

**$(\gamma, \beta)$ - $P_S$ -IRRESOLUTE AND  
 $(\gamma, \beta)$ - $P_S$ -CONTINUOUS FUNCTIONS**

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**Abstract:** This paper introduces some new types of functions called  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -continuous by using  $\gamma$ - $P_S$ -open sets in topological spaces  $(X, \tau)$ . From  $\gamma$ - $P_S$ -open and  $\gamma$ - $P_S$ -closed sets, some other types of  $\gamma$ - $P_S$ -functions can also be defined. Moreover, some basic properties and preservation theorems of these functions are obtained. In addition, we investigate basic characterizations and properties of these  $\gamma$ - $P_S$ - functions. Finally, some compositions of these  $\gamma$ - $P_S$ - functions are given.

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**Key Words:**  $\gamma$ - $P_S$ -open set,  $\gamma$ - $P_S$ -closed set,  $(\gamma, \beta)$ - $P_S$ -irresolute function,  $(\gamma, \beta)$ - $P_S$ -continuous function

## 1. Introduction

Kasahara [7] defined the concept of  $\alpha$ -closed graphs of an operation on  $\tau$ . Later, Ogata [10] renamed the operation  $\alpha$  as  $\gamma$  operation on  $\tau$ . He defined  $\gamma$ -open sets and introduced the notion of  $\tau_\gamma$  which is the class of all  $\gamma$ -open sets in a topological space  $(X, \tau)$ . Further study by Krishnan and Balachandran ([8], [9]) defined two types of sets called  $\gamma$ -preopen and  $\gamma$ -semiopen sets. Recently, Asaad, Ahmad and Omar [1] introduced the notion of  $\gamma$ -regular-open sets. They

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also introduced the notion of  $\gamma$ - $P_S$ -open sets [2] which lies strictly between the classes of  $\gamma$ -regular-open set and  $\gamma$ -preopen set. By using this set, they defined a new type of function called  $\gamma$ - $P_S$ -continuous and studies some of its basic properties [3].

In the present paper, we define some new types of  $\gamma$ - $P_S$ - functions called  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -continuous by using  $\gamma$ - $P_S$ -open sets in topological spaces  $(X, \tau)$ . In addition, we give some basic characterizations and properties of these  $\gamma$ - $P_S$ - functions by using  $\gamma$ - $P_S$ -open and  $\gamma$ - $P_S$ -closed sets are introduced. Finally, some compositions of these  $\gamma$ - $P_S$ - functions are given.

## 2. Preliminaries

Throughout this paper, spaces  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) will always means topological spaces on which no separation axioms are assumed unless explicitly stated. An operation  $\gamma$  on the topology  $\tau$  on  $X$  is a mapping  $\gamma: \tau \rightarrow P(X)$  such that  $U \subseteq \gamma(U)$  for each  $U \in \tau$ , where  $P(X)$  is the power set of  $X$  and  $\gamma(U)$  denotes the value of  $\gamma$  at  $U$  [10]. A nonempty subset  $A$  of a space  $(X, \tau)$  with an operation  $\gamma$  on  $\tau$  is said to be  $\gamma$ -open if for each  $x \in A$ , there exists an open set  $U$  such that  $x \subseteq U$  and  $\gamma(U) \subseteq A$  [10]. The complement of a  $\gamma$ -open set is called a  $\gamma$ -closed. The  $\tau_\gamma$ -closure of a subset  $A$  of  $X$  with an operation  $\gamma$  on  $\tau$  is defined as the intersection of all  $\gamma$ -closed sets of  $X$  containing  $A$  and it is denoted by  $\tau_\gamma\text{-Cl}(A)$  [10], and the  $\tau_\gamma$ -interior of a subset  $A$  of  $X$  with an operation  $\gamma$  on  $\tau$  is defined as the union of all  $\gamma$ -open sets of  $X$  contained in  $A$  and it is denoted by  $\tau_\gamma\text{-Int}(A)$  [9]. A topological space  $(X, \tau)$  is said to be  $\gamma$ -regular if for each  $x \in X$  and for each open neighborhood  $V$  of  $x$ , there exists an open neighborhood  $U$  of  $x$  such that  $\gamma(U) \subseteq V$  [10]. Throughout of this paper,  $\gamma$  and  $\beta$  be operations on  $\tau$  and  $\sigma$  respectively.

Now we begin to recall some known notions which are useful in the sequel.

**Definition 2.1.** A subset  $A$  of a topological space  $(X, \tau)$  is said to be:

1.  $\gamma$ -regular-open if  $A = \tau_\gamma\text{-Int}(\tau_\gamma\text{-Cl}(A))$  and  $\gamma$ -regular-closed if  $A = \tau_\gamma\text{-Cl}(\tau_\gamma\text{-Int}(A))$  [1].
2.  $\gamma$ -preopen if  $A \subseteq \tau_\gamma\text{-Int}(\tau_\gamma\text{-Cl}(A))$  and  $\gamma$ -preclosed if  $\tau_\gamma\text{-Cl}(\tau_\gamma\text{-Int}(A)) \subseteq A$  [8].
3.  $\gamma$ -semiopen if  $A \subseteq \tau_\gamma\text{-Cl}(\tau_\gamma\text{-Int}(A))$  and  $\gamma$ -semiclosed if  $\tau_\gamma\text{-Int}(\tau_\gamma\text{-Cl}(A)) \subseteq A$  [9].

4.  $\gamma$ -dense if  $\tau_\gamma\text{-Cl}(A) = X$  [6].

**Definition 2.2.** [2] A  $\gamma$ -preopen subset  $A$  of a topological space  $(X, \tau)$  is called  $\gamma$ - $P_S$ -open if for each  $x \in A$ , there exists a  $\gamma$ -semiclosed set  $F$  such that  $x \in F \subseteq A$ . The complement of a  $\gamma$ - $P_S$ -open set of  $X$  is called  $\gamma$ - $P_S$ -closed.

The class of all  $\gamma$ - $P_S$ -open and  $\gamma$ -preopen subsets of a topological space  $(X, \tau)$  are denoted by  $\tau_\gamma\text{-}P_S O(X)$  and  $\tau_\gamma\text{-}P O(X)$  respectively.

**Lemma 2.3.** [2] A subset  $A$  of  $X$  is  $\gamma$ - $P_S$ -open if and only if  $A$  is  $\gamma$ -preopen set and it is a union of  $\gamma$ -semiclosed sets.

**Definition 2.4.** A subset  $N$  of a topological space  $(X, \tau)$  is called a  $\gamma$ - $P_S$ -neighbourhood of a point  $x \in X$ , if there exists a  $\gamma$ - $P_S$ -open set  $U$  in  $X$  containinig  $x$  such that  $U \subseteq N$ .

**Definition 2.5.** [2] For any subset  $A$  of a space  $X$ . Then:

1. the  $\gamma$ - $P_S$ -boundary of  $A$  is defined as  $\tau_\gamma\text{-}P_S Cl(A) \setminus \tau_\gamma\text{-}P_S Int(A)$  and it is denoted by  $\tau_\gamma\text{-}P_S Bd(A)$ .
2. the  $\gamma$ - $P_S$ -derived set of  $A$  is defined as  $\{x : \text{for every } \gamma\text{-}P_S\text{-open set } U \text{ containing } x, U \cap A \setminus \{x\} \neq \emptyset\}$  and it is denoted by  $\tau_\gamma\text{-}P_S D(A)$ .

