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VAGUE MAGNIFIED TRANSLATION IN Γ-SEMIRINGS

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Abstract: In this paper, we introduce and study the concept of vague magnified translation of a vague set in Γ-semiring and we characterized vague Γ-semiring, left(resp. right) vague ideal, vague bi-ideal, vague quasi ideal in terms of vague magnified translation.

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Key Words: vague Γ-semiring, vague magnified translation, left (resp. right) vague ideal, vague bi-ideal, vague quasi ideal

1. Introduction

The concept of vague set theory was introduced by Gau W.L and Buehrer D.J[4] in 1993, as a improvement of the theory of fuzzy sets by Zadeh L.A[9] in approximating the real life situations. The idea of fuzzy magnified translation has been introduced by Majumder S.K and Sardar S.K[7]. In 1995, M.K.Rao[6] introduced the notion of Γ-semiring as a generalization of Γ-ring as well as semiring and studied the concepts of Γ-semirings and its sub Γ-semirings with a left(resp. right) unity. Moreover the concept of Γ-semiring not only generalizes the concepts of semiring and Γ-ring but also the notion of ternary semiring. In this paper we introduce and study the concept of vague magnified translation of a

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vague set in Γ-semiring with membership and non membership functions taking values in unit interval of real numbers and established some of the properties. Further we prove that, if A is a left(resp. right) vague ideal of a Γ-semiring R then the vague magnified translation $A^c_{\beta\alpha}$ of A is a vague bi-ideal of R and if A is a left(resp. right) vague ideal of a left(resp. right) zero Γ-semiring R, then $A^c_{\beta\alpha}$ is a constant vague set.

Throughout this paper, R stands for Γ-semiring. That is for two additive commutative semigroups R and Γ and there exists a mapping $R \times \Gamma \times R \to R$ image to be denoted by aαb for a, b $\in R$ and $\alpha \in \Gamma$ satisfying the following conditions.
1. $a\alpha(b + c) = a\alpha b + a\alpha c$
2. $(a + b)\alpha c = a\alpha c + b\alpha c$

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3. $a(\alpha + \beta)c = a\alpha c + a\beta c$
4. $a\alpha(b\beta c) = (a\alpha b)\beta c, \forall\ a, b, c \in R; \alpha, \beta \in \Gamma$.

Also, δ stands for the characteristic set of R.

2. Preliminaries

In this section we recall some of the fundamental concepts and definitions, which are necessary for this paper.

Definition 2.1: A Γ-semiring R is called left-zero(resp. right-zero) Γ-semiring if $x\gamma y = x(resp. x\gamma y = y), \forall\ x, y \in R; \gamma \in \Gamma$.

Definition 2.2: A Γ-semiring R is said to be regular if for all $x \in R$, there exists $a \in R\ and\ \alpha, \beta \in \Gamma$ such that $x = x\alpha a\beta x$.

Definition 2.3: A Γ-semiring R is said to be intra-regular if for all $x \in R$, there exists $a, b \in R\ and\ \alpha, \beta, \gamma \in \Gamma$ such that $x = a\alpha x\beta x\gamma b$.

Definition 2.4: Let μ be a non-empty fuzzy subset of X and $\alpha \in [0, 1 - sup\{\mu(x)/\ x \in X\}]$ and $\beta \in [0, 1]$. A mapping $\mu^c_{\beta\alpha} : X \to [0, 1]$ is called a fuzzy magnified translation of μ if $\mu^c_{\beta\alpha}(x) = \beta\ \mu(x) + \alpha,\ \forall\ x \in X$.

Definition 2.5: A vague set A in the universe of discourse U is a pair (t_A, f_A), where t_A: $U \to$[0,1] and f_A: $U \to$[0,1] are mappings such that $t_A(u) + f_A(u) \leq 1, \forall\ u \in U$. The functions t_A and f_A are called true membership function and false membership function respectively.

Definition 2.6: A vague set A of a Γ-semiring R is called a constant vague set if $V_A(x) = V_A(y)$, $\forall x, y \in R$.

