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NEW JULIA AND MANDELBROT SETS FOR
A NEW FASTER ITERATIVE PROCESS

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Abstract: Fixed point iterative procedures are the backbones of fractal geometry. In existing literature Julia sets, Mandelbrot sets and their variants have been studied using one - step, two - step, three - step and four - step iterative process. Recently, M. Abbas and T. Nazir [12] introduced a new iterative process (a four-step iterative process) which is faster than all of Picard, Mann and Agarwal processes. In this paper, we obtain further generalizations of Julia and Mandelbrot sets using this faster iterative process for quadratic, cubic and higher degree polynomials. Further, we analyze that few Julia and Mandelbrot sets took the shape of Lord Ganesha (name of Hindu God), Dragon and Urn.

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Key Words: Julia set, Mandelbrot set, four-step feedback process, escape criterion, complex polynomials.

1. Introduction

Complex analytic dynamics attains a landmark in the field of fractal theory during the time of great French mathematician Gaston Julia. Julia set is of vital importance in the study of the complex dynamics of functions because it is a place where the chaotic behavior of complex functions occur [10]. In 1975,

Mandelbrot extended the work of Gaston Julia and introduced Mandelbrot set. During that time, various Julia and Mandelbrot sets have been introduced for quadratic [3, 21, 24, 10], cubic [21, 1, 2, 11, 22] and higher degree polynomials [26] using Picard iterative process (a one step feedback process) [21].

Many researchers have studied Julia and Mandelbrot sets from different aspects. In 2009, D. Rochon [9] studied generalized Mandelbrot sets in bicomplex plane. Later on the work of Rochon was extended by Wang [25] jointly with others and carried further analysis of generalized Julia and Mandelbrot sets. Also, they have studied the fractal structure and discontinuity law of the generalized Julia sets generated from the extended complex mapping $z^n + c$, where $n \in R$ [25]. Further, Julia and Mandelbrot sets have been studied under the effect of noises [19, 4, 14, 15, 27]. In 2004, Rani and Kumar [17, 18] introduced the superior iterate and created superior Julia and Mandelbrot sets for quadratic [17, 18] and cubic [17, 18] polynomials. Later on, they also presented Julia and Mandelbrot sets for higher [13, 16] degree complex polynomials. Further, in 2008, Rani and Negi [5] studied the midgets of Mandelbrot sets. In 2011, Singh, Mishra and Sinkala [23] presented a complete literature review of superior fractals existing in the literature, see also[20].

Recently, Ashish et al. [8] used Noor iterative process (an example of four-step feedback process) for creating new Julia and Mandelbrot sets for quadratic, cubic and higher degree polynomials. Further they have shown anti Julia, Multicorns [6] and cubic Julia sets [7] using Noor iterative process.

In this paper, we create Julia and Mandelbrot sets using a new faster iterative process [12], which is an example of a four-step feedback process. In Section 2, we give some definitions, which are the basis of our work. In Section 3, we discuss escape criterions for quadratic, cubic and nth degree polynomials under the new iteration process. Several Julia and Mandelbrot sets have been presented in Sections 4 and 5 respectively. Finally, the paper has been concluded in Section 6.

2. Preliminaries

In 2014, M. Abbas and T. Nazir [12] introduced the following new iterative process which is faster than all of Picard, Mann and Agarwal et al. processes and is independent of these iterative processes.

Definition 2.1. Let us consider a sequence $\{x_n\}$ of iterates for initial point $x_0 \in X$ such that

$$\{x_{n+1} : x_{n+1} = (1 - \alpha_n)Ty_n + \alpha_n Tz_n;$$

$$y_n = (1 - \beta_n)Tx_n + \beta_n Tz_n;$$
$$z_n = (1 - \gamma_n)x_n + \gamma_n Tx_n;$$
$$n = 0, 1, 2, ..., \}$$

where $\alpha_n, \beta_n, \gamma_n \in [0, 1]$ and $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ are sequences of positive numbers. For the sake of simplicity, we take $\alpha_n = \alpha, \beta_n = \beta$ and $\gamma_n = \gamma$.

Definition 2.2. (Julia Set). The filled in Julia set of the function Q is defined as

$$K(Q) = \{z \in C : Q^k(z) \text{does not tend to infinity}\},$$

where C is the complex space, $Q^k(z)$ is k^{th} iterate of function Q and $K(Q)$ denotes the filled Julia set. The Julia set of the function Q is defined to be the boundary of $K(Q)$, i.e.,

$$J(Q) = \alpha K(Q),$$

where $J(Q)$ denotes the Julia set. The set of points whose orbits are bounded under the Picard orbit $Q_c(z) = z^2 + c$ is called the Julia set. We choose the initial point 0, as 0 is the only critical point of Q_c [21] .