**Lemma 2.6.** [2] For any subset  $A$  of a space  $X$ . Then the following statements are true:

1.  $\tau_\gamma\text{-}P_S Cl(A)$  is the smallest  $\gamma$ - $P_S$ -closed set of  $X$  containing  $A$ .
2.  $\tau_\gamma\text{-}P_S Int(A)$  is the largest  $\gamma$ - $P_S$ -open set of  $X$  contained in  $A$ .
3.  $A$  is  $\gamma$ - $P_S$ -closed if and only if  $\tau_\gamma\text{-}P_S Cl(A) = A$ , and  $A$  is  $\gamma$ - $P_S$ -open if and only if  $\tau_\gamma\text{-}P_S Int(A) = A$ .
4.  $\tau_\gamma\text{-}P_S Cl(A) = X \setminus \tau_\gamma\text{-}P_S Int(X \setminus A)$  and  $\tau_\gamma\text{-}P_S Int(A) = X \setminus \tau_\gamma\text{-}P_S Cl(X \setminus A)$ .
5.  $A$  is  $\gamma$ - $P_S$ -closed if and only if  $\tau_\gamma\text{-}P_S Bd(A) \subseteq A$ .
6.  $\tau_\gamma\text{-}P_S D(A) \subseteq \tau_\gamma\text{-}P_S Cl(A)$ .
7.  $A$  is  $\gamma$ - $P_S$ -closed if and only if  $\tau_\gamma\text{-}P_S D(A) \subseteq A$ .

**Definition 2.7.** [4] A subset  $A$  of a space  $(X, \tau)$  is said to be  $\gamma$ - $P_S$ -generalized closed ( $\gamma$ - $P_S$ - $g$ -closed) if  $\tau_\gamma\text{-}P_S Cl(A) \subseteq G$  whenever  $A \subseteq G$  and  $G$  is a  $\gamma$ - $P_S$ -open set in  $X$ .

**Definition 2.8.** A topological space  $(X, \tau)$  is said to be:

1.  $\gamma$ -locally indiscrete if every  $\gamma$ -open subset of  $X$  is  $\gamma$ -closed, or every  $\gamma$ -closed subset of  $X$  is  $\gamma$ -open [1].
2.  $\gamma$ -hyperconnected if every nonempty  $\gamma$ -open subset of  $X$  is  $\gamma$ -dense [1].
3.  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$  if every  $\gamma$ - $P_S$ - $g$ -closed set of  $X$  is  $\gamma$ - $P_S$ -closed [4].
4.  $\gamma$ -semi $T_1$  if for each pair of distinct points  $x, y$  in  $X$ , there exist two  $\gamma$ -semiopen sets  $U$  and  $V$  such that  $x \in U$  but  $y \notin U$  and  $y \in V$  but  $x \notin V$  [9].

**Theorem 2.9.** *The following statements are true for any space  $(X, \tau)$ :*

1. If  $X$  is  $\gamma$ -locally indiscrete, then  $\tau_\gamma$ - $P_S$  $O(X) = \tau_\gamma$  [2].
2. If  $X$  is  $\gamma$ -semi $T_1$ , then  $\tau_\gamma$ - $P_S$  $O(X) = \tau_\gamma$ - $PO(X)$  [2].
3. If  $X$  is  $\gamma$ -hyperconnected if and only if  $\tau_\gamma$ - $P_S$  $O(X) = \{\phi, X\}$  [2].
4.  $X$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$  if and only if for each element  $x \in X$ , the set  $\{x\}$  is  $\gamma$ - $P_S$ -closed or  $\gamma$ - $P_S$ -open [4].
5.  $X$  is  $\gamma$ -regular, then  $\tau_\gamma = \tau$  [10].

**Definition 2.10.** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called:

1.  $\gamma$ - $P_S$ -continuous if for each  $P_S$ -open set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$  [3].
2.  $(\gamma, \beta)$ -precontinuous if for each  $\beta$ -preopen set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ -preopen set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$  [8].
3.  $\gamma$ -continuous if for each open set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$  [5].
4.  $\beta$ - $P_S$ -open (resp.,  $\beta$ -open and  $\beta$ - $P_S$ -closed) if for every open (resp., open and closed) set  $V$  of  $X$ ,  $f(V)$  is  $\beta$ - $P_S$ -open (resp.,  $\beta$ -open and  $\beta$ - $P_S$ -closed) set in  $Y$  [3].

### 3. $(\gamma, \beta)$ - $P_S$ -Irresolute and $(\gamma, \beta)$ - $P_S$ -Continuous Functions

In this section, we introduce three types of  $\gamma$ - $P_S$ - functions called  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -continuous by using  $\gamma$ - $P_S$ -open set. Also we give relations between these functions and  $\gamma$ - $P_S$ -continuous function.

**Definition 3.1.** Let  $(X, \tau)$  and  $(Y, \sigma)$  be two topological spaces. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $(\gamma, \beta)$ - $P_S$ -irresolute (resp.,  $(\gamma, \beta)$ - $P_S$ -continuous) at a point  $x \in X$  if for each  $\beta$ - $P_S$ -open (resp.,  $\beta$ -open) set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ . If  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute (resp.,  $(\gamma, \beta)$ - $P_S$ -continuous) at every point  $x$  in  $X$ , then  $f$  is said to be  $(\gamma, \beta)$ - $P_S$ -irresolute (resp.,  $(\gamma, \beta)$ - $P_S$ -continuous).

**Remark 3.2.** It is clear from the Definition 2.10 (1) and Definition 3.1 that every  $\gamma$ - $P_S$ -continuous function is  $(\gamma, \beta)$ - $P_S$ -continuous since every  $\beta$ -open set is open, where  $\beta$  is an operation on  $\sigma$ . However, the converse is not true in general as it can be seen from the following example.

**Example 3.3.** Let  $X = \{a, b, c\}$  with the topology  $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$  and  $Y = \{1, 2, 3\}$  with the topology  $\sigma = \{\phi, Y, \{2\}, \{3\}, \{2, 3\}\}$ . Define operations  $\gamma: \tau \rightarrow P(X)$  and  $\beta: \sigma \rightarrow P(Y)$  as follows: for every  $A \in \tau$  and  $B \in \sigma$

$$\gamma(A) = \begin{cases} A & \text{if } a \in A \\ Cl(A) & \text{if } a \notin A \end{cases}$$

$$\beta(B) = \begin{cases} B & \text{if } B = \{2\} \\ Cl(B) & \text{if } B \neq \{2\} \end{cases}$$

Then  $\sigma_\beta = \{\phi, \{2\}, Y\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined as follows:

$$f(x) = \begin{cases} 2 & \text{if } x = a \\ 3 & \text{if } x = b \\ 1 & \text{if } x = c \end{cases}$$

Clearly,  $\tau = \{\phi, X, \{a\}, \{a, b\}, \{b, c\}\}$ ,  $\tau_{\gamma-P_S}O(X) = \{\phi, \{a\}, \{a, c\}, \{b, c\}, X\}$  and  $\sigma_\beta = \{\phi, \{2\}, Y\}$ . Let  $f: (X, \tau) \rightarrow (X, \sigma)$  be a function defined as follows:

$$f(x) = \begin{cases} 2 & \text{if } x = a \\ 3 & \text{if } x = b \\ 1 & \text{if } x = c \end{cases}$$

Then  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous, but it is not  $\gamma$ - $P_S$ -continuous since  $\{3\}$  is an open set in  $(Y, \sigma)$  containing  $f(b) = 3$ , but there exist no  $\gamma$ - $P_S$ -open set  $U$  in  $(X, \tau)$  containing  $b$  such that  $f(U) \subseteq \{3\}$ .

**Remark 3.4.** The relation between  $(\gamma, \beta)$ - $P_S$ -irresolute function and  $(\gamma, \beta)$ - $P_S$ -continuous function are independent. Similarly the relation between  $(\gamma, \beta)$ - $P_S$ -irresolute function and  $\gamma$ - $P_S$ -continuous function are independent, as shown from the following examples.