Definition 2.7[1]: A vague set $A = (t_A, f_A)$ on R is said to be vague Γ-semiring if the following conditions are true:

For all $x, y \in R; \gamma \in \Gamma$,
$V_A(x + y) \geq min\{V_A(x), V_A(y)\}$ and
$V_A(x\gamma y) \geq min\{V_A(x), V_A(y)\}$
i.e.,
(i). $t_A(x + y) \geq min\{t_A(x), t_A(y)\}$,
$1 - f_A(x + y) \geq min\{1 - f_A(x), 1 - f_A(y)\}$ and
(ii). $t_A(x\gamma y) \geq min\{t_A(x), t_A(y)\}$,
$1 - f_A(x\gamma y) \geq min\{1 - f_A(x), 1 - f_A(y)\}$.

Definition 2.8[2]: A vague set $A = (t_A, f_A)$ on R is said to be left(resp. right) vague ideal of R if the following conditions are true:
For all $x, y \in R; \gamma \in \Gamma$,
$V_A(x + y) \geq min\{V_A(x), V_A(y)\}$ and
$V_A(x\gamma y) \geq V_A(y)$ (resp. $V_A(x\gamma y) \geq V_A(x)$)
i.e.,
(i). $t_A(x + y) \geq min\{t_A(x), t_A(y)\}$,
$1 - f_A(x + y) \geq min\{1 - f_A(x), 1 - f_A(y)\}$ and
(ii).$t_A(x\gamma y) \geq t_A(y)(t_A(x\gamma y) \geq t_A(x))$,
$1 - f_A(x\gamma y) \geq 1 - f_A(y)$ (resp. $1 - f_A(x\gamma y) \geq 1 - f_A(x)$).

Definition 2.9[3]: A vague Γ-semiring $A = (t_A, f_A)$ of R is said to be vague bi-ideal of R if for all $x, y, z \in R; \alpha, \beta \in \Gamma$, $V_A(x\alpha y\beta z) \geq min\{V_A(x), V_A(z)\}$
i.e.,
$t_A(x\alpha y\beta z) \geq min\{t_A(x), t_A(z)\}$,
$1 - f_A(x\alpha y\beta z) \geq min\{1 - f_A(x), 1 - f_A(z)\}$.

Definition 2.10[3]: A vague set $A = (t_A, f_A)$ of R is said to be vague quasi ideal of R if for all $x, y \in R$,
1.$V_A(x + y) \geq min\{V_A(x), V_A(y)\}$
2. $(A\Gamma\delta) \cap (\delta\Gamma A) \subseteq A$, where δ is a vague characteristic set of R.

3.Vague Magnified Translation of a Vague set

We introduce the concept of vague magnified translation of a vague set in Γ-semiring. We prove that, if A is a left(resp. right) vague ideal of a Γ-semiring R then the vague magnified translation $A^c_{\beta\alpha}$ of A is a vague bi-ideal of R and if A is a left(resp. right) vague ideal of a left(resp. right) zero Γ-semiring R, then $A^c_{\beta\alpha}$ is a constant vague set.

We begin with the following.

Definition 3.1: Let A be a non-empty vague set of R and $\alpha \in [0, 1 - sup\{t_A(x) + f_A(x)/\ x \in R\}]$ and $\beta \in [0, 1]$. The vague magnified translation of A, $A^c_{\beta\alpha}$ is a pair $(t_{A^c_{\beta\alpha}}, f_{A^c_{\beta\alpha}})$, where $t_{A^c_{\beta\alpha}} : R \to [0, 1]$ and $f_{A^c_{\beta\alpha}} : R \to [0, 1]$ are mappings such that $t_{A^c_{\beta\alpha}}(x) = \beta\ t_A(x) + \alpha$ and $f_{A^c_{\beta\alpha}}(x) = \beta\ f_A(x) - \alpha$, $\forall\ x \in R$.

Verification 3.2: Vague magnified translation is also a vague set.
Let $A = (t_A,\ f_A)$be a vague set of a R.
Let $\alpha \in$ [0, 1- sup $\{t_A(x)\ +\ f_A(x)$/x $\in R\}]$ and $\beta \in$ [0,1].
The vague magnified translation of A is $A^c_{\beta\alpha} = (t_{A^c_{\beta\alpha}},\ f_{A^c_{\beta\alpha}})$.
Let $x \in R$.
Now, $t_{A^c_{\beta\alpha}}(x)\ +\ f_{A^c_{\beta\alpha}}(x) = \beta\ t_A(x)\ +\ \alpha\ +\ \beta\ f_A(x)$ - α
$= \beta[t_A(x)\ +\ f_A(x)]$
≤ 1.
Thus $A^c_{\beta\alpha}$ is a vague set.

