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SOLUTION OF HEAT CONDUCTION PROBLEM WITH DISCONTINUOUS BOUNDARY CONDITIONS IN NONHOMOGENOUS MOVING CYLINDER USING MAPLE

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Abstract: This paper presents a heat conduction problem with discontinuous boundary conditions in nonhomogeneous moving entire cylinder, which moves on the axis oz with any movement law $z = S(t)$. The temperature field is determined in the cylindrical coordinates system linked with motionless cylinder as a system in a single movement. Using the method inversion of the unit function $\eta(t - S^{-1}(z))$, method a sequence of integral transformations like Fourier transform with respect to z, Hankel transform with respect to r, Bessel functions theory,general integral transforms theory, a solution in the form of the series is obtained.

And in order to illustrate theoretical results in this paper,we wrote special programming in Maple program and for a special cases, where numerical solutions were presented with explained graphics and discussed.

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1. Introduction

The thermal properties of a solid very with position, exact solutions can be found in a limited number of special,in this paper studying the case of one of these cases. Thermal conductivity problem of the semi-infinite nonhomogeneous solids,it has been studied by Carslaw and Jaeger [2]. Combining the traditional presentation of fundamental mathematical concepts with the contemporary computational benefits of Maple software submitted by Articolo [4]. In recent years,a number of the papers investigated analytical heat flow initiate within a hollow infinite cylinder using Maple program and the numerical solutions obtained and showed graphic animation, has been submitted by Shahout et al [7]. Solution of the problems thermal conductivity of the homogeneous moving bodies with finite size is obtained by Lotarev [8].Kholodovskii [9] considers boundary value problems for linear differential in piecewise homogenous cylinders into half cylinders by multilayer film. Studied thermal conductivity in the moving composite cylinder is obtained by [6]. Our motivation in this paper is to investigate the solution of heat conduction problem in physically nonhomogeneous moving entire cylinder,on the direction of the radius with discontinuous boundary conditions on the surface of the cylinder, which moves along the axis oz. Using the method insertion of the unit function,general integral transforms theory, method sequential of integral transformations like Fourier and Hankel transformation with corresponding inverse transformations respectively, a solution to the problem of the spread heat in the form of the series is obtained.As numerical solutions of the problem under study are achieved by using Maple18 package, where results are obtained and clear graphics with discussion. And these problems have a great importance in many engineering fields which intervention in the design of internal combustion engines, material in aviation, and the factories of the production of military.

2. Formulation of the Problem

Consider the temperature field of moving cylinder with discontinuous boundary conditions and initially at temperature $\varphi(r, z)$ for entire cylinder,which moves along the z-axis with any movement law.The temperature field is determined in the cylindrical coordinates system linked with motionless cylinder as a system in a single movement. This coordinate system moves from one environment in other environment with different thermophyscial descriptions.

The formulation of this problem is given as:

$$\frac{\partial}{\partial r}(k(r)\frac{\partial}{\partial r}T(r,z,t)) + \frac{\partial^2}{\partial z^2}T(r,z,t) - \rho c\frac{\partial}{\partial t}T(r,z,t) = 0,$$

$$0 < r < a\ , \quad 0 < z < \ell, z = S(t)\ , \quad t > 0 \tag{1}$$

$$T(r,z,0) = \varphi(r,z), \tag{2}$$

$$[\alpha_1\frac{\partial}{\partial r}T + \alpha_2 T]|_{r=a} = \begin{cases} f_0(z,t), & \text{if } z < S(t), \\ f_1(z,t), & \text{if } z > S(t), \end{cases} \tag{3}$$

$$T(r,z,t)|_{r=0} < \infty, \tag{4}$$

$$[\beta_1\frac{\partial}{\partial z}T - \beta_2 T]|_{z=0} = -g_0(r,t), \quad [\beta_3\frac{\partial}{\partial z}T + \beta_4 T]|_{z=\ell} = g_1(r,t), \tag{5}$$