**Example 3.5.** Let  $(X, \tau)$  be a topological space as in Example 3.3. Suppose that  $Y = \{1, 2, 3\}$  and  $\sigma = \{\phi, Y, \{2\}, \{2, 3\}\}$  be a topology on  $Y$ . Define an operation  $\beta$  on  $\sigma$  such that  $\beta: \sigma \rightarrow P(Y)$  by  $\beta(B) = B$  for all  $B \in \sigma$ . Then  $\sigma_\beta$ - $P_S O(Y) = \{\phi, Y\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined as follows:

$$f(x) = \begin{cases} 3 & \text{if } x = a \\ 2 & \text{if } x = b \\ 1 & \text{if } x = c \end{cases}$$

Then the function  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute, but  $f$  is not  $(\gamma, \beta)$ - $P_S$ -continuous since  $\{2\}$  is a  $\beta$ -open set in  $(Y, \sigma)$  containing  $f(b) = 2$ , but there exist no  $\gamma$ - $P_S$ -open set  $U$  in  $(X, \tau)$  containing  $b$  such that  $f(U) \subseteq \{2\}$ . By Remark 3.2,  $f$  is not  $\gamma$ - $P_S$ -continuous.

**Example 3.6.** Consider the space  $X = \{a, b, c\}$  with the topologies  $\tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$  and  $\sigma = \{\phi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ . Define the operations  $\gamma$  and  $\beta$  on  $\tau$  and  $\sigma$  respectively as follows: For every  $A \in \tau$ ,  $\gamma(A) = A$  and for every  $B \in \sigma$

$$\beta(B) = \begin{cases} B & \text{if } c \in B \\ Cl(B) & \text{if } c \notin B \end{cases}$$

Obviously,  $\tau_\gamma = \tau = \tau_\gamma$ - $P_S O(X)$ ,  $\sigma_\beta = \{\phi, X, \{c\}, \{a, b\}, \{b, c\}\}$  and  $\sigma_\beta$ - $P_S O(X) = \{\phi, X, \{c\}, \{a, b\}, \{a, c\}\}$ .

Define a function  $f: (X, \tau) \rightarrow (X, \sigma)$  as follows:

$$f(x) = \begin{cases} b & \text{if } x \in \{a, c\} \\ a & \text{if } x = b \end{cases}$$

So, the function  $f$  is both  $(\gamma, \beta)$ - $P_S$ -continuous and  $\gamma$ - $P_S$ -continuous, but  $f$  is not  $(\gamma, \beta)$ - $P_S$ -irresolute since  $\{a, c\}$  is a  $\beta$ - $P_S$ -open set in  $(X, \sigma)$  containing  $f(b) = a$ , there exist no  $\gamma$ - $P_S$ -open set  $U$  in  $(X, \tau)$  containing  $c$  such that  $f(U) \subseteq \{a, c\}$ .

#### 4. Characterizations

We start with the most important characterizations of  $(\gamma, \beta)$ - $P_S$ -irresolute functions.

**Theorem 4.1.** *For any function  $f: (X, \tau) \rightarrow (Y, \sigma)$ . The following properties of  $f$  are equivalent:*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.
2. The inverse image of every  $\beta$ - $P_S$ -open set of  $Y$  is  $\gamma$ - $P_S$ -open set in  $X$ .
3. The inverse image of every  $\beta$ - $P_S$ -closed set of  $Y$  is  $\gamma$ - $P_S$ -closed set in  $X$ .
4.  $f(\tau_\gamma\text{-}P_S\text{Cl}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$ , for every subset  $A$  of  $X$ .
5.  $\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(B))$ , for every subset  $B$  of  $Y$ .
6.  $f^{-1}(\sigma_\beta\text{-}P_S\text{Int}(B)) \subseteq \tau_\gamma\text{-}P_S\text{Int}(f^{-1}(B))$ , for every subset  $B$  of  $Y$ .
7.  $\sigma_\beta\text{-}P_S\text{Int}(f(A)) \subseteq f(\tau_\gamma\text{-}P_S\text{Int}(A))$ , for every subset  $A$  of  $X$ .

*Proof.* (1)  $\Rightarrow$  (2) Let  $V$  be any  $\beta$ - $P_S$ -open set in  $Y$ . We have to show that  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ . Let  $x \in f^{-1}(V)$ . Then  $f(x) \in V$ . By (1), there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ . Which implies that  $x \in U \subseteq f^{-1}(V)$ . Therefore,  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ .

(2)  $\Rightarrow$  (3) Let  $F$  be any  $\beta$ - $P_S$ -closed set of  $Y$ . Then  $Y \setminus F$  is a  $\beta$ - $P_S$ -open set of  $Y$ . By (2),  $f^{-1}(Y \setminus F) = X \setminus f^{-1}(F)$  is  $\gamma$ - $P_S$ -open set in  $X$  and hence  $f^{-1}(F)$  is  $\gamma$ - $P_S$ -closed set in  $X$ .

(3)  $\Rightarrow$  (4) Let  $A$  be any subset of  $X$ . Then  $f(A) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$  and hence  $A \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(f(A)))$ . Since  $\sigma_\beta\text{-}P_S\text{Cl}(f(A))$  is  $\beta$ - $P_S$ -closed set in  $Y$ . Then by (3), we have  $f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(f(A)))$  is  $\gamma$ - $P_S$ -closed set in  $X$ . Therefore,  $\tau_\gamma\text{-}P_S\text{Cl}(A) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(f(A)))$ . Hence  $f(\tau_\gamma\text{-}P_S\text{Cl}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$ .

(4)  $\Rightarrow$  (5) Let  $B$  be any subset of  $Y$ . Then  $f^{-1}(B)$  is a subset of  $X$ . By (4), we have  $f(\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B))) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(f^{-1}(B))) = \sigma_\beta\text{-}P_S\text{Cl}(B)$ . Hence  $\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(B))$ .

(5)  $\Leftrightarrow$  (6) Let  $B$  be any subset of  $Y$ . Then apply (5) to  $Y \setminus B$  we obtain  $\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(Y \setminus B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(Y \setminus B)) \Leftrightarrow \tau_\gamma\text{-}P_S\text{Cl}(X \setminus f^{-1}(B)) \subseteq f^{-1}(Y \setminus \sigma_\beta\text{-}P_S\text{Int}(B)) \Leftrightarrow X \setminus \tau_\gamma\text{-}P_S\text{Int}(f^{-1}(B)) \subseteq X \setminus f^{-1}(\sigma_\beta\text{-}P_S\text{Int}(B)) \Leftrightarrow f^{-1}(\sigma_\beta\text{-}P_S\text{Int}(B)) \subseteq \tau_\gamma\text{-}P_S\text{Int}(f^{-1}(B))$ . Therefore,  $f^{-1}(\sigma_\beta\text{-}P_S\text{Int}(B)) \subseteq \tau_\gamma\text{-}P_S\text{Int}(f^{-1}(B))$ .

(6)  $\Rightarrow$  (7) Let  $A$  be any subset of  $X$ . Then  $f(A)$  is a subset of  $Y$ . By (6), we have  $f^{-1}(\sigma_{\beta}\text{-}P_S\text{Int}(f(A))) \subseteq \tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(f(A))) = \tau_{\gamma}\text{-}P_S\text{Int}(A)$ . Therefore,  $\sigma_{\beta}\text{-}P_S\text{Int}(f(A)) \subseteq f(\tau_{\gamma}\text{-}P_S\text{Int}(A))$ .