Definition 2.3. (Mandelbrot Set). The Mandelbrot set M consists of all parameters c for which the filled Julia set of Q_c is connected, that is

$$M = \{c \in C : K(Q_c) \text{is connected}\}.$$

In fact, M contains an enormous amount of information about the structure of Julia sets. The Mandelbrot set M for the Quadratic $Q_c(z) = z^2 + c$ is defined as the collection of all $c \in C$ for which the orbit of the point 0 is bounded, that is

$$M = \{c \in C; n = 0, 1, 2, ...\} \text{is bounded.}$$

We choose the initial point 0 as 0 is the only critical point of Q_c [21].

3. Escape Criterions for Complex Polynomials using Faster Iterative Process

The escape criterion plays a vital role in the generation and analysis of Julia sets, Mandelbrot sets and their variants. We need the following escape criterions for the quadratic, cubic and higher degree polynomials.

3.1. Escape Criterions for Quadratic Polynomials

Throughout this section, we assume that $T(x_n) = Q_c(z)$, $T(z_n) = Q_c(z)$ and $T(y_n) = Q_c(z)$.

Theorem 3.1. *Suppose* $\mid z \mid \geq \mid c \mid > 2/\alpha$, $\mid z \mid \geq \mid c \mid > 2/\beta$ *and* $\mid z \mid \geq \mid c \mid > 2/\gamma$, *where* $0 < \alpha < 1$, $0 < \beta < 1$ *and* $0 < \gamma < 1$ *and* c *is a complex number. Define*

$$z_1 = \alpha Q_c(z) + (1 - \alpha)z ,$$
$$z_2 = \alpha Q_c(z_1) + (1 - \alpha)z_1 ,$$
$$\dots ,$$
$$\dots ,$$
$$\dots ,$$
$$z_n = \alpha Q_c(z_{n-1}) + (1 - \alpha)z_{n-1} ,$$

where $Q_c(z)$ *can be a quadratic, cubic or biquadratic polynomial in terms of* γ *and* $n = 2, 3, \dots$ *then* $\mid z_n \mid \to \infty$ *as* $n \to \infty$.

Proof. For $Q_c''(z) = z^2 + c$, we consider

$$
\begin{aligned}
\mid Q_c(z) \mid &= \mid (1 - \gamma)z + \gamma Q_c(z) \mid \\
&= \mid \gamma z^2 + (1 - \gamma)z + \gamma c \mid \\
&\geq \mid \gamma z^2 + (1 - \gamma)z \mid - \mid \gamma c \mid \\
&\geq \mid z \mid (\mid \gamma z + (1 - \gamma) \mid) - \gamma \mid z \mid \quad (\text{since } \mid z \mid \geq \mid c \mid) \\
\mid Q_c(z) \mid &\geq \mid z \mid (\mid \gamma z \mid -1).
\end{aligned}
\tag{1}
$$

Also

$$
\begin{aligned}
\mid Q_c(z) \mid &= \mid (1 - \beta)Q_c(z) + \beta Q_c(z) \mid \\
&= \mid (1 - \beta)(z^2 + c) + \beta \mid z \mid (\mid \gamma z \mid -1) \mid \\
&= \mid \beta\gamma \mid z \mid^2 + (1 - \beta)z^2 + (1 - \beta)c - \beta \mid z \mid\mid \\
&\geq \beta\gamma \mid z \mid^2 + \mid (1 - \beta)z^2 \mid -(1 - \beta) \mid z \mid -\beta \mid z \mid \\
&\geq \beta\gamma \mid z \mid^2 + (1 - \beta) \mid z^2 \mid - \mid z \mid +\beta \mid z \mid -\beta \mid z \mid \\
&\geq \mid z \mid [(\beta\gamma - \beta + 1) \mid z \mid -1] \\
i.e. \quad \mid Q_c(z) \mid &\geq \mid z \mid [(\beta\gamma - \beta + 1) \mid z \mid -1] .
\end{aligned}
\tag{2}
$$

Now, we have

$$z_n = (1 - \alpha)Ty_n + \alpha Tz_n$$

$$| z_n | = | (1 - \alpha)Q_c(z) + \alpha Q_c(z) |$$
$$| z_1 | = | (1 - \alpha)Q_c(z) + \alpha Q_c(z) | . \tag{3}$$

On substituting (3.1) and (3.2) in (3.3), we get

$$
\begin{aligned}
| z_1 | &= | (1 - \alpha)[| z | [(\beta\gamma - \beta + 1) | z | -1]] + \alpha | z | (\gamma | z | -1) | \\
&= | (| z | -\alpha | z |)[(\beta\gamma - \beta + 1) | z | -1] + \alpha\gamma | z |^2 -\alpha | z |) | \\
&\geq | \beta\gamma | z |^2 -\beta | z |^2 + | z |^2 -\alpha\beta\gamma | z |^2 +\alpha\beta | z |^2 -\alpha | z |^2 \\
&\quad - | z | +\alpha | z | +\alpha\gamma | z |^2 -\alpha | z || \\
&\geq | z | [\beta\gamma | z | -\beta | z | + | z | -\alpha\beta\gamma | z | \\
&\quad + \alpha\beta | z | -\alpha | z | -1 + \alpha\gamma | z |] \\
&\geq | z | [(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1) | z | -1]
\end{aligned}
$$
$$i.e. \quad | z_1 | \geq | z | [(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1) | z | -1] \tag{4}$$