$$k(r) = k_0 r^m\ (k_0 - const, 0 \le m \le 1), \ \rho\ - const, c\ - const,$$

$$\alpha_1^2 + \alpha_2^2 > 0, \beta_1^2 + \beta_2^2 > 0, \beta_3^2 + \beta_4^2 > 0, \beta_2^2 + \beta_4^2 > 0,$$

where a is the radius of the cylinder. $T, k(r), \rho$ and c are the temperature, thermal conductivity, density and specific heat of the cylinder, and $\alpha_1, \alpha_2, \beta_1, \beta_2$, β_3, β_4 are the coefficient of surface heat transfer are constants, and $\varphi(r,z)$, $f_0(z,t)$, $f_1(z,t), g_0(r,t), g_1(r,t)$are given functions.

3. Solving Method

The boundary condition (3) is discontinuous,we can write its as follows:

$$[\alpha_1\frac{\partial}{\partial r}T + \alpha_2 T]|_{r=a} = f_1(z,t) - f_2(z,t)\eta(t - S^{-1}(z)) \equiv f(z,t); \tag{6}$$

where

$$\eta(\tau) = \begin{cases} 1, & \text{if } \tau > 0, \\ 0, & \text{if } \tau < 0, \end{cases}$$

$S^{-1}(z)$ is inverse function to $z = S(t)$, $f_2(z,t) = f_1(z,t) - f_0(z,t)$.

We choose a solution of this problem in the form:

$$T(r,z,t) = T_0(r,z,t) + T_1(r,t) + T_2(r,t)z, \tag{7}$$

where $T_0(r,z,t)$ satisfies homogeneous boundary conditions as follows:

$$[\beta_1 \frac{\partial}{\partial z} T_0 - \beta_2 T_0]|_{z=0} = 0, \quad [\beta_3 \frac{\partial}{\partial z} T_0 + \beta_4 T_0]|_{z=\ell} = 0. \tag{8}$$

Using (7) and boundary conditions (5) we get:

$$\begin{gathered} [\beta_1(\tfrac{\partial}{\partial z}T_0 + T_2) - \beta_2(T_0 + T_1 + T_2 z)]|_{z=0} = -g_0(r,t), \\ [\beta_3(\tfrac{\partial}{\partial z}T_0 + T_2) + \beta_4(T_0 + T_1 + T_2 z]|_{z=\ell} = g_1(r,t). \end{gathered} \tag{9}$$

This system (9) we rewrite as following

$$\begin{gathered} [\beta_1 \tfrac{\partial}{\partial z} T_0 - \beta_2 T_0]|_{z=0} = (\beta_2 T_1 - \beta_1 T_2 - g_0), \\ [\beta_3 \tfrac{\partial}{\partial z} T_0 + \beta_4 T_0]|_{z=\ell} = (-\beta_4 T_1 - (\beta_4 \ell + \beta_3) T_2 + g_1). \end{gathered} \tag{10}$$

In order to satisfies conditions of (8), the most T_1, T_2 satisfies the following system:

$$\begin{gathered} \beta_2 T_1 - \beta_1 T_2 = g_0, \\ \beta_4 T_1 + (\beta_4 \ell + \beta_3) T_2 = g_1. \end{gathered} \tag{11}$$

The determinant this system

$$\Delta_1 = \begin{vmatrix} \beta_2 & -\beta_1 \\ \beta_4 & \beta_4 \ell + \beta_3 \end{vmatrix};$$

also $\Delta_1 = \beta_1 \beta_4 + \beta_2 (\beta_4 \ell + \beta_3) \neq 0$, we find that the system (11) with respect to T_1, T_2 there are non- trivial solution:

$$\begin{aligned} T_1(r,t) &= \frac{1}{\Delta_1}[g_0(r,t)(\beta_4 \ell + \beta_3) + \beta_1 g_1(r,t)], \\ T_2(r,t) &= \frac{1}{\Delta_1}[\beta_2 g_1(r,t) - \beta_4 g_0(r,t)]. \end{aligned}$$