(7)  $\Rightarrow$  (1) Let  $x \in X$  and let  $V$  be any  $\beta\text{-}P_S$ -open set of  $Y$  containing  $f(x)$ . Then  $x \in f^{-1}(V)$  and  $f^{-1}(V)$  is a subset of  $X$ . By (7), we have  $\sigma_{\beta}\text{-}P_S\text{Int}(f(f^{-1}(V))) \subseteq f(\tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(V)))$ . Then  $\sigma_{\beta}\text{-}P_S\text{Int}(V) \subseteq f(\tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(V)))$ . Since  $V$  is a  $\beta\text{-}P_S$ -open set. Then  $V \subseteq f(\tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(V)))$  implies that  $f^{-1}(V) \subseteq \tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(V))$ . Therefore,  $f^{-1}(V)$  is  $\gamma\text{-}P_S$ -open set in  $X$  which contains  $x$  and clearly  $f(f^{-1}(V)) \subseteq V$ . Hence  $f$  is  $(\gamma, \beta)\text{-}P_S$ -irresolute function.  $\square$

**Theorem 4.2.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function. Then the following statements are equivalent:*

1.  $f$  is  $(\gamma, \beta)\text{-}P_S$ -continuous.
2.  $f^{-1}(V)$  is  $\gamma\text{-}P_S$ -open set in  $X$ , for every  $\beta$ -open set  $V$  in  $Y$ .
3.  $f^{-1}(F)$  is  $\gamma\text{-}P_S$ -closed set in  $X$ , for every  $\beta$ -closed set  $F$  in  $Y$ .
4.  $f(\tau_{\gamma}\text{-}P_S\text{Cl}(A)) \subseteq \sigma_{\beta}\text{-Cl}(f(A))$ , for every subset  $A$  of  $X$ .
5.  $\sigma_{\beta}\text{-Int}(f(A)) \subseteq f(\tau_{\gamma}\text{-}P_S\text{Int}(A))$ , for every subset  $A$  of  $X$ .
6.  $f^{-1}(\sigma_{\beta}\text{-Int}(B)) \subseteq \tau_{\gamma}\text{-}P_S\text{Int}(f^{-1}(B))$ , for every subset  $B$  of  $Y$ .
7.  $\tau_{\gamma}\text{-}P_S\text{Cl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\beta}\text{-Cl}(B))$ , for every subset  $B$  of  $Y$ .

*Proof.* Similar to Theorem 4.1 and hence it is omitted.  $\square$

**Theorem 4.3.** *The following properties are equivalent for any function  $f: (X, \tau) \rightarrow (Y, \sigma)$ :*

1.  $f$  is  $(\gamma, \beta)\text{-}P_S$ -irresolute.
2. For every  $x \in X$  and for every  $\beta\text{-}P_S$ -neighbourhood  $N$  of  $Y$  such that  $f(x) \in N$ , there exists a  $\gamma\text{-}P_S$ -neighbourhood  $M$  of  $X$  such that  $x \in M$  and  $f(M) \subseteq N$ .
3. The inverse image of every  $\beta\text{-}P_S$ -neighbourhood of  $f(x)$  is  $\gamma\text{-}P_S$ -neighbourhood of  $x \in X$ .

*Proof.* It is clear and hence it is omitted.  $\square$



**Lemma 4.4.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a  $\gamma$ -continuous and  $\beta$ -open function, then the following statements are true:*

1. *If  $V$  is  $\beta$ -preopen set of  $Y$ , then  $f^{-1}(V)$  is  $\gamma$ -preopen set in  $X$ .*
2. *If  $F$  is  $\beta$ -semiclosed set of  $Y$ , then  $f^{-1}(F)$  is  $\gamma$ -semiclosed set in  $X$ .*

**Lemma 4.5.** *If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a  $\gamma$ -continuous and  $\beta$ -open function and  $V$  is  $\beta$ - $P_S$ -open set of  $Y$ , then  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ .*

*Proof.* Let  $V$  be a  $\beta$ - $P_S$ -open set of  $Y$ , then  $V$  is a  $\beta$ -preopen set of  $Y$  and  $V = \bigcup_{i \in I} F_i$  where  $F_i$  is  $\beta$ -semiclosed set in  $Y$  for each  $i$ . Then  $f^{-1}(V) = f^{-1}(\bigcup_{i \in I} F_i) = \bigcup_{i \in I} f^{-1}(F_i)$  where  $F_i$  is  $\beta$ -semiclosed set in  $Y$  for each  $i$ . Since  $f$  is a  $\gamma$ -continuous and  $\beta$ -open function. Then by Lemma 4.4 (1),  $f^{-1}(V)$  is  $\gamma$ -preopen set of  $X$  and by Lemma 4.4 (2),  $f^{-1}(F_i)$  is  $\gamma$ -semiclosed set of  $X$  for each  $i$ . Hence by Lemma 2.3,  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ .  $\square$

**Corollary 4.6.** *If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a  $\gamma$ -continuous and  $\beta$ -open function and  $F$  is  $\beta$ - $P_S$ -closed set of  $Y$ , then  $f^{-1}(F)$  is  $\gamma$ - $P_S$ -closed set in  $X$ .*

**Lemma 4.7.** *If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is both  $\gamma$ -continuous and  $\beta$ -open, then  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.*

*Proof.* The proof follows directly from Lemma 4.5 and Theorem 4.1.  $\square$

Some other characterizations of  $(\gamma, \beta)$ - $P_S$ -irresolute functions are mentioned in the following.

**Theorem 4.8.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function. Then the following properties are equivalent:*

1.  *$f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.*
2.  *$\tau_\gamma$ - $P_S$  $Bd(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta$ - $P_S$  $Bd(B))$ , for each subset  $B$  of  $Y$ .*
3.  *$f(\tau_\gamma$ - $P_S$  $Bd(A)) \subseteq \sigma_\beta$ - $P_S$  $Bd(f(A))$ , for each subset  $A$  of  $X$ .*

*Proof.* (1)  $\Rightarrow$  (2). Let  $f$  be a  $(\gamma, \beta)$ - $P_S$ -irresolute function and  $B$  be any subset of  $(Y, \sigma)$ . Then by Theorem 4.1 (2) and (5), we have  $\tau_\gamma$ - $P_S$  $Bd(f^{-1}(B)) = \tau_\gamma$ - $P_S$  $Cl(f^{-1}(B)) \setminus \tau_\gamma$ - $P_S$  $Int(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta$ - $P_S$  $Cl(B)) \setminus \tau_\gamma$ - $P_S$  $Int(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta$ - $P_S$  $Cl(B)) \setminus \tau_\gamma$ - $P_S$  $Int(f^{-1}(\sigma_\beta$ - $P_S$  $Int(B))) = f^{-1}(\sigma_\beta$ - $P_S$  $Cl(B)) \setminus f^{-1}(\sigma_\beta$ - $P_S$  $Int(B)) = f^{-1}(\sigma_\beta$ - $P_S$  $Cl(B)) \setminus \sigma_\beta$ - $P_S$  $Int(B) = f^{-1}(\sigma_\beta$ - $P_S$  $Bd(B))$ . Therefore,  $\tau_\gamma$ - $P_S$  $Bd(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta$ - $P_S$  $Bd(B))$ .

(2)  $\Rightarrow$  (3). Let  $A$  be any subset of  $X$ . Then  $f(A)$  is a subset of  $Y$ . Then by (2), we have  $\tau_\gamma$ - $P_S$  $Bd(f^{-1}(f(A))) \subseteq f^{-1}(\sigma_\beta$ - $P_S$  $Bd(f(A)))$  implies

that  $\tau_\gamma\text{-}P_S\text{Bd}(A) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Bd}(f(A)))$  and hence  $f(\tau_\gamma\text{-}P_S\text{Bd}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Bd}(f(A))$ . This completes the proof.

(3)  $\Rightarrow$  (1). Let  $E$  be any  $\beta\text{-}P_S$ -closed set in  $Y$ . Then  $f^{-1}(E)$  is a subset of  $X$ . So by using part (3), we have  $f(\tau_\gamma\text{-}P_S\text{Bd}(f^{-1}(E))) \subseteq \sigma_\beta\text{-}P_S\text{Bd}(f(f^{-1}(E))) = \sigma_\beta\text{-}P_S\text{Bd}(E)$  implies that  $f(\tau_\gamma\text{-}P_S\text{Bd}(f^{-1}(E))) \subseteq \sigma_\beta\text{-}P_S\text{Bd}(E) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(E) = E$  and hence  $f(\tau_\gamma\text{-}P_S\text{Bd}(f^{-1}(E))) \subseteq E$ . This implies that  $\tau_\gamma\text{-}P_S\text{Bd}(f^{-1}(E)) \subseteq f^{-1}(E)$ . Thus, by Lemma 2.6 (5),  $f^{-1}(E)$  is  $\gamma\text{-}P_S$ -closed set in  $X$ . Consequently by Theorem 4.1,  $f$  is  $(\gamma, \beta)\text{-}P_S$ -irresolute function.  $\square$

**Theorem 4.9.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function. Then the following properties are equivalent:*

1.  $f$  is  $(\gamma, \beta)\text{-}P_S$ -continuous.
2. For each subset  $A$  in  $X$ ,  $f(\tau_\gamma\text{-}P_S\text{Bd}(A)) \subseteq \sigma_\beta\text{-}Bd(f(A))$ .
3. For each subset  $B$  in  $Y$ ,  $\tau_\gamma\text{-}P_S\text{Bd}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}Bd(B))$ .