Example 3.3: Let R be the set of natural numbers including zero and Γ be the set of positive even integers.
Define aγb = a.γ.b, where '.' is the usual multiplication on R, for all a, b $\in R$;$\gamma \in \Gamma$.
Therefore R is a Γ-semiring.
Let $A = (t_A,\ f_A)$, where t_A: $R \to$ [0,1] and f_A: $R \to$ [0,1] such that

$$t_A(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.6 & \text{if } x\ is\ even \\ 0.4 & \text{if } x\ is\ odd \end{cases} \text{ and } f_A(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.3 & \text{if } x\ is\ even \\ 0.5 & \text{if } x\ is\ odd \end{cases}$$

Therefore A is a vague set.
Now, $A^c_{\beta\alpha} = (t_{A^c_{\beta\alpha}}, f_{A^c_{\beta\alpha}})$, where $\beta \in$ [0,1] and
$\alpha \in$ [0,1- sup{1,0.9,0.9}]= [0,1-1]= 0.
put $\beta = 0.4$
Then

$$t_{A^c_{\beta\alpha}}(x) = \begin{cases} 0.32 & \text{if } x = 0 \\ 0.24 & \text{if } x\ is\ even \\ 0.16 & \text{if } x\ is\ odd \end{cases} \text{ and } f_{A^c_{\beta\alpha}}(x) = \begin{cases} 0.08 & \text{if } x = 0 \\ 0.12 & \text{if } x\ is\ even \\ 0.2 & \text{if } x\ is\ odd \end{cases}$$

Therefore $A^c_{\beta\alpha} = (t_{A^c_{\beta\alpha}}, f_{A^c_{\beta\alpha}})$ is a vague set.

Theorem 3.4: Let $A = (t_A, f_A)$ and $B = (t_B, f_B)$ be two vague sets of R. Then

1. $(A \cap B)^c_{\beta\alpha} = A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$
2. $(A \cup B)^c_{\beta\alpha} = A^c_{\beta\alpha} \cup B^c_{\beta\alpha}$.

Proof. : Let $x \in R$.
1.Now, $t_{(A\cap B)^c_{\beta\alpha}}(x) = \beta\ t_{A\cap B}(x) + \alpha$
$= \beta\ min\{t_A(x), t_B(x)\} + \alpha$
$= min\{\beta\ t_A(x) + \alpha, \beta\ t_B(x) + \alpha\}$
$= min\{t_{A^c_{\beta\alpha}}(x), t_{B^c_{\beta\alpha}}(x)\}$
$= t_{A^c_{\beta\alpha}\cap B^c_{\beta\alpha}}(x)$.
Again
$f_{(A\cap B)^c_{\beta\alpha}}(x) = \beta\ f_{A\cap B}(x) - \alpha$
$= \beta\ max\{f_A(x), f_B(x)\} - \alpha$
$= max\{\beta\ f_A(x) - \alpha, \beta\ f_B(x) - \alpha\}$
$= max\{f_{A^c_{\beta\alpha}}(x), f_{B^c_{\beta\alpha}}(x)\}$
$= f_{A^c_{\beta\alpha}\cap B^c_{\beta\alpha}}(x)$.
Hence $(A \cap B)^c_{\beta\alpha} = A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$.

2. proof of 2 follows from 1. □

Theorem 3.5: Let $A = (t_A, f_A)$ be a vague set of R. Then A is a vague Γ-semiring of R if and only if the vague magnified translation of A, $A^c_{\beta\alpha}$ is vague Γ-semiring of R.