Now we use integral transform with respect to z; on the basis of the theory of integral transformations [5], the kernel of this transformation assignment as solution of Sturm- Liouville problem in the following form:

$$\frac{d^2}{dz^2} K(z) + \gamma^2 K = 0, \quad 0 < z < \ell, \tag{12}$$

$$[\beta_1 \frac{d}{dz}K - \beta_2 K]|_{z=0} = 0, \quad [\beta_3 \frac{d}{dz}K + \beta_4 K]|_{z=\ell} = 0. \tag{13}$$

The general solution of (12) is

$$K(z) = C_1 \cos \gamma z + C_2 \sin \gamma z,$$

where C_1 *and* C_2 are arbitrary constants, from the condition (2), we can be found C_1 *and* C_2 as
follows:

$$\begin{aligned} &-\beta_2 C_1 + \beta_1 \gamma C_2 = 0, \\ &\left(-\beta_3 \gamma \sin \gamma \ell + \beta_4 \cos \gamma \ell\right) C_1 + \left(\beta_3 \gamma \cos \gamma \ell + \beta_4 \sin \gamma \ell\right) C_2 = 0. \end{aligned} \tag{14}$$

For the existence non trivial solution to this system a necessary and sufficient condition the determinant of this system equal to zero:

$$\begin{vmatrix} -\beta_2 & \beta_1 \gamma \\ -\beta_3 \gamma \sin \gamma \ell + \beta_4 \cos \gamma \ell & \beta_3 \gamma \cos \gamma \ell + \beta_4 \sin \gamma \ell \end{vmatrix} = 0.$$

For parameter γ we get the equation as follows:

$$(\beta_1 \beta_4 + \beta_2 \beta_3)\, \gamma \cos \gamma \ell = \left(\beta_1 \beta_3 \gamma^2 - \beta_2 \beta_4\right) \sin \gamma \ell$$

or

$$tg \gamma \ell = \frac{\gamma (\beta_1 \beta_4 + \beta_2 \beta_3)}{\gamma^2 \beta_1 \beta_3 - \beta_2 \beta_4}. \tag{15}$$

The roots of (15) are all real, simple, and have an arithmetic numbers: $\gamma_n, n = 1, 2, ...$ are the eigenvalues of the problem (12)-(13), and the corresponding solutions are eigenfunctions for this problem.

Assign the eigenfunctions of the first equation for the system (14)

$$-\beta_2 C_1 + \beta_1 \gamma_n C_2 = 0;$$

from here we choose $C_1 = \beta_1 \gamma_n, C_2 = \beta_2$.
Then

$$K_n(z) = \beta_1 \gamma_n \cos \gamma_n z + \beta_2 \sin \gamma_n z; n = 1, 2, ...$$

and

$$\widetilde{K}_n(z) = \frac{1}{c_n} K_n(z), \quad c_n = \int_0^\ell K_n^2(z) dz,$$

are the kernel of transform with respect to z. This transformation is integral transform Fourier. We apply Fourier transform by kernel $\widetilde{K}_n(z)$ for (1), and with respect to Fourier representation, we get on the following equation:

$$\rho c\frac{\partial}{\partial t}\widetilde{T}_{0,n}(r,t) = \frac{\partial}{\partial r}(k(r)\frac{\partial}{\partial r}\widetilde{T}_{0,n}) - \gamma_n^2\widetilde{T}_{0,n} + \widetilde{T}_n^{\square}(r,t), \quad n = 1, 2, ... \tag{16}$$

where

$$\widetilde{T}_{0,n}(r,t) = \int_0^{\ell} T_0(r,z,t)\widetilde{K}_n(z)dz,$$

$$\begin{aligned}\widetilde{T}_n^{\square}(r,t) \quad = \quad & [\frac{\partial}{\partial r}(k(r)\frac{\partial}{\partial r}T_1(r,t)) - \rho c\frac{\partial}{\partial t}T_1(r,t)]\int_0^{\ell}\widetilde{K}_n(z)dz \\ & +[\frac{\partial}{\partial r}(k(r)\frac{\partial}{\partial r}T_2(r,t)) - \rho c\frac{\partial}{\partial t}T_2(r,t)]\int_0^{\ell} z\widetilde{K}_n(z)dz.\end{aligned}$$