Since $| z | \geq | c | > 2/\alpha$, $| z | \geq | c | > 2/\beta$ and $| z | \geq | c | > 2/\gamma$, so that $| z | > 2/\alpha\beta\gamma > 2/(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1)$. Therefore there exists $\lambda > 0$, such that $(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1) | z | -1 > 1 + \lambda$, consequently

$$| z_1 | > (1 + \lambda) | z | .$$

In particular, $| z_n | > | z |$. So we may apply the same argument repeatedly to find

$$| z_n | > (1 + \lambda)^n | z | .$$

Thus, the orbit of z tends to infinity. This completes the proof. $\square$

Corollary 3.2. *Suppose that $| c | > 2/\alpha$, $| c | > 2/\beta$ and $| c | > 2/\gamma$, then the orbit $O(Q_c, 0, \alpha, \beta, \gamma)$ escapes to infinity.*

The proof of the escape criterion actually gives us a little more information. In the proof, we used the only fact that $| z | > | c |$ and $| c | > 2/\alpha$, $| c | > 2/\beta$ and also $| c | > 2/\gamma$. Hence we have the following refinement of the escape criterion:

Corollary 3.3. *(Escape Criterion). Let $| z | > max\{| c |, 2/\alpha, 2/\beta, 2/\gamma\}$, then $| z_n | > (1 + \lambda)^n | z |$ as $| z_n | \to \infty$ as $n \to \infty$.*

We observe that if $| z_k | > max\{| c |, 2/\alpha, 2/\beta, 2/\gamma\}$ for some $k \geq 0$, then we may apply Corollary 3.2 to $| z_k |$ to have the following result.

Corollary 3.4. *Let for some $k \geq 0$, we have $\mid z_k \mid > max\{\mid c \mid, 2/\alpha, 2/\beta, 2/\gamma\}$. Then $\mid z_{k+1} \mid > (1 + \lambda)^n \mid z_k \mid$ so that $\mid z_n \mid \to \infty$ as $n \to \infty$.*

Note that this corollary gives us an algorithm for computing the filled Julia set of quadratics for any complex constant c. For any point z which satisfies $\mid z \mid \leq \mid c \mid$, we find out the orbit of z. If, for any n, $\mid z_n \mid$ lies outside the circle of radius $max\{\mid c \mid, 2/\alpha, 2/\beta, 2/\gamma\}$, then we say that the orbit escapes to infinity that means z is not in the filled Julia set. On the other hand, if $\mid z_n \mid$ never exceeds this bound, then z is by definition in Julia set.

3.2. Escape Criterion for Cubic Polynomials

We consider the following form of a cubic polynomial as it is conjugate to all the cubic polynomials:

$$C_c(z) = z^3 + c, \quad \text{where } c \text{ are complex numbers.}$$

Also, throughout this section, we assume that $T(x_n) = C_c''(z)$, $T(z_n) = C_c'(z)$ and $T(y_n) = C_c(z)$.

Theorem 3.5. *Suppose $\mid z \mid \geq \mid c \mid > (2/\alpha)^{1/2}$, $\mid z \mid \geq \mid c \mid > (2/\beta)^{1/2}$ and $\mid z \mid \geq \mid c \mid > (2/\gamma)^{1/2}$, where $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ and c is a complex number. Define*

$$z_1 = \alpha C_c(z) + (1 - \alpha)z \ ,$$
$$z_2 = \alpha C_c(z_1) + (1 - \alpha)z_1 \ ,$$
$$\dots \ ,$$
$$\dots \ ,$$
$$\dots \ ,$$
$$z_n = \alpha C_c(z_{n-1}) + (1 - \alpha)z_{n-1} \ ,$$

where $C_c(z)$ is a polynomial in terms of γ and $n = 2, 3, \dots$ then $\mid z_n \mid \to \infty$ as $n \to \infty$.

Proof. For $C_c''(z) = z^3 + c$, we have considered

$$\mid C_c(z) \mid = \mid (1 - \gamma)z + \gamma C_c(z) \mid$$
$$= \mid \gamma z^3 + (1 - \gamma)z + \gamma c \mid$$
$$\geq \gamma \mid z \mid^3 - (1 - \gamma) \mid z \mid - \gamma \mid c \mid$$
$$\text{i.e.} \quad \mid C_c(z) \mid \geq \mid z \mid (\gamma \mid z \mid^2 - 1). \tag{5}$$

Also

$$| C_c(z) | = | (1 - \beta)C_c(z) + \beta C_c(z) |$$
$$\geq | (1 - \beta)(z^3 + c) + \beta | z | (\gamma | z |^2 - 1) |$$
$$\geq (1 - \beta) | z |^3 - (1 - \beta) | c | + \beta \gamma | z |^2 - \beta | z |$$
$$\geq | z | [(\beta \gamma - \beta + 1) | z |^2 - 1]$$

i.e. $\quad | C_c(z) | \geq | z | [(\beta \gamma - \beta + 1) | z |^2 - 1]$. $\qquad (6)$