Proof. : Suppose A is a vague Γ-semiring of R.
Let $x, y \in R$; $\gamma \in \Gamma$.
Now, $t_{A^c_{\beta\alpha}}(x+y) = \beta\ t_A(x+y)+\alpha \geq \beta\ min\{t_A(x), t_A(y)\}+\alpha = min\{\beta\ t_A(x)+\alpha,\ \beta\ t_A(y) + \alpha\} = min\{t_{A^c_{\beta\alpha}}(x),\ t_{A^c_{\beta\alpha}}(y)\}$
and
$f_{A^c_{\beta\alpha}}(x + y) = \beta\ f_A(x + y) - \alpha \leq \beta\ max\{f_A(x), f_A(y)\} - \alpha = max\{\beta\ f_A(x) - \alpha, \beta\ f_A(y) - \alpha\} = max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}$.
Similarly, we can prove that $t_{A^c_{\beta\alpha}}(x\gamma y) \geq min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(y)\}$ and
$f_{A^c_{\beta\alpha}}(x\gamma y) \leq max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}$
Hence $A^c_{\beta\alpha}$ is a vague Γ-semiring of R.
Conversely suppose that $A^c_{\beta\alpha}$ is a vague Γ-semiring of R.
Let $x, y \in R; \gamma \in \Gamma$.
Now, $t_A(x + y) = \frac{1}{\beta}(t_{A^c_{\beta\alpha}}(x + y) - \alpha) \geq \frac{1}{\beta}(min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(y)\} - \alpha) = \frac{1}{\beta}(min\{t_{A^c_{\beta\alpha}}(x) - \alpha, t_{A^c_{\beta\alpha}}(y) - \alpha\}) = min\{\frac{1}{\beta}(t_{A^c_{\beta\alpha}}(x) - \alpha), \frac{1}{\beta}(t_{A^c_{\beta\alpha}}(y) - \alpha)\}$
$= min\{t_A(x), t_A(y)\}$
and

$f_A(x+y) = \frac{1}{\beta}(f_{A^c_{\beta\alpha}}(x+y)+\alpha) \leq \frac{1}{\beta}(max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}+\alpha) = \frac{1}{\beta}(max\{f_{A^c_{\beta\alpha}}(x)+\alpha, f_{A^c_{\beta\alpha}}(y)+\alpha\}) = max\{\frac{1}{\beta}(f_{A^c_{\beta\alpha}}(x)+\alpha), \frac{1}{\beta}(f_{A^c_{\beta\alpha}}(y)+\alpha)\} = max\{f_A(x), f_A(y)\}$.
Similarly we can prove that $t_A(x\gamma y) \geq min\{t_A(x), t_A(y)\}$ and $f_A(x\gamma y) \leq max\{f_A(x), f_A(y)\}$.
Hence A is a vague Γ-semiring of R. □

The following two theorems follows theorem:3.5.

Theorem 3.6: Let $A = (t_A, f_A)$ be a vague set of R. Then A is a left(resp. right) vague ideal of R if and only if the vague magnified translation of A, $A^c_{\beta\alpha}$ is left(right) vague ideal of R.

Theorem 3.7: Let $A = (t_A, f_A)$ be a vague set of R. Then A is a vague bi-ideal of R if and only if the vague magnified translation of A, $A^c_{\beta\alpha}$ is vague bi-ideal of R.

Theorem 3.8: If A is a left(resp. right) vague ideal of R, then $A^c_{\beta\alpha}$ is a vague bi-ideal of R.

Proof. : Let $x, y, z \in R; \gamma, \eta \in \Gamma$.
1. $t_{A^c_{\beta\alpha}}(x+y) = \beta\ t_A(x+y) + \alpha \geq \beta\ min\{t_A(x), t_A(y)\} + \alpha = min\{\beta\ t_A(x) + \alpha, \beta\ t_A(y) + \alpha\} = min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(y)\}$.
2. $t_{A^c_{\beta\alpha}}(x\gamma y) = \beta\ t_A(x\gamma y) + \alpha \geq \beta\ t_A(y) + \alpha$ (resp. $\beta\ t_A(x) + \alpha$) $= t_{A^c_{\beta\alpha}}(y)$ (resp. $= t_{A^c_{\beta\alpha}}(x)$) $\geq min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(y)\}$.
3. $t_{A^c_{\beta\alpha}}(x\gamma y\eta z) = \beta\ t_A(x\gamma y\eta z) + \alpha \geq \beta\ t_A(z) + \alpha$ (resp. $\beta\ t_A(x) + \alpha$) $= t_{A^c_{\beta\alpha}}(z)$ (resp. $t_{A^c_{\beta\alpha}}(x)$) $= min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(z)\}$.
Similarly we can prove $f_{A^c_{\beta\alpha}}(x+y) \leq max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}$,
$f_{A^c_{\beta\alpha}}(x\gamma y) \leq max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}$ and
$f_{A^c_{\beta\alpha}}(x\gamma y\eta z) \leq max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(z)\}$.
Hence $A^c_{\beta\alpha}$ is a vague bi-ideal of R.

