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CYCLICITY ON SOME BK SPACES

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Abstract: In this paper, we give some sufficient conditions for cyclicity of adjoint of the multiplication operator acting on a space of formal power series with coefficients in some BK spaces with BK property.

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1. Introduction

We write ω for the set of all complex sequences $x = (x_k)_{k=0}^{\infty}$. Let ϕ, l_∞ and c_0 denote the set of all finite, bounded and null sequences. We write $l_p = \{x \in \omega : \sum_{k=0}^{\infty} |x_k|^p < \infty\}$ for $1 \leq p < \infty$. By $e^{(n)}$ $(n \in N_0)$, we denote the sequence with $e_n^{(n)} = 1$ and $e_k^{(n)} = 0$ whenever $k \neq n$. For any sequence $x = (x_k)_{k=0}^{\infty}$, let $x^{[n]} = \sum_{k=0}^{n} x_k e^{(k)}$ be its n-section. Given any subset F of ω, we write $\hat{F}$ for the set of all formal power series $\hat{f}$ with $\hat{f}(z) = \sum_{k=0}^{\infty} f_k z^k$ where $f = (f_k)_{k=0}^{\infty} \in F$, regardless of whether or not the series converges for any value of z. Let $\hat{M}_z : \hat{F} \to \hat{\omega}$ be defined by $(\hat{M}_z \hat{f}) = \sum_{k=0}^{\infty} f_k z^{k+1}$.

A BK space is a Banach sequence space with the property that convergence implies coordinatewise convergence. A BK space F containing ϕ is said to have AK if every sequence $f = (f_k)_{k=0}^{\infty} \in F$ has a unique representation $f =$

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$\sum_{k=0}^{\infty} f_k e^{(k)}$, that is $f = \lim_{n\to\infty} f^{[n]}$; it is said to have AD, if ϕ is dense in F. Given any subset F of ω, the set $F^{\beta} = \{a \in \omega : \sum_{k=0}^{\infty} a_k f_k \quad \text{converges for all } f \in F\}$ is called the β-dual of F.

Let F be a normed sequence space and $\hat{F}$ be the space of formal power series with coefficients in F endowed with the norm of F. Then F and $\hat{F}$ are norm isomorphic.

We say that a vector x in a Banach space X is a cyclic vector of a bounded operator A on X if $X = \operatorname{span}\{A^n x : n = 0, 1, 2, \ldots\}$. Here $\operatorname{span}\{\cdot\}$ is the closed linear span of the set $\{\cdot\}$. It is convenient and helpful to introduce the notation $< x, x^* >$ to stand for $x^*(x)$, for $x \in X$ and $x^* \in X^*$.

In this paper, we study the cyclicity of adjoint of the multiplication operator acting on spaces of formal power series with coefficients in some BK spaces with AK property. For some sources on sequence spaces, see [1–5].

2. Main Results

In the main theorem of this paper we give some sufficient conditions for cyclicity of the adjoint of weighted composition operators on BK spaces of formal power series.

Consider $f = \{f_k\}_{k=0}^{\infty}$ and $g = \{g_k\}_{k=0}^{\infty}$ in ω and let $E \subset \omega$. Define $fg = \{f_k g_k\}_{k=0}^{\infty}$ and $g^{-1} \star E = \{f \in \omega : fg \in E\}$. If $\alpha = \{\alpha_k\}_{k=0}^{\infty} \in \omega$ is a given sequence with $\alpha_k \neq 0$ for all k, then by $1/\alpha$ we mean $1/\alpha = \{1/\alpha_k\}_{k=0}^{\infty}$. Write $\hat{F}(\alpha) = (\alpha^{-1} \star F)$ for any subset F of ω. From now on we suppose that $\alpha = \{\alpha_k\}_{k=0}^{\infty} \in \omega$ satisfying $\alpha_0 = 1$ and $\alpha_k \neq 0$ for all $k \geq 1$.

Lemma 2.1. *Let $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then $(l_p(\alpha))^* = l_q(\alpha^{-1})$ where $\alpha^{-1} = \{\alpha_n^{-1}\}_{n=0}^{\infty}$.*

Proof. Since l_p is a BK space with AK property, thus $l_p(\alpha)$ is also a Bk and AK space with respect to the norm $\|f\|_{l_p(\alpha)} = \|\alpha f\|_{l_p}$ for all $f \in l_p(\alpha)$. Furthermore, l_p^* and l_p^{β} are isomorphic. Thus we get $(\alpha^{-1} \star l_p)^{\beta} = (1/\alpha)^{-1} \star l_p^{\beta} = (1/\alpha)^{-1} \star l_p^*$. Hence $(l_p(\alpha))^* = (l_p(\alpha))^{\beta} = l_q(\alpha^{-1})$.