Now, we have

$$| z_n | = | (1 - \alpha)C_c(z) + \alpha C_c(z) |$$
$$| z_1 | = | (1 - \alpha)C_c(z) + \alpha C_c(z) | . \qquad (7)$$

Using (3.5) and (3.6), we get

$$| z_1 | \geq | (1 - \alpha[| z | ((\beta \gamma - \beta + 1) | z |^2 - 1)] + \alpha[| z | (\gamma | z |^2 - 1)] |$$
$$\geq | z | [(\alpha \beta + \beta \gamma + \alpha \gamma - \alpha \beta \gamma - \alpha - \beta + 1) | z |^2 - 1]$$

i.e. $\quad | z_1 | \geq | z | [(\alpha \beta + \beta \gamma + \alpha \gamma - \alpha \beta \gamma - \alpha - \beta + 1) | z |^2 - 1] \qquad (8)$

Since $| z | \geq | c | > (2/\alpha)^{1/2}$, $| z | \geq | c | > (2/\beta)^{1/2}$ and $| z | \geq | c | > (2/\gamma)^{1/2}$, so that $| z | > (2/\alpha\beta\gamma)^{1/2} > (2/(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1))^{1/2}$. Therefore there exists $\lambda > 0$, such that $(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta + 1) | z |^2 - 1 > 1 + \lambda$, consequently

$$| z_1 | > (1 + \lambda) | z | \ .$$

In particular, $| z_n | > | z |$. So we may apply the same argument repeatedly to find

$$| z_n | > (1 + \lambda)^n | z | \ .$$

Thus, the orbit of z tends to infinity. This completes the proof. $\qquad \square$

Corollary 3.6. (**Escape Criterion**). Let

$$| z | > max\{| c |, (2/\alpha)^{1/2}, (2/\beta)^{1/2}, (2/\gamma)^{1/2}\},$$

then $| z_n | > (1 + \lambda)^n | z |$ as $| z_n | \to \infty$ as $n \to \infty$.

3.3. Escape Criterion for n^{th} Degree Polynomials

Now, we discuss the general escape criterion for polynomials of the form

$$G_c(z) = z^n + c, \quad \text{where } c \text{ is a complex numbers.}$$

Also taking $T(x_n) = G_c''(z)$, $T(z_n) = G_c'(z)$ and $T(y_n) = G_c(z)$.

Theorem 3.7. *Let $G_c(z) = z^n + c$, $n = 1, 2, 3...$ be a general polynomial, where $0 < \alpha < 1$, $0 < \beta < 1$ and $0 < \gamma < 1$ and c is a complex number. Define*

$$z_1 = \alpha G_c(z) + (1 - \alpha)z \ ,$$
$$z_2 = \alpha G_c(z_1) + (1 - \alpha)z_1 \ ,$$
$$\dots \ ,$$
$$\dots \ ,$$
$$\dots \ ,$$
$$z_n = \alpha G_c(z_{n-1}) + (1 - \alpha)z_{n-1} \ ,$$

where $n = 2, 3, \dots$ then $max(\mid c \mid, (2/\alpha)^{1/n-1}, (2/\beta)^{1/n-1}, (2/\gamma)^{1/n-1})$ is the general escape criterion.

Proof. Using induction method on n,
For $n = 1$, $G_c(z) = z + c$,

$$\mid z \mid > max(\mid c \mid, 0, 0, 0)$$

For $n = 2$, $G_c(z) = z^2 + c$,

$$\mid z \mid > max(\mid c \mid, 2/\alpha, 2/\beta, 2/\gamma)$$

and for $n = 3$, $G_c(z) = z^3 + c$,

$$\mid z \mid > max(\mid c \mid, (2/\alpha)^{1/2}, (2/\beta)^{1/2}, (2/\gamma)^{1/2})$$

Now, for $n + 1$, let $G_c(z) = z^{n+1} + c$ and $\mid z \mid \geq \mid c \mid > (2/\alpha)^{1/n}$, $\mid z \mid \geq \mid c \mid > (2/\beta)^{1/n}$ and $\mid z \mid \geq \mid c \mid > (2/\gamma)^{1/n}$. Then

$$
\begin{aligned}
\mid G_c(z) \mid &= \mid (1 - \gamma)z + \gamma G_c(z) \mid, \quad \text{where } \ G_c(z) = z^{n+1} + c \\
&= \mid (1 - \gamma)z + \gamma(z^{n+1} + c) \mid \\
&\geq \gamma \mid z \mid^{n+1} - (1 - \gamma) \mid z \mid - \gamma \mid z \mid \quad (\text{since } \mid z \mid \geq \mid c \mid) \\
&= \mid z \mid (\gamma \mid z^n \mid - 1),
\end{aligned}
$$

$$i.e. \quad | \, G_c(z) \, | \geq | \, z \, | \, (\gamma \, | \, z^n \, | -1). \tag{9}$$