□

Theorem 3.9: The vague magnified translation of the intersection of an arbitrary collection of vague bi-ideals of R is a vague bi-ideal of R if it is not empty.

Proof. : Let A be the intersection of arbitrary collection of vague bi-ideals of R.
We have arbitrary collection of vague bi-ideals of R is a vague bi-ideal of R.
Hence from theorem:3.7, $A^c_{\beta\alpha}$ is a vague bi-ideal of R.

□

Theorem 3.10: Let R be a regular and intra regular Γ-semiring. Then
1. $A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha} \supseteq A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$
2. $(A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha}) \cap (B^c_{\beta\alpha}\Gamma A^c_{\beta\alpha}) \supseteq A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$, where $A = (t_A, f_A)$, $B = (t_B, f_B)$ are vague bi-ideals of R.

Proof. : Let $x \in R$.
Since R is regular and intra regular, we have
$x = x\gamma_1 a\gamma_2 x$ and $x = p\gamma_3 x\gamma_4 x\gamma_5 q$, for some $a, p, q \in R$ and $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5 \in \Gamma$.
That implies $x = x\gamma_1 a\gamma_2 x$
$= x\gamma_1 a\gamma_2 x\gamma_1 a\gamma_2 x$
$= x\gamma_1 a\gamma_2(p\gamma_3 x\gamma_4 x\gamma_5 q)\gamma_1 a\gamma_2 x$
$= (x\gamma_1 a\gamma_2 p\gamma_3 x)\gamma_4(x\gamma_5 q\gamma_1 a\gamma_2 x)$.
Now, $t_A(x\gamma_1 a\gamma_2 p\gamma_3 x) \geq min\{t_A)x), t_A(x)\} = t_A(x)$ and
$t_B(x\gamma_5 q\gamma_1 a\gamma_2 x) \geq min\{t_B(x), t_B(x)\} = t_B(x)$.
Now, $t_{A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha}}(x) = sup\{min\{t_{A^c_{\beta\alpha}}(x\gamma_1 a\gamma_2 p\gamma_3 x), t_{B^c_{\beta\alpha}}(x\gamma_5 q\gamma_1 a\gamma_2 x)\}\}$
$= sup\{min\{\beta\ t_A(x\gamma_1 a\gamma_2 p\gamma_3 x) + \alpha,\ \beta\ t_B(x\gamma_5 q\gamma_1 a\gamma_2 x) + \alpha\}\}$
$\geq sup\{min\{\beta\ t_A(x) + \alpha, \beta\ t_B(x) + \alpha\}\}$
$= min\{t_{A^c_{\beta\alpha}}(x), t_{B^c_{\beta\alpha}}(x)\}$
$= t_{A^c_{\beta\alpha}\cap B^c_{\beta\alpha}}(x)$.
Again
$f_{A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha}}(x) = inf\{max\{f_{A^c_{\beta\alpha}}(x\gamma_1 a\gamma_2 p\gamma_3 x), f_{B^c_{\beta\alpha}}(x\gamma_5 q\gamma_1 a\gamma_2 x)\}\}$
$= inf\{max\{\beta\ f_A(x\gamma_1 a\gamma_2 p\gamma_3 x) - \alpha,\ \beta\ f_B(x\gamma_5 q\gamma_1 a\gamma_2 x) - \alpha\}\}$
$\geq inf\{max\{\beta\ f_A(x) - \alpha, \beta\ f_B(x) - \alpha\}\}$
$= max\{f_{A^c_{\beta\alpha}}(x), f_{B^c_{\beta\alpha}}(x)\}$
$= f_{A^c_{\beta\alpha}\cap B^c_{\beta\alpha}}(x)$.
Hence $A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha} \supseteq A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$.
Similarly we can prove $B^c_{\beta\alpha}\Gamma A^c_{\beta\alpha} \supseteq A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$.
Combining these two, we get $(A^c_{\beta\alpha}\Gamma B^c_{\beta\alpha}) \cap (B^c_{\beta\alpha}\Gamma A^c_{\beta\alpha}) \supseteq A^c_{\beta\alpha} \cap B^c_{\beta\alpha}$.

□

Theorem 3.11: Let $A = (t_A, f_A)$ be a vague set of R. Then A is a vague quasi ideal of R if and only if the vague magnified translation of A, $A^c_{\beta\alpha}$ is vague quasi ideal of R.