Note that the space $\hat{l}_p(\alpha)$ is a reflexive Banach space and the dual of $\hat{l}_p(\alpha)$ is $\hat{l}_q(\alpha^{-1})$. Here for simplicity we use $||\hat{g}||$ instead of $||\hat{g}||_{\hat{l}_q(\alpha^{-1})}$.

Theorem 2.2. *Let the sequence $\alpha = \{\alpha_n\}$ satisfy $\alpha_n \geq 1$ for all n and $f = \{f_n\}_{n=0}^{\infty}$ be a vector in the Banach space $l_q(\alpha^{-1})$ with infinitely many*

$f_n \neq 0$. If $\lim_{n\to\infty} \sum_{k=n+1}^{\infty} |f_{k+m}/f_{n+m}|^q = 0$ for all $m \geq 0$, then $\hat{f}$ is a cyclic vector of $\hat{M}_z^*$ acting on $\hat{l}_q(\alpha^{-1})$.

Proof. Put $M = \operatorname{span}\{(\hat{M}_z^*)^n \hat{f} : n = 0, 1, 2, \ldots\}$. We have $(\hat{M}_z^*)^n \hat{f} = \sum_{k=0}^{\infty} f_{k+n} z^k$. Now for $f_n \neq 0$,

$$\|(\hat{M}_z^*)^n \hat{f} - f_n\|^q = \|\sum_{k=1}^{\infty} f_{k+n} z^k\|^q = \sum_{k=1}^{\infty} |f_{k+n}|^q \alpha_k^{-q} = \sum_{k=n+1}^{\infty} |f_k|^q \alpha_{k-n}^{-q}.$$

Therefore

$$\|\frac{(\hat{M}_z^*)^n \hat{f}}{f_n} - 1\|^q = \sum_{k=n+1}^{\infty} |\frac{f_k}{f_n}|^q \alpha_{k-n}^{-q} \leq \sum_{k=n+1}^{\infty} |\frac{f_k}{f_n}|^q.$$

Now by condition of the theorem $\lim_n (\hat{M}_z^*)^n \hat{f}/f_n = 1$. Thus $1 \in M$ and so for each n we get

$$(\hat{M}_z^*)^n \hat{f} - f_n = \sum_{k=n+1}^{\infty} f_k z^{k-n} = f_{n+1} z + \sum_{k=n+1}^{\infty} f_k z^{k-n}$$

is in M. Thus whenever $f_{n+1} \neq 0$, we have $z + \sum_{k=n+2}^{\infty} (f_k/f_{n+1}) z^{k-n} \in M$. Now by induction we prove that $z^m \in M$ for all m: let $1, z, \ldots, z^{m-1} \in M$ and note that

$$(\hat{M}_z^*)^n \hat{f} = \sum_{k=0}^{m-1} f_{k+n} z^k + f_{m+n} z^m + \sum_{k=m+1}^{\infty} f_{k+n} z^k.$$

For $f_{m+n} \neq 0$, we have

$$\begin{aligned} \|(\hat{M}_z^*)^n \hat{f} = \sum_{k=0}^{m-1} f_{k+n} z^k / f_{m+n} - z^m\|^q &= \|\sum_{k=m+1}^{\infty} \frac{f_{k+n}}{f_{m+n}} z^k\|^q \\ &= \sum_{k=m+1}^{\infty} |\frac{f_{k+n}}{f_{m+n}}|^q \alpha_k^{-q} \\ &\leq \sum_{k=n+1}^{\infty} |\frac{f_{k+m}}{f_{m+n}}|^q. \end{aligned}$$

By letting $n \to \infty$, we conclude that $z^m \in M$. Thus indeed $z^m \in M$ for all m, hence $M = l_q(\alpha^{-1})$ and so f is a cyclic vector of $\hat{f}$ is a cyclic vector of $\hat{M}_z^*$. $\square$

References

[1] L. Bagheri and B. Yousefi, Reflexivity of the shift operator on some BK spaces, *Rendiconti Del Circolo Matematico Di Palermo*, Volume **2013**, DOI 10.1007/s12215-013-0143-5 (2013).

[2] M. Mursaleen and A. K. Noman, On some new difference sequence spaces of non-absolute type, *Math. Comput. Modelling*, **52** (2010), 603-617.

[3] M. Mursaleen and A. K. Noman, Applications of the Hausdorff measure of noncompactness in some sequence spaces of weighted means, *Comput. Math. Appl.*, **60** (2010), 1245-1258.

[4] E. Malkowsky, Linear operators in certain BK spaces, *Bolyai Society Mathematical Studies*, **5** (1996), 259-273.

[5] A. Wilansky, *Summability through functional analysis*, Mathematics Studies 85, North-Holland, 1984.