Therefore

$$\begin{aligned}
| \, G_c(z) \, | &= | \, (1-\beta)G_c(z) + \beta G_c(z) \, | \\
&\geq | \, (1-\beta)(z^{n+1}+c) + \beta \, | \, z \, | \, (\gamma \, | \, z^n \, | -1) \, | \\
&\geq (1-\beta) \, | \, z \, |^{n+1} -(1-\beta) \, | \, z \, | + \beta\gamma \, | \, z \, |^{n+1} -\beta \, | \, z \, | \\
&\geq | \, z \, | \, [(\beta\gamma - \beta + 1) \, | \, z \, |^n -1]
\end{aligned}$$

$$i.e. \quad | \, G_c(z) \, | \geq | \, z \, | \, [(\beta\gamma - \beta + 1) \, | \, z \, |^n -1]. \tag{10}$$

Since $z_n = (1-\alpha)G_c(z_{n-1}) + \alpha G_c(z_{n-1})$, we obtain

$$\begin{aligned}
| \, z_1 \, | &= | \, (1-\alpha)G_c(z) + \alpha G_c(z) \, | \\
&= | \, (1-\alpha) \, | \, z \, | \, [(\beta\gamma - \beta + 1) \, | \, z \, |^n -1] + \alpha \, | \, z \, | \, (\gamma \, | \, z^n \, | -1) \, | \\
&\geq (| \, z \, | -\alpha \, | \, z \, |)[\beta\gamma \, | \, z \, |^n -\beta \, | \, z \, |^n + | \, z \, |^n -1] + \alpha\gamma \, | \, z \, |^{n+1} -\alpha \, | \, z \, | \, | \\
&\geq | \, z \, | \, [(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \beta - \alpha + 1) \, | \, z \, |^n -1],
\end{aligned}$$

$$| \, z_1 \, | \geq | \, z \, | \, [(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \beta - \alpha + 1) \, | \, z \, |^n -1]. \tag{11}$$

Since $| \, z \, | > (2/\alpha)^{1/n}$, $| \, z \, | > (2/\beta)^{1/n}$, and $| \, z \, | > (2/\gamma)^{1/n}$, so that $| \, z \, | > (2/\alpha\beta\gamma)^{1/n} > (2/(\alpha\beta + \beta\gamma + \alpha\gamma - \alpha\beta\gamma - \alpha - \beta - \gamma + 1))^{1/n}$.
Hence there exits a $\lambda > 1$, such that $| \, z_1 \, | > (1+\lambda) \, | \, z \, |$. Applying the inequality n times , we find $| \, z_n \, | > (1+\lambda)^n \, | \, z \, |$. Hence the result.

Corollary 3.8. (Escape Criterion). *Suppose*

$$| \, z_k \, | > max\{| \, c \, |, (2/\alpha)^{1/k-1}, (2/\beta)^{1/k-1}, (2/\gamma)^{1/k-1}\},$$

for some $k \geq 0$, then $| \, z_k \, | > \lambda \, | \, z_{k-1} \, |$ and $| \, z_n \, | \to \infty$ as $| \, n \, | \to \infty$.

$\square$

4. Julia Sets

In this section, various Julia sets have been generated for quadratic and cubic polynomials using new faster iteration process .

4.1. Julia sets for Quadratic Polynomials

Following results have been observed from the graphical representation (see on page no. 15) of quadratic Julia sets:

1. It is observed that Figs 1-3, took the shape of Dragon Julia sets which has reflectional symmetry along x-axis.

2. The Dragon Julia sets get fattier with decrease in the parameters α, β and γ and fixing the value of parameter c.

3. Further, Figs 4-6 shows connected Julia sets which have the reflectional symmetry about x-axis and y-axis.

4. Julia set in Fig. 7, shows the disconnectivity of orbits of Julia sets.

4.2. Julia Sets for Cubic Polynomial

In this section we study few cubic Julia sets. It is interesting to see that in Figs 10-12 (see on Page 16), cubic Julia sets took the shape of Lord Ganesha (name of a Hindu God). Further, it is observed that cubic Julia set becomes more connected (i.e. fattier) by decreasing the parameters α, β and γ and fixing the value of parameter c.

5. Mandelbrot Sets

5.1. Mandelbrot Sets for Quadratic Polynomials

In this section we present few Mandelbrot sets using faster iteration process for quadratic, cubic and higher order polynomials. Following results have been analyzed from the graphical representation of Mandelbrot sets: Figs 16-24 (see on Page no. 17) shows the graphical representation of the quadratic Mandelbrot sets generated in computational software Mathematica. Here, we observed that the quadratic Mandelbrot sets have reflectional symmetry along x-axis (called line of symmetry). Further from Figs 16-18 (see on Page no. 17), it was analyzed that when any two parameters out of α, β and γ are fixed then Mandelbrot sets get fattier with increase in the third parameter and becomes slimmer with decrease in the third parameter. From Figs 19-21 (see on Page no. 17), it was observed that when the value of parameter beta is increased, the lobes of the Mandelbrot sets get fattier.