For $\widetilde{T}_{0,n}(r,t)$ we find initial and boundary conditions as follows:

$$\widetilde{T}_{0,n}(r,0) = \widetilde{\varphi}_n(r) - T_1(r,0)\int_0^{\ell}\widetilde{K}_n(z)dz - T_2(r,0)\int_0^{\ell} z\widetilde{K}_n(z)dz, \tag{17}$$

$$[\alpha_1\frac{\partial}{\partial r} + \alpha_2]\widetilde{T}_{0,n}(r,t)|_{r=a} = \widetilde{f}_n(t) - [\alpha_1\frac{\partial}{\partial r} + \alpha_2]T_1(r,t)|_{r=a}\int_0^{\ell}\widetilde{K}_n(z)dz$$

$$- [\alpha_1\frac{\partial}{\partial r} + \alpha_2]T_2(r,t)|_{r=a}\int_0^{\ell} z\widetilde{K}_n(z)dz \equiv \widetilde{f}_{0,n}(t), \tag{18}$$

where

$$\widetilde{\varphi}_n(r) = \int_0^{\ell}\varphi(r,z)\widetilde{K}_n(z)dz,$$

$$\widetilde{f}_n(t) = \int_0^{\ell} f(z,t)\widetilde{K}_n(z)dz.$$

In order to be converges uniformly with respect to r, we write the solution (16)-(18) on the following formula:

$$\widetilde{T}_{0,n}(r,t) = \widetilde{T}_{0,n}^{\square}(r,t) + \frac{1}{\alpha_2}\widetilde{f}_n(t),$$

thus $\widetilde{T}^{\square}_{0,n}(r,t)$ satisfies boundary condition:

$$[\alpha_1 \frac{\partial}{\partial r} + \alpha_2]\widetilde{T}^{\square}_{0,n}(r,t)|_{r=a} = \underbrace{[\alpha_1 \frac{\partial}{\partial r} + \alpha_2]\widetilde{T}_{0,n}(r,t)|_{r=a}}_{\widetilde{f}_{0,n}(t)} - \alpha_2 \cdot \frac{1}{\alpha_2}\widetilde{f}_{0,n}(t) = 0.$$

With respect to $\widetilde{T}^{\square}_{0,n}(r,t)$ we get the following equation:

$$\rho c \frac{\partial}{\partial t}\widetilde{T}^{\square}_{0,n}(r,t) = \frac{\partial}{\partial r}(k(r)\frac{\partial}{\partial r}\widetilde{T}^{\square}_{0,n}(r,t)) - \gamma_n^2 \widetilde{T}^{\square}_{0,n}(r,t) + \widetilde{T}^{\square}_{n}(r,t)$$

$$-\frac{1}{\alpha_2}\left[\rho c \frac{d}{dt} + \gamma_n^2\right]\widetilde{f}_{0,n}(t), \tag{19}$$

$$\widetilde{T}^{\square}_{0,n}(r,0) = \widetilde{\varphi}^{\square}_{0,n}(r), \tag{20}$$

$$\widetilde{T}^{\square}_{0,n}(r,t)|_{r=0} = \widetilde{T}_{0,n}(r,t)|_{r=0} - \frac{1}{\alpha_2}\widetilde{f}_{0,n}(t) < \infty, \tag{21}$$

$$[\alpha_1 \frac{\partial}{\partial r} + \alpha_2]\widetilde{T}^{\square}_{0,n}(r,t)|_{r=a} = 0, \tag{22}$$

where

$$\widetilde{\varphi}^{\square}_{0,n}(r) = \widetilde{\varphi}_n(r) - T_1(r,0)\int_0^{\ell}\widetilde{K}_n(z)dz - T_2(r,0)\int_0^{\ell} z\widetilde{K}_n(z)dz.$$

For problem (19)-(22), we use integral transform with respect to r. So that the kernel and weight of this integral be found by using eigenfunctions for Sturm-Liouvill problem following:

$$(k(r)Q^{\square}(r))^{\square} + \lambda^2 Q(r) = 0, \quad 0 < r < a, \tag{23}$$

$$Q(0) < \infty, \quad [\alpha_1 \frac{d}{dr} + \alpha_2]Q(r)|_{r=a} = 0, \tag{24}$$

the solution of this problem (23) and (24) are the kernel of integral transform with respect to r.