Proof. : Suppose A is a vague quasi ideal of R.
Let $x, y \in R$.
Now, $t_{A^c_{\beta\alpha}}(x + y) \geq min\{t_{A^c_{\beta\alpha}}(x), t_{A^c_{\beta\alpha}}(y)\}$ and
$f_{A^c_{\beta\alpha}}(x + y) \leq max\{f_{A^c_{\beta\alpha}}(x), f_{A^c_{\beta\alpha}}(y)\}$

Now, $t_{(A^c_{\beta\alpha}\Gamma\delta)\cap(\delta\Gamma A^c_{\beta\alpha})}(x) = min\{t_{A^c_{\beta\alpha}\Gamma\delta}(x), t_{\delta\Gamma A^c_{\beta\alpha}}(x)\}$
$= min\{sup\{min\{t_{A^c_{\beta\alpha}}(a), t_\delta(b)\}\}, sup\{min\{t_\delta(a), t_{A^c_{\beta\alpha}}(b)\}\}/x = a\gamma b\}$
$= min\{t_{A^c_{\beta\alpha}}(a), t_{A^c_{\beta\alpha}}(b)\}$
$= min\{\beta\ t_A(a) + \alpha, \beta\ t_A(b) + \alpha\}$
$= \beta\ min\{t_A(a), t_A(b)\} + \alpha$
$= \beta\ t_{(A\Gamma\delta)\cap(\delta\Gamma A)}(x) + \alpha$
$\leq \beta\ t_A(x) + \alpha$
$= t_{A^c_{\beta\alpha}}(x).$
Therefore $(A^c_{\beta\alpha}\Gamma\delta) \cap (\delta\Gamma A^c_{\beta\alpha}) \subseteq A^c_{\beta\alpha}$.
Hence $A^c_{\beta\alpha}$ is a vague quasi ideal of R.
Conversely suppose that $A^c_{\beta\alpha}$ is a vague quasi ideal of R.
Let $x, y \in R$.
Then $t_A(x + y) \geq min\{t_A(x), t_A(y)\}$ and
$f_A(x + y) \leq max\{f_A(x), f_A(y)\}$.
Now, $t_{(A\Gamma\delta)\cap(\delta\Gamma A)}(x) = min\{t_{A\Gamma\delta}(x), t_{\delta\Gamma A}(x)\}$
$= min\{sup\{min\{t_A(a), t_\delta(b)\}\}, sup\{min\{t_\delta(a), t_A(b)\}\}/x = a\gamma b\}$
$= mi\{t_A(a), t_A(b)\}$
$= \frac{1}{\beta}(t_{(A^c_{\beta\alpha}\Gamma\delta)\cap(\delta\Gamma A^c_{\beta\alpha})}(x) - \alpha)$
$\leq \frac{1}{\beta}(t_{A^c_{\beta\alpha}}(x) - \alpha)$
$= t_A(x).$
Therefore $(A\Gamma\delta) \cap (\delta\Gamma A) \subseteq A$.
Hence A is a vague quasi ideal of R.

□

Theorem 3.12: Let A be a left(resp. right) vague ideal of a left(right) zero Γ-semiring R.Then $A^c_{\beta\alpha}$ is a constant vague set.

Proof. : Let $x, y \in R$; $\gamma \in \Gamma$.
Since R is a left zero Γ-semiring, we have $x\gamma y = x$ *and* $y\gamma x = y$.
Now, $t_{A^c_{\beta\alpha}}(x) = \beta\ t_A(x) + \alpha = \beta\ t_A(x\gamma y) + \alpha \geq \beta\ t_A(y) + \alpha = t_{A^c_{\beta\alpha}}(y)$.
Again $t_{A^c_{\beta\alpha}}(y) = \beta\ t_A(y) + \alpha = \beta\ t_A(y\gamma x) + \alpha \geq \beta\ t_A(x) + \alpha = t_{A^c_{\beta\alpha}}(x)$.
Therefore $t_{A^c_{\beta\alpha}}(x) = t_{A^c_{\beta\alpha}}(y)$
Similarly, $f_{A^c_{\beta\alpha}}(x) = f_{A^c_{\beta\alpha}}(y)$
Thus $A^c_{\beta\alpha}$ is a constant vague set.
Similarly we can prove other case also.

□

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