5.2. Mandelbrot Sets for Cubic and Higher Order Polynomials

In case of cubic polynomials, it was observed that there is a reflectional symmetry along x-axis and y-axis (see Figs 25-28 on Page no. 18). Further, we show that Figs 25-27 took the shape of fascinating Coupled Urn. As we decrease the parameters α, β and γ they become more decorated Coupled Urns. In case of higher order polynomials, we observe that the polynomial $z^n + c$, $n > 2$ generates the Mandelbrot sets that have $(n - 1)$ lobes and also have reflectional (along x-axis and y-axis) and rotational symmetry (along center) (see Figs 30-32 on Page no. 18).

6. Conclusions

In this paper, a new faster iterative process have been studied for generalizations of Julia and Mandelbrot sets for complex quadratic, cubic and higher order polynomials. In Section 3, first we have presented new escape criterions for complex quadratic, cubic and higher order polynomials and then new Julia and Mandelbrot sets have been generated. From the graphical representations of Julia sets we have analyzed that few quadratic and cubic Julia sets took the shape of Dragon and Lord Ganesha (Hindu God name) respectively. Further, some Mandelbrot sets also took the shape of fascinating Urn.

7. Acknowledgments
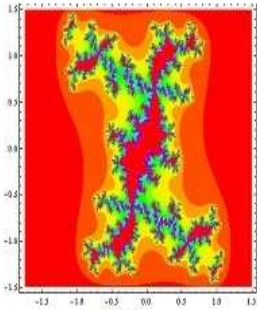
Fig-1
Dragon Julia set (A) for
$\alpha = 0.9$, $\beta = 0.8$, $\gamma = 0.7$,
$c = 0.5+0.5I$
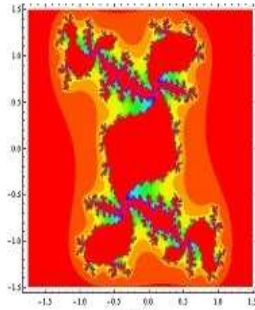
Fig-2
Dragon Julia set (B) for
$\alpha = 0.8$, $\beta = 0.9$, $\gamma = 0.7$,
$c = 0.5+0.5I$
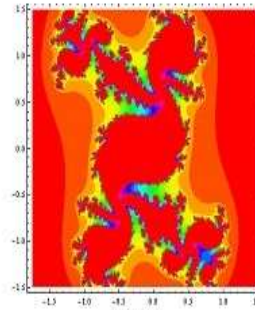
Fig-3
Dragon Julia set (C) for
$\alpha = 0.7$, $\beta = 0.8$, $\gamma = 0.6$,
$c = 0.5+0.5I$
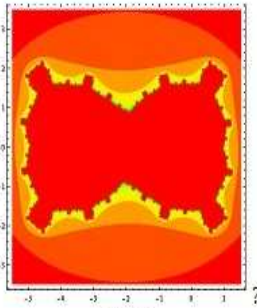
Fig-4
Quadratic Julia set for
$\alpha = 0.2$, $\beta = 1$, $\gamma = 1$,
$c = 0.05+0.05I$
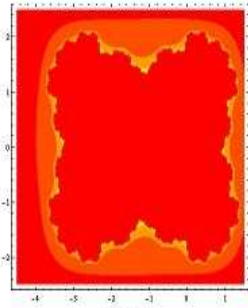
Fig-5
Quadratic Julia set for
$\alpha = 0.3$, $\beta = 1$, $\gamma = 0.8$,
$c = 0.1+0.1I$
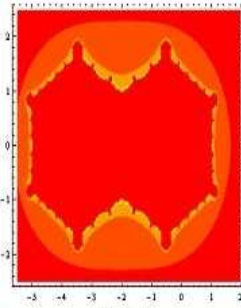
Fig-6
Quadratic Julia set for
$\alpha = 0.2$, $\beta = 0.9$, $\gamma = 0.2$,
$c = 0.01+0.01I$
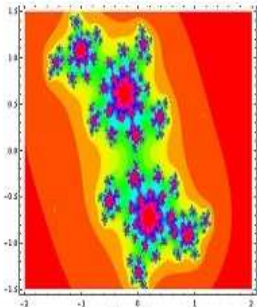
Fig-7
Quadratic Julia set for