*Proof.* The proof is similar to Theorem 4.8, and hence it is omitted.  $\square$

**Theorem 4.10.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function. Then the following properties are equivalent:*

1.  $f$  is  $(\gamma, \beta)\text{-}P_S$ -irresolute.
2.  $f(\tau_\gamma\text{-}P_S\text{D}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$ , for each subset  $A$  of  $X$ .
3.  $\tau_\gamma\text{-}P_S\text{D}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(B))$ , for each subset  $B$  of  $Y$ .

*Proof.* (1)  $\Rightarrow$  (2). Let  $f$  be a  $(\gamma, \beta)\text{-}P_S$ -continuous function and  $A$  be any subset of  $X$ . Then by Theorem 4.1 (4), we have  $f(\tau_\gamma\text{-}P_S\text{Cl}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$ . Then by Lemma 2.6 (6), we obtain  $f(\tau_\gamma\text{-}P_S\text{D}(A)) \subseteq f(\tau_\gamma\text{-}P_S\text{Cl}(A))$  which implies that  $f(\tau_\gamma\text{-}P_S\text{D}(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(A))$ .

(2)  $\Rightarrow$  (3). Let  $B$  be any subset of  $Y$ . Then  $f^{-1}(B)$  is a subset of  $X$ . Then by hypothesis, we get  $f(\tau_\gamma\text{-}P_S\text{D}(f^{-1}(B))) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(f^{-1}(B))) = \sigma_\beta\text{-}P_S\text{Cl}(B)$  and hence  $\tau_\gamma\text{-}P_S\text{D}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(B))$ . This completes the proof.

(3)  $\Rightarrow$  (1). Let  $F$  be any  $\beta\text{-}P_S$ -closed set in  $Y$ . Then by (3), we have  $\tau_\gamma\text{-}P_S\text{D}(f^{-1}(F)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(F)) = f^{-1}(F)$  and hence  $\tau_\gamma\text{-}P_S\text{D}(f^{-1}(F)) \subseteq f^{-1}(F)$ . So by Lemma 2.6 (7), we get  $f^{-1}(F)$  is  $\gamma\text{-}P_S$ -closed set in  $X$ . Therefore, by Theorem 4.1,  $f$  is  $(\gamma, \beta)\text{-}P_S$ -irresolute function.  $\square$

**Theorem 4.11.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function. Then the following properties are equivalent:*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous.
2.  $f(\tau_\gamma\text{-}P_S D(A)) \subseteq \sigma_\beta\text{-}Cl(f(A))$ , for each subset  $A$  of  $X$ .
3.  $\tau_\gamma\text{-}P_S D(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}Cl(B))$ , for each subset  $B$  of  $Y$ .

*Proof.* The proof is similar to Theorem 4.10, and hence it is omitted.  $\square$

**Proposition 4.12.** *If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute, then for each  $x \in X$  and each  $\beta$ - $P_S$ -open set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ -semiclosed set  $F$  in  $X$  such that  $x \in F$  and  $f(F) \subseteq V$ . Furthermore, if  $f$  is  $(\gamma, \beta)$ -precontinuous, then the converse also holds.*

*Proof.* Suppose  $f$  be a  $(\gamma, \beta)$ - $P_S$ -irresolute function and let  $V$  be any  $\beta$ - $P_S$ -open set of  $Y$  such that  $f(x) \in V$ , for each  $x \in X$ . Then there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  such that  $x \in U$  and  $f(U) \subseteq V$ . Since  $U$  is  $\gamma$ - $P_S$ -open set. Then for each  $x \in U$ , there exists a  $\gamma$ -semiclosed set  $F$  of  $X$  such that  $x \in F \subseteq U$ . Therefore, we have  $f(F) \subseteq V$ .

Now suppose that  $f$  is  $(\gamma, \beta)$ -precontinuous function. Let  $V$  be any  $\beta$ - $P_S$ -open set of  $Y$ . We have to show that  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ . Since every  $\beta$ - $P_S$ -open set is  $\beta$ -preopen, then  $f^{-1}(V)$  is  $\gamma$ -preopen set in  $X$ . Let  $x \in f^{-1}(V)$ . Then  $f(x) \in V$ . By hypothesis, there exists a  $\gamma$ -semiclosed set  $F$  of  $X$  containing  $x$  such that  $f(F) \subseteq V$ . Which implies that  $x \in F \subseteq f^{-1}(V)$ . Therefore, by Definition 2.2,  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set in  $X$ . Hence by Theorem 4.1,  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute. This completes the proof.  $\square$

## 5. Properties

**Theorem 5.1.** *Let  $(X, \tau)$  be  $\gamma$ -semi $T_1$  space and  $(Y, \sigma)$  be  $\beta$ -semi $T_1$  space. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute if and only if  $f$  is  $(\gamma, \beta)$ -precontinuous.*

*Proof.* This is an immediate consequence of Theorem 2.9 (2).  $\square$

**Theorem 5.2.** *Let  $(Y, \sigma)$  be  $\beta$ -locally indiscrete space. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute if and only if  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous.*

*Proof.* Follows directly from Theorem 2.9 (1).  $\square$

**Theorem 5.3.** *Let  $(Y, \sigma)$  be  $\beta$ -regular space. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $\gamma$ - $P_S$ -continuous if and only if  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous.*

*Proof.* Follows directly from Theorem 2.9 (5).  $\square$

**Theorem 5.4.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function. If  $(Y, \sigma)$  is  $\beta$ -hyperconnected space, then  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.*

*Proof.* This is an immediate consequence of Theorem 2.9 (3).  $\square$

**Proposition 5.5.** *If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute (resp.  $(\gamma, \beta)$ - $P_S$ -continuous), then the following properties are true:*

1. *for each  $x \in X$  and each  $\beta$ - $P_S$ -open (resp.  $\beta$ -open) set  $V$  of  $Y$  such that  $f(x) \in V$ , there exists a  $\gamma$ -preopen set  $U$  of  $X$  such that  $x \in U$  and  $f(U) \subseteq V$ .*
2. *for each  $x \in X$  and each  $\beta$ -regular-open set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ .*

*Proof.* 1) Since every  $\gamma$ - $P_S$ -open set of  $X$  is  $\gamma$ -preopen, then by using this in Definition 3.1 we get the proof.

2) Since every  $\beta$ -regular-open set of  $Y$  is both  $\beta$ -open and  $\beta$ - $P_S$ -open, then the proof follows directly from Definition 3.1.  $\square$

**Proposition 5.6.** *A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute (resp.  $(\gamma, \beta)$ - $P_S$ -continuous), if the following properties are true:*

1. *for each  $x \in X$  and each  $\beta$ -preopen set  $V$  of  $Y$  such that  $f(x) \in V$ , there exists a  $\gamma$ - $P_S$ -open set  $U$  of  $X$  such that  $x \in U$  and  $f(U) \subseteq V$ .*
2. *for each  $x \in X$  and each  $\beta$ - $P_S$ -open (resp.  $\beta$ -open) set  $V$  of  $Y$  containing  $f(x)$ , there exists a  $\gamma$ -regular-open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ .*

*Proof.* 1) The proof is clear since every  $\beta$ - $P_S$ -open (resp.  $\beta$ -open) set of  $Y$  is  $\beta$ -preopen.

