma \in \Gamma$.
1. $\mu(x+y) = \bigcap_{i \in I} \mu_i(x+y)$
$= \inf_{i \in I} \ \mu_i(x+y)$
$\geq \inf_{i \in I} \ \{\min \mu_i(x), \mu_i(y)\}$
$= \min\{\inf_{i \in I}\{\mu_i(x), \inf_{i \in I} \mu_i(y)\}\}$
$= \min\{\{\inf\{\mu_i(x)/i \in I\}, \inf\{\mu_i(y)/i \in I\}\}$
$= \min\{\bigcap_{i \in I} \mu_i(x), \bigcap_{i \in I} \mu_i(y)\}$
$= \min\{\mu(x), \mu(y)\}$

2. $\mu(x\gamma y) = \bigcap_{i \in I} \mu_i(x\gamma y)$
$= \inf_{i \in I} \ \mu_i(x\gamma y)$
$\geq \inf_{i \in I} \mu_i(y)$
$= \bigcap_{i \in I} \mu_i(y)$
$= \mu(y)$
Hence $\bigcap_{i \in I} \mu_i$ is fuzzy left(right) ideal of R. □

5. Conclusion

In this paper, the concept of fuzzy Γ-semirings is introduced and we established a one-one correspondence between fuzzy Γ-semiring and its level set. Further we investigated a one-one correspondence between fuzzy left(right) ideal of a Γ-semiring and its level set left(right) ideal of a Γ-semiring. We are expecting that these structures are useful in developing fuzzy prime ideals, fuzzy maximal ideals and fuzzy semiprime ideals of a Γ-semiring.

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References

[1] Dutta T. K, Sujit Kumar Sardar, Sarbani Goswami "Fuzzy ideals extension in Γ-semirings", International Mathematical Forum, Vol.6, no.18, 2011, 857-866.

[2] Dutta and Chanda, "Structures of Fuzzy ideals of Γ-ring", Bull.Malays.Math.Sci.Soc.(2)28(1), 2005, 9-18.

[3] Hedayati. H, Shum K.P, "An Introduction to Γ-semirings ", International Journal of Algebra, Vol. 5, no.15, 2011, 709-726.

[4] Jun, Y.B. and Lee, C.Y., "Fuzzy Γ-rings ", Pusan Kyongnam Math. J. 8, 1992, 163-170.

[5] John N Mordeson and D.S.Malik, "Fuzzy Commutative Algebra", World Scientific Publishing Co. Pte. Ltd.

[6] M.K.Rao, "Γ-semiring 1", Southeast Asian Bulletin of Maths, 19, 1995, 49-54.

[7] Nobusawa. N, "On generalization of the ring theory ", Osaka J. Math. 1, 1978, 185-190.

[8] Rajesh Kumar, "Fuzzy Algebra ", University Press, University of Delhi, Delhi-110007.

[9] Zadeh. L.A., "Fuzzy sets", Information and Control 8, 1965, 338-353.