$\alpha = 0.3$, $\beta = 0.3$, $\gamma = 0.3$,
$c = 0.1+0.9I$
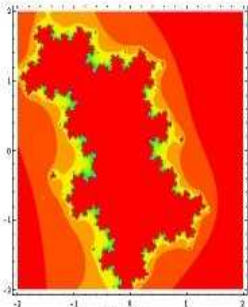
Fig-8
Quadratic Julia set for
$\alpha = 0.4$, $\beta = 0.4$, $\gamma = 0.4$,
$c = 0.1+0.9I$
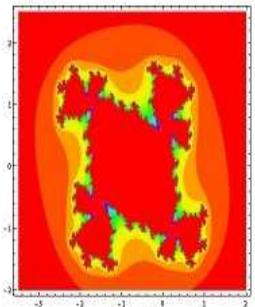
Fig-9
Quadratic Julia set for
$\alpha = 0.4$, $\beta = 1.0$, $\gamma = 1.0$,
$c = 0.1+0.9I$
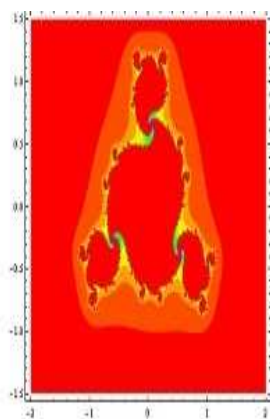
Fig-10
Lord Ganesha (A) Julia set for
$\alpha = 0.3$, $\beta = 0.1$, $\gamma = 0.1$,
$c = 0.1 + 0.9I$
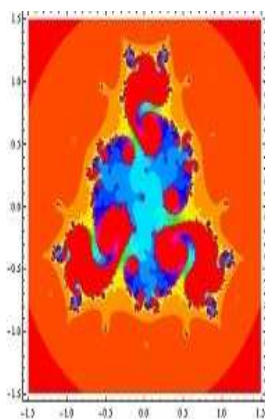
Fig-11
Lord Ganesha(B) Julia set for
$\alpha = 0.9$, $\beta = 0.7$, $\gamma = 0.9$,
$c = 0.1 + 0.9I$
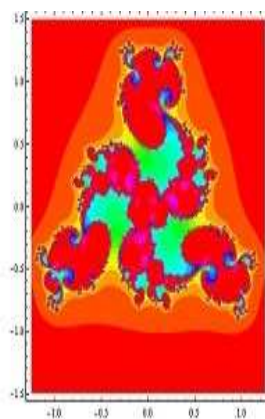
Fig-12
Lord Ganesha(C) Julia set for
$\alpha = 0.26$, $\beta = 0.16$, $\gamma = 0.26$,
$c = 0.11 + 0.95I$
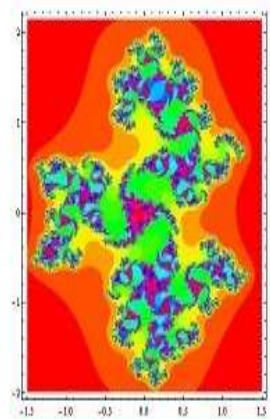
Fig-13
Cubic Julia set for
$\alpha = 0.1$, $\beta = 0.7$, $\gamma = 0.9$,
$c = 0.7 + 0.2I$
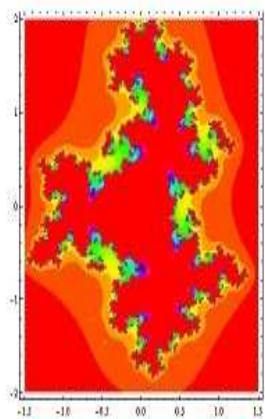
Fig-14
Cubic Julia set for
$\alpha = 0.1$, $\beta = 0.7$, $\gamma = 0.9$,
$c = 0.7 + 0.7I$
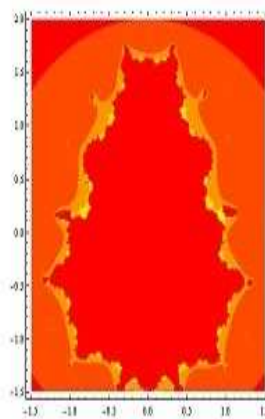
Fig-15
Cubic Julia set for
$\alpha = 0.1$, $\beta = 0.7$, $\gamma = 0.9$,
$c = 0.1 + 0.9I$
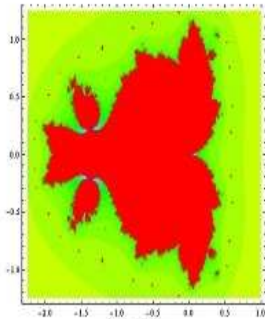
Fig.-16
Quadratic Mandelbrot set for
$\alpha = 0.3$, $\beta = 0.1$, $\gamma = 0.3$
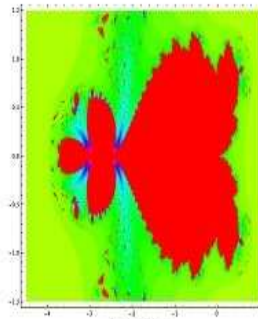
Fig.-17
Quadratic Mandelbrot set for
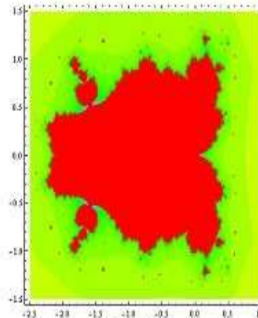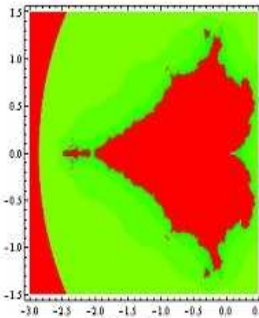$\alpha = 0.3$, $\beta = 0.6$, $\gamma = 0.3$
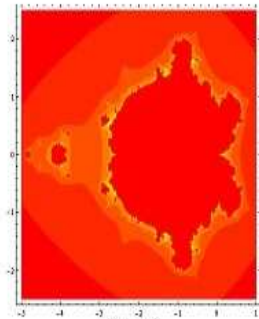
Fig.-18
Quadratic Mandelbrot set for
$\alpha = 0.3$, $\beta = 0.9$, $\gamma = 0.3$
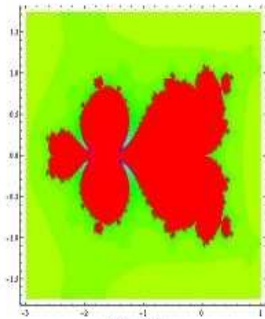
Fig.-19
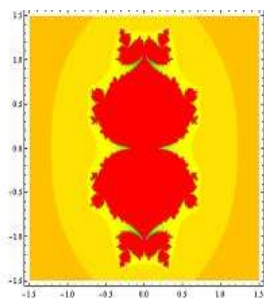Quadratic Mandelbrot set for
$\alpha = 0.1$, $\beta = 0.8$, $\gamma = 0.1$
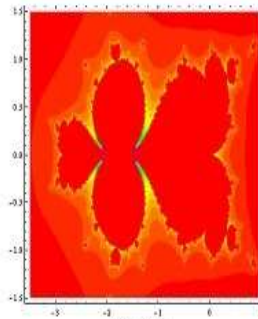
Fig.-20
Quadratic Mandelbrot set for
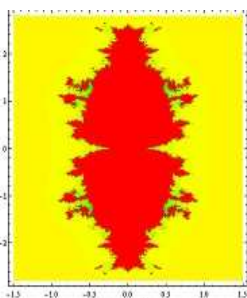$\alpha = 0.1$, $\beta = 0.9$, $\gamma = 0.1$
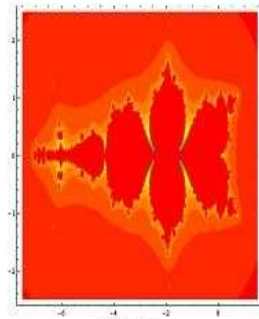
Fig.-21
Quadratic Mandelbrot set for
$\alpha = 0.1$, $\beta = 1.0$, $\gamma = 0.1$
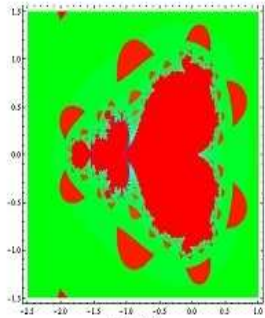
Fig.22
Quadratic Mandelbrot set for
$\alpha = 1$, $\beta = 0.1$, $\gamma = 1$