2) Obvious since every  $\gamma$ -regular-open set of  $X$  is  $\gamma$ - $P_S$ -open and hence it is omitted.  $\square$

**Theorem 5.7.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function, then:  $X \setminus \tau_{\gamma}$ - $P_S C(f) = \cup \{ \tau_{\gamma}$ - $P_S B d(f^{-1}(V)) : V \text{ is a } \beta$ - $P_S$ -open in  $(Y, \sigma)$  such that  $f(x) \in V$  for each  $x \in X \}$ , where  $\tau_{\gamma}$ - $P_S C(f)$  denotes the set of points at which  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute function.*

*Proof.* Let  $x \in \tau_\gamma\text{-}P_S C(f)$ . Then there exists  $\beta$ - $P_S$ -open set  $V$  in  $(Y, \sigma)$  containing  $f(x)$  such that  $f(U) \not\subseteq V$  for every  $\gamma$ - $P_S$ -open set  $U$  of  $(X, \tau)$  containing  $x$ . Hence  $U \cap X \setminus f^{-1}(V) \neq \emptyset$  for every  $\gamma$ - $P_S$ -open set  $U$  of  $(X, \tau)$  containing  $x$ . Therefore,  $x \in \tau_\gamma\text{-}P_S Cl(X \setminus f^{-1}(V))$ . Then  $x \in f^{-1}(V) \cap \tau_\gamma\text{-}P_S Cl(X \setminus f^{-1}(V)) \subseteq \tau_\gamma\text{-}P_S Cl(f^{-1}(V)) \cap \tau_\gamma\text{-}P_S Cl(X \setminus f^{-1}(V)) = \tau_\gamma\text{-}P_S Bd(f^{-1}(V))$ . Then  $X \setminus \tau_\gamma\text{-}P_S C(f) \subseteq \cup \{ \tau_\gamma\text{-}P_S Bd(f^{-1}(V)) : V \text{ is } \beta\text{-}P_S\text{-open in } (Y, \sigma) \text{ such that } f(x) \in V \text{ for each } x \in X \}$ .

Conversely, let  $x \notin X \setminus \tau_\gamma\text{-}P_S C(f)$ . Then for each  $\beta$ - $P_S$ -open set  $V$  in  $(Y, \sigma)$  containing  $f(x)$ ,  $f^{-1}(V)$  is  $\gamma$ - $P_S$ -open set of  $(X, \tau)$  containing  $x$ . So  $x \in \tau_\gamma\text{-}P_S Int(f^{-1}(V))$  and hence  $x \notin \tau_\gamma\text{-}P_S Bd(f^{-1}(V))$  for every  $\beta$ - $P_S$ -open set  $V$  in  $(Y, \sigma)$  containing  $f(x)$ . Therefore,  $X \setminus \tau_\gamma\text{-}P_S C(f) \supseteq \cup \{ \tau_\gamma\text{-}P_S Bd(f^{-1}(V)) : V \text{ is } \beta\text{-}P_S\text{-open in } (Y, \sigma) \text{ such that } f(x) \in V \text{ for each } x \in X \}$ .  $\square$

The proof of the following theorem is similar to Theorem 5.7 and is thus omitted.

**Theorem 5.8.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be any function, then:  $X \setminus \tau_\gamma\text{-}P_S C(f) = \cup \{ \tau_\gamma\text{-}P_S Bd(f^{-1}(V)) : V \text{ is a } \beta\text{-open in } (Y, \sigma) \text{ such that } f(x) \in V \text{ for each } x \in X \}$ , where  $\tau_\gamma\text{-}P_S C(f)$  denotes the set of points at which  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous function.*

Now, we will define more types of  $\gamma$ - $P_S$ - functions by using  $\gamma$ - $P_S$ -open set which are defined as follows.

**Definition 5.9.** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $(\gamma, \beta)$ - $P_S$ -open (resp.  $(\gamma, \beta P_S)$ -open) if for every  $\gamma$ - $P_S$ -open (resp.  $\gamma$ -open) set  $V$  of  $X$ ,  $f(V)$  is  $\beta$ - $P_S$ -open set in  $Y$ .

**Definition 5.10.** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called  $(\gamma, \beta)$ - $P_S$ -closed (resp.  $(\gamma, \beta P_S)$ -open) if for every  $\gamma$ - $P_S$ -closed ( $\gamma$ -closed) set  $F$  of  $X$ ,  $f(F)$  is  $\beta$ - $P_S$ -closed set in  $Y$ .

**Theorem 5.11.** *A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -open if and only if for every  $x \in X$  and for every  $\gamma$ - $P_S$ -neighbourhood  $N$  of  $x$ , there exists a  $\beta$ - $P_S$ -neighbourhood  $M$  of  $Y$  such that  $f(x) \in M$  and  $M \subseteq f(N)$ .*

*Proof.* Obvious.  $\square$

**Theorem 5.12.** *The following statements are equivalent for a function  $f: (X, \tau) \rightarrow (Y, \sigma)$ :*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -open.
2.  $f(\tau_\gamma\text{-}P_S Int(A)) \subseteq \sigma_\beta\text{-}P_S Int(f(A))$ , for every  $A \subseteq X$ .

3.  $\tau_\gamma\text{-}P_S\text{Int}(f^{-1}(B)) \subseteq f^{-1}(\sigma_\beta\text{-}P_S\text{Int}(B))$ , for every  $B \subseteq Y$ .

*Proof.* The proof is similar to Theorem 4.1. □

**Theorem 5.13.** *The following properties of  $f$  are equivalent for a function  $f: (X, \tau) \rightarrow (Y, \sigma)$ :*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed.
2.  $f^{-1}(\sigma_\beta\text{-}P_S\text{Cl}(B)) \subseteq \tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B))$ , for every  $B \subseteq Y$ .
3.  $\sigma_\beta\text{-}P_S\text{Cl}(f(A)) \subseteq f(\tau_\gamma\text{-}P_S\text{Cl}(A))$ , for every  $A \subseteq X$ .
4.  $\sigma_\beta\text{-}P_S\text{D}(f(A)) \subseteq f(\tau_\gamma\text{-}P_S\text{Cl}(A))$ , for every  $A \subseteq X$ .

*Proof.* The proof is similar to Theorem 4.1. □

**Definition 5.14.** Let  $id: \tau \rightarrow P(X)$  be the identity operation. If  $f$  is  $(id, \beta)$ - $P_S$ -closed, then for every  $\gamma$ - $P_S$ -closed set  $F$  of  $X$ ,  $f(F)$  is  $\beta$ - $P_S$ -closed set in  $Y$ .

**Theorem 5.15.** *If a function  $f$  is bijective and  $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$  is  $(id, \beta)$ - $P_S$ -irresolute, then  $f$  is  $(id, \beta)$ - $P_S$ -closed.*

*Proof.* Follows from Definition 5.14 and Definition 3.1. □

**Theorem 5.16.** *Suppose that a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is both  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed, then:*

1. For every  $\gamma$ - $P_S$ - $g$ -closed set  $A$  of  $(X, \tau)$ , the image  $f(A)$  is  $\beta$ - $P_S$ - $g$ -closed in  $(Y, \sigma)$ .
2. For every  $\beta$ - $P_S$ - $g$ -closed set  $B$  of  $(Y, \sigma)$  the inverse set  $f^{-1}(B)$  is  $\gamma$ - $P_S$ - $g$ -closed in  $(X, \tau)$ .

*Proof.* (1) Let  $G$  be any  $\beta$ - $P_S$ -open set in  $(Y, \sigma)$  such that  $f(A) \subseteq G$ . Since  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute function, then by Theorem 4.1 (2),  $f^{-1}(G)$  is  $\gamma$ - $P_S$ -open set in  $(X, \tau)$ . Since  $A$  is  $\gamma$ - $P_S$ - $g$ -closed and  $A \subseteq f^{-1}(G)$ , we have  $\tau_\gamma\text{-}P_S\text{Cl}(A) \subseteq f^{-1}(G)$ , and hence  $f(\tau_\gamma\text{-}P_S\text{Cl}(A)) \subseteq G$ . Since  $\tau_\gamma\text{-}P_S\text{Cl}(A)$  is  $\gamma$ - $P_S$ -closed set and  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed, then  $f(\tau_\gamma\text{-}P_S\text{Cl}(A))$  is  $\beta$ - $P_S$ -closed set in  $Y$ . Therefore,  $\sigma_\beta\text{-}P_S\text{Cl}(f(A)) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(f(\tau_\gamma\text{-}P_S\text{Cl}(A))) = f(\tau_\gamma\text{-}P_S\text{Cl}(A)) \subseteq G$ . This implies that  $f(A)$  is  $\beta$ - $P_S$ - $g$ -closed in  $(Y, \sigma)$ .