Using permutation variables:

$$r \to y: \quad y = \frac{2}{2-m}r^{\frac{2-m}{2}}; r = (\frac{2-m}{2}y)^{\frac{2}{2-m}},$$

and when the equation (23) takes the following form

$$\frac{\partial^2}{\partial y^2}Q + \frac{m}{y}\frac{\partial}{\partial y}Q + \left(\frac{\lambda}{\sqrt{k_0}}\right)^2 Q = 0, 0 < r < a.$$

The solution that bounded when $r = 0$, appointed in the following form:

$$J_\nu\left(\frac{\lambda}{\beta\sqrt{k_0}}r^\beta\right)\cdot r^{(1-m)/2}; \beta = \frac{2-m}{2}; \nu = (1-m)/(2-m).$$

Using a second condition of (24), we get

$$\frac{\alpha_1}{\sqrt{k_0}}\lambda J^{\square}_\nu\left(\frac{\lambda}{\beta\sqrt{k_0}}a^\beta\right) + a^{\frac{m}{2}-1}\left((1-m)\frac{\alpha_1}{2} + a\alpha_2\right)J_\nu\left(\frac{\lambda}{\beta\sqrt{k_0}}a^\beta\right) = 0. \quad (25)$$

The roots of (25) are all real, simple and have an arithmetic numbers, λ_η , $\eta = 1, 2, \ldots$ are the roots of the eigenvalues for Sturm-Liouville problem (23) and (24).

Using eigenfunctions corresponding, we find kernel of integral transform with respect to r.

$$K^{\square}_\eta(r) = \frac{1}{N_\eta}J_\nu\left(\frac{\lambda_\eta}{\beta\sqrt{k_0}}r^\beta\right)r^{(1-m)/2},$$

where

$$N_\eta = \int_0^a \left(J_\nu\left(\frac{\lambda_\eta}{\beta\sqrt{k_0}}r^\beta\right)r^{(1-m)/2}\right)^2 dr.$$

Now using integeral transform with respect to r,by kernel $K^{\square}_\eta(r)$ and weight 1, we get on problem for representation the function $\widetilde{T}^{\square}_{0,n}(r,t)$

$$\rho c\frac{d}{dt}\widetilde{T}^{\square}_{0,n,\eta}(t) = -\mu^2_{n,\eta}\widetilde{T}^{\square}_{0,n,\eta}(t) + \widetilde{T}^{\square}_{n,\eta}(t) - \widetilde{F}_{0,n,\eta}(t), \quad (26)$$

$$\widetilde{T}^{\square}_{0,n,\eta}(t)|_{t=0} = \widetilde{\varphi}^{\square}_{0,n,\eta}, \quad (27)$$

where

$$\begin{aligned}
\mu^2_{n,\eta} &= (\lambda^2_\eta + \gamma^2_n),\\
\widetilde{T}^{\square}_{n,\eta}(t) &= \int_0^a \widetilde{T}^{\square}_n(r,t)K^{\square}_\eta(r)dr,\\
\widetilde{F}_{0,n,\eta}(t) &= \frac{1}{\alpha_2}\left[\rho c\frac{d}{dt} + \gamma^2_n\right]\widetilde{f}_{0,n}(t)\cdot\int_0^a K^{\square}_\eta(r)dr,
\end{aligned}$$

$$\widetilde{\varphi}^{\square}_{0,n,\eta} = \int_0^a \widetilde{\varphi}^{\square}_{0,n}(r) K^{\square}_{\eta}(r) dr.$$