(2) Let  $H$  be any  $\gamma$ - $P_S$ -open set of  $(X, \tau)$  such that  $f^{-1}(B) \subseteq H$ . Let  $C = \tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B)) \cap (X \setminus H)$ , then  $C$  is  $\gamma$ - $P_S$ -closed set in  $(X, \tau)$ . Since  $f$  is

$(\gamma, \beta)$ - $P_S$ -closed function. Then  $f(C)$  is  $\beta$ - $P_S$ -closed in  $(Y, \sigma)$ . Since  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute function, then by using Theorem 4.1 (4), we have  $f(C) = f(\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B))) \cap f(X \setminus H) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(B) \cap f(X \setminus H) \subseteq \sigma_\beta\text{-}P_S\text{Cl}(B) \cap (Y \setminus B)$ . This implies that  $f(C) = \phi$ , and hence  $C = \phi$ . So  $\tau_\gamma\text{-}P_S\text{Cl}(f^{-1}(B)) \subseteq H$ . Therefore,  $f^{-1}(B)$  is  $\gamma$ - $P_S$ - $g$ -closed in  $(X, \tau)$ .  $\square$

**Theorem 5.17.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an injective,  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed function. If  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$ , then  $(X, \tau)$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$ .*

*Proof.* Let  $G$  be any  $\gamma$ - $P_S$ - $g$ -closed set of  $(X, \tau)$ . Since  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed function. Then by Theorem 5.16 (1),  $f(G)$  is  $\beta$ - $P_S$ - $g$ -closed in  $(Y, \sigma)$ . Since  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$ , then  $f(G)$  is  $\beta$ - $P_S$ -closed in  $Y$ . Again, since  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute, then by Theorem 4.1,  $f^{-1}(f(G))$  is  $\gamma$ - $P_S$ -closed in  $X$ . Hence  $G$  is  $\gamma$ - $P_S$ -closed in  $X$  since  $f$  is injective. Therefore, a space  $(X, \tau)$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$ .  $\square$

**Theorem 5.18.** *Let a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective,  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed. If  $(X, \tau)$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$ , then  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$ .*

*Proof.* Let  $H$  be a  $\beta$ - $P_S$ - $g$ -closed set of  $(Y, \sigma)$ . Since a function  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed. Then by Theorem 5.16 (2),  $f^{-1}(H)$  is  $\gamma$ - $P_S$ - $g$ -closed in  $(X, \tau)$ . Since  $(X, \tau)$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$ , then we have,  $f^{-1}(H)$  is  $\gamma$ - $P_S$ -closed set in  $X$ . Again, since  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed function, then  $f(f^{-1}(H))$  is  $\beta$ - $P_S$ -closed in  $Y$ . Therefore,  $H$  is  $\beta$ - $P_S$ -closed in  $Y$  since  $f$  is surjective. Hence  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$  space.  $\square$

**Remark 5.19.** Every  $\beta$ - $P_S$ -open (resp.,  $\beta$ - $P_S$ -closed) function is  $(\gamma, \beta P_S)$ -open (resp.,  $(\gamma, \beta P_S)$ -closed), but the converse is not true as it is shown in the following example.

**Example 5.20.** Let  $X = \{a, b, c\}$  with the topology  $\tau = \{\phi, \{c\}, \{b, c\}, X\}$  and  $Y = \{1, 2, 3\}$  with the topology  $\sigma = \{\phi, Y, \{2\}, \{1, 3\}\}$ . Define operations  $\beta$  on  $\sigma$  by  $\beta(B) = B$  for all  $B \in \sigma$  and  $\gamma$  on  $\tau$  as follows: For every  $A \in \tau$

$$\gamma(A) = \begin{cases} A & \text{if } A = \{c\} \\ X & \text{otherwise} \end{cases}$$

Then  $\tau_\gamma = \{\phi, X, \{c\}\}$  and hence  $\tau_\gamma\text{-}P_S O(X) = \{\phi, X\}$ .

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function defined as follows:

$$f(x) = \begin{cases} 3 & \text{if } x = a \\ 1 & \text{if } x = b \\ 2 & \text{if } x = c \end{cases}$$

So  $f$  is  $(\gamma, \beta P_S)$ -open (resp.,  $(\gamma, \beta P_S)$ -closed) function, but  $f$  is not  $\beta$ - $P_S$ -open (resp.,  $\beta$ - $P_S$ -closed) since  $\{c\} \in \tau$ , but  $f(\{b, c\}) = \{1, 2\}$  is not  $\beta$ - $P_S$ -open set in  $(Y, \sigma)$ . Again since  $\{a\}$  is closed set in  $(X, \tau)$ , but  $f(\{a\}) = \{3\}$  is not  $\beta$ - $P_S$ -closed set in  $(Y, \sigma)$ .

**Theorem 5.21.** *Let  $(X, \tau)$  be  $\gamma$ -semi $T_1$  space and  $(Y, \sigma)$  be  $\beta$ -semi $T_1$  space. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -closed if and only if  $f$  is  $(\gamma, \beta)$ -preclosed.*

*Proof.* This is an immediate consequence of Theorem 2.9 (2). □

**Theorem 5.22.** *Let  $(X, \tau)$  be  $\gamma$ -locally indiscrete space and  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a function, then the following properties of  $f$  are equivalent:*

1.  $(\gamma, \beta)$ - $P_S$ -open.
2.  $(\gamma, \beta)$ - $P_S$ -closed.
3.  $(\gamma, \beta P_S)$ -closed.
4.  $(\gamma, \beta P_S)$ -open.

*Proof.* Follows directly from Theorem 2.9 (1). □

**Theorem 5.23.** *Let  $(X, \tau)$  be  $\gamma$ -regular space. A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta P_S)$ -open (resp.,  $(\gamma, \beta P_S)$ -closed) if and only if  $f$  is  $\beta$ - $P_S$ -open (resp.,  $\beta$ - $P_S$ -closed).*

*Proof.* This is an immediate consequence of Theorem 2.9 (5). □

**Theorem 5.24.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective function. If  $(X, \tau)$  is  $\gamma$ -hyperconnected space, then  $f$  is  $(\gamma, \beta)$ - $P_S$ -open.*

*Proof.* This is an immediate consequence of Theorem 2.9 (3). □

**Theorem 5.25.** *A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -closed if and only if for each subset  $S$  of  $Y$  and each  $\gamma$ - $P_S$ -open set  $O$  in  $X$  containing  $f^{-1}(S)$ , there exists a  $\beta$ - $P_S$ -open set  $R$  in  $Y$  containing  $S$  such that  $f^{-1}(R) \subseteq O$ .*

*Proof.* Suppose that  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed function and let  $O$  be a  $\gamma$ - $P_S$ -open set in  $X$  containing  $f^{-1}(S)$ , where  $S$  is any subset in  $Y$ . Then  $f(X \setminus O)$  is  $\beta$ - $P_S$ -open set in  $Y$ . If we put  $R = Y \setminus f(X \setminus O)$ . Then  $R$  is  $\beta$ - $P_S$ -closed set in  $Y$  such that  $S \subseteq R$  and  $f^{-1}(R) \subseteq O$ .



Conversely, let  $F$  be a  $\gamma$ - $P_S$ -closed set in  $X$ . Let  $S = Y \setminus f(F) \subseteq Y$ . Then  $f^{-1}(S) \subseteq X \setminus F$  and  $X \setminus F$  is  $\gamma$ - $P_S$ -open set in  $X$ . By hypothesis, there exists a  $\beta$ - $P_S$ -open set  $R$  in  $Y$  such that  $S = Y \setminus f(F) \subseteq R$  and  $f^{-1}(R) \subseteq X \setminus F$ . For  $f^{-1}(R) \subseteq X \setminus F$  implies  $R \subseteq f(X \setminus F) \subseteq Y \setminus f(F)$ . Hence  $R = Y \setminus f(F)$ . Since  $R$  is  $\beta$ - $P_S$ -open set in  $Y$ . Then  $f(F)$  is  $\beta$ - $P_S$ -closed set in  $Y$ . Therefore,  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed function.  $\square$

**Theorem 5.26.** *A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta P_S)$ -closed if and only if for each subset  $S$  of  $Y$  and each  $\gamma$ -open set  $O$  in  $X$  containing  $f^{-1}(S)$ , there exists a  $\beta$ - $P_S$ -open set  $R$  in  $Y$  containing  $S$  such that  $f^{-1}(R) \subseteq O$ .*

*Proof.* The proof is similar to Theorem 5.25.  $\square$

**Definition 5.27.** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be  $(\gamma, \beta)$ - $P_S$ -homeomorphism, if  $f$  is bijective,  $(\gamma, \beta)$ - $P_S$ -irresolute and  $f^{-1}$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.

**Theorem 5.28.** *For a bijective function  $f: (X, \tau) \rightarrow (Y, \sigma)$ . The following properties of  $f$  are equivalent:*

1.  $f^{-1}$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.
2.  $f$  is  $(\gamma, \beta)$ - $P_S$ -open.
3.  $f$  is  $(\gamma, \beta)$ - $P_S$ -closed.

*Proof.* Obvious.  $\square$

**Theorem 5.29.** *The following conditions of  $f$  are equivalent for a bijective function  $f: (X, \tau) \rightarrow (Y, \sigma)$ :*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -homeomorphism.
2.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -open.
3.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -closed.
4.  $f(\tau_\gamma\text{-}P_S\text{Cl}(A)) = \sigma_\beta\text{-}P_S\text{Cl}(f(A))$  for each subset  $A$  of  $X$ .

*Proof.* Straightforward.  $\square$

**Proposition 5.30.** *Assume that a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a  $(\gamma, \beta)$ - $P_S$ -homeomorphism. If  $(X, \tau)$  is  $\gamma$ - $P_S$ - $T_{\frac{1}{2}}$ , then  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$ .*

*Proof.* Let  $\{y\}$  be any singleton set of  $(Y, \sigma)$ . Then there exists an element  $x$  of  $X$  such that  $y = f(x)$ . So by hypothesis and Theorem 2.9 (4), we have  $\{x\}$  is  $\gamma$ - $P_S$ -closed or  $\gamma$ - $P_S$ -open set in  $X$ . By using Theorem 4.1,  $\{y\}$  is  $\beta$ - $P_S$ -closed or  $\beta$ - $P_S$ -open set. Then by Theorem 2.9 (4),  $(Y, \sigma)$  is  $\beta$ - $P_S$ - $T_{\frac{1}{2}}$  space.  $\square$

In the end of this section, we shall obtain some conditions under which the composition of two functions is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $(\gamma, \beta)$ - $P_S$ -continuous.

**Theorem 5.31.** *Let  $\alpha$  be an operation on the topological space  $(Z, \rho)$ . Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  be functions. Then the composition function  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -continuous if  $f$  and  $g$  satisfy one of the following conditions:*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $g$  is  $(\beta, \alpha)$ - $P_S$ -continuous.
2.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $g$  is  $\beta$ - $P_S$ -continuous.

*Proof.* (1) Let  $V$  be an  $\alpha$ -open subset of  $(Z, \rho)$ . Since  $g$  is  $(\beta, \alpha)$ - $P_S$ -continuous, then  $g^{-1}(V)$  is  $\beta$ - $P_S$ -open in  $(Y, \sigma)$ . Since  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute, then by Theorem 4.1,  $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $\gamma$ - $P_S$ -open in  $X$ . Therefore,  $g \circ f$  is  $(\gamma, \alpha)$ - $P_S$ -continuous.

(2) The proof follows directly from the part (1) since every  $\beta$ - $P_S$ -continuous is  $(\beta, \alpha)$ - $P_S$ -continuous.  $\square$

**Theorem 5.32.** *Let  $\alpha$  be an operation on the topological space  $(Z, \rho)$ . If the functions  $f: (X, \tau) \rightarrow (Y, \sigma)$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  is  $(\beta, \alpha)$ - $P_S$ -irresolute. Then  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -irresolute.*

*Proof.* It is clear.  $\square$

**Theorem 5.33.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  be functions. Then the composition function  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $\gamma$ - $P_S$ -continuous if  $f$  and  $g$  satisfy one of the following conditions:*

1.  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and  $g$  is  $\beta$ - $P_S$ -continuous.
2.  $f$  is  $(\gamma, \beta)$ - $P_S$ -continuous and  $g$  is  $\beta$ -continuous.

*Proof.* The proof is similar to Theorem 5.31.  $\square$

**Proposition 5.34.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  be any functions and  $\alpha$  be an operation on  $\rho$ . Then the following are holds:*

1. If  $f$  is  $\beta$ -open ( $\beta$ -closed) and  $g$  is  $(\beta, \alpha)$ - $P_S$ -open ( $(\beta, \alpha)$ - $P_S$ -closed), then  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $\alpha$ - $P_S$ -open ( $\alpha$ - $P_S$ -closed).

2. If  $f$  is  $(\gamma, \beta)$ - $P_S$ -open ( $(\gamma, \beta)$ - $P_S$ -closed) and  $g$  is  $(\beta, \alpha)$ - $P_S$ -open ( $(\beta, \alpha)$ - $P_S$ -closed), then  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -open ( $(\gamma, \alpha)$ - $P_S$ -closed).
3. If  $f$  is  $(\gamma, \beta P_S)$ -open ( $(\gamma, \beta P_S)$ -closed) and  $g$  is  $(\beta, \alpha)$ - $P_S$ -open ( $(\beta, \alpha)$ - $P_S$ -closed), then  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha P_S)$ -open ( $(\gamma, \alpha P_S)$ -closed).

*Proof.* It is clear. □

**Proposition 5.35.** *If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a function,  $g: (Y, \sigma) \rightarrow (Z, \rho)$  is  $(\beta, \alpha)$ - $P_S$ -open and injective, and  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -irresolute. Then  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.*

*Proof.* Let  $V$  be an  $\beta$ - $P_S$ -open subset of  $Y$ . Since  $g$  is  $(\beta, \alpha)$ - $P_S$ -open,  $g(V)$  is  $\alpha$ - $P_S$ -open subset of  $Z$ . Since  $g \circ f$  is  $(\gamma, \alpha)$ - $P_S$ -irresolute and  $g$  is injective, then  $f^{-1}(V) = f^{-1}(g^{-1}(g(V))) = (g \circ f)^{-1}(g(V))$  is  $\gamma$ - $P_S$ -open in  $X$ , which proves that  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute. □

**Proposition 5.36.** *Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \rho)$  be any functions and  $\alpha$  be an operation on  $\rho$ . Then the following are holds:*

1. If  $f$  is  $(\gamma, \beta)$ - $P_S$ -open and surjective, and  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -irresolute, then  $g$  is  $(\gamma, \beta)$ - $P_S$ -irresolute.
2. If  $f$  is  $(\gamma, \beta)$ - $P_S$ -irresolute and surjectiv, and  $g \circ f: (X, \tau) \rightarrow (Z, \rho)$  is  $(\gamma, \alpha)$ - $P_S$ -open, then  $g$  is  $(\beta, \alpha)$ - $P_S$ -open.

*Proof.* Similar to Proposition 5.35. □

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