The solution of (26) and (27) as:

$$\widetilde{T}^{\square}_{0,n,\eta}(t) = e^{-\mu^2_{n,\eta}/(\rho c)\cdot t}[\widetilde{\varphi}^{\square}_{0,n,\eta} + \frac{1}{\rho c}\int_0^t (\widetilde{T}^{\square}_{n,\eta}(\tau) - \widetilde{F}_{0,n,\eta}(\tau))e^{\mu^2_{n,\eta}/(\rho c)\cdot\tau} d\tau]. \quad (28)$$

Using the inverse Hankel transform with respect to r , we obtain:

$$\widetilde{T}^{\square}_{0,n}(r,t) = \sum_{\eta=1}^{\infty} e^{-\mu^2_{n,\eta}/(\rho c)\cdot t}[\widetilde{\varphi}^{\square}_{0,n,\eta} + \frac{1}{\rho c}\int_0^t (\widetilde{T}^{\square}_{n,\eta}(\tau) - \widetilde{F}_{0,n,\eta}(\tau)) \cdot e^{\mu^2_{n,\eta}/(\rho c)\cdot\tau} d\tau] J_{\nu}(\frac{\lambda_{\eta}}{\beta\sqrt{k_0}} r^{\beta}) r^{(1-m)/2}.$$

Then

$$\widetilde{T}_{0,n}(r,t) = \frac{1}{\alpha_2}\widetilde{f}_{0,n}(t) + \sum_{\eta=1}^{\infty} e^{-\mu^2_{n,\eta}/(\rho c)\cdot t}[\widetilde{\varphi}^{\square}_{0,n,\eta} + \frac{1}{\rho c}\int_0^t (\widetilde{T}^{\square}_{n,\eta}(\tau) - \widetilde{F}_{0,n,\eta}(\tau))e^{\mu^2_{n,\eta}/(\rho c)\cdot\tau} d\tau] J_{\nu}(\frac{\lambda_{\eta}}{\beta\sqrt{k_0}} r^{\beta}) r^{(1-m)/2}.$$

Finally, applying the inverse Fourier transform with respect to z ,we find the relation to $T(r,z,t)$:

$$T(r,z,t) = T_1(r,t) + zT_2(r,t) + \sum_{n=1}^{\infty}\{\frac{1}{\alpha_2}\widetilde{f}_{0,n}(t) + \sum_{\eta=1}^{\infty} e^{-\mu^2_{n,\eta}/(\rho c)\cdot t} \cdot [\widetilde{\varphi}^{\square}_{0,n,\eta} + \frac{1}{\rho c}\int_0^t (\widetilde{T}^{\square}_{n,\eta}(\tau) - \widetilde{F}_{0,n,\eta}(\tau))e^{\mu^2_{n,\eta}/(\rho c)\cdot\tau} d\tau] J_{\nu}(\frac{\lambda_{\eta}}{\beta\sqrt{k_0}} r^{\beta}) r^{(1-m)/2}\} K_n(z).$$

4. Steps Solution using Program Maple 18

To consider that the outer surface of cylinder is silver: $k_0 = 1, \rho = 10.49$, $c = 0.0556$,(using the units are c.g.s, caloric, and ^0c).

5. Conclusion

In this paper, the spread heat in nonhomogeneous moving entire cylinder is studied, the solution behavior with discontinuous boundary conditions of the boundary problem is given as an infinite series, using two sequences of integral transformations, Bessel functions theory, we get the solution to the problem of the spread heat in the form of the series,and with writing special programming in Maple program as for a special cases,we constructed to a spread heat as a function of r as a function of z.A numerical solutions were obtained and tables as curves defined in 2D,3D. It can be noticed that the inner surface of the cylinder under the effect of two waves be a convex curve and direction from the top to the bottom,as temperatures regularly drop.As for the cylinder which be under the effect one wave, be a convex curve and direction to top. And also through the comparison can be seen that the curves are the different as structure is different.This problem has numerous engineering applications, such as,machining, welding, grinding,are all others practical examples.

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