Fig.-23
Quadratic Mandelbrot set for
$\alpha = 0.2$, $\beta = 0.3$, $\gamma = 0.4$

Fig.-24
Quadratic Mandelbrot set for
$\alpha = 0.7$, $\beta = 0.8$, $\gamma = 0.9$

Fig.-25
Coupled urn(A) Mandelbrot set
for α =0.1, β = 0.1, γ =0.1

Fig.-26
Coupled urn(B) Mandelbrot set
For α =0.2, β = 0.9, γ = 0.2
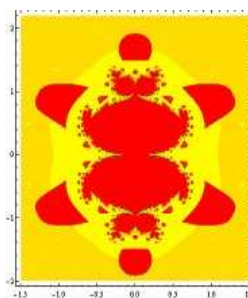
Fig.-27
Coupled urn(C) Mandelbrot set for
α =0.9, β= 0.1, γ = 0.9
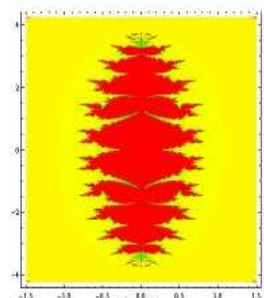
Fig.-28
Cubic Mandelbrot set
for α=0.1, β = 0.9, γ = 0.1
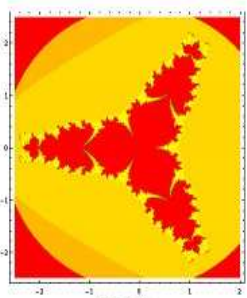
Fig.-29
Mandelbrot set for
α =0.1, β=0.9 , γ =0.1 for n=4
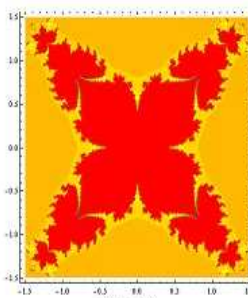
Fig.-30
Mandelbrot set for
α =0.1, β =0.9 , γ = 0.1 for n=5
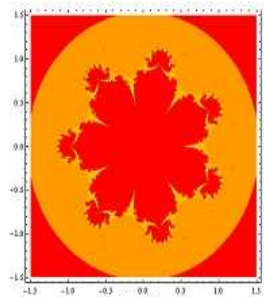
Fig.-31
Mandelbrot set for
α =0.1, β =0.3 , γ =0.1 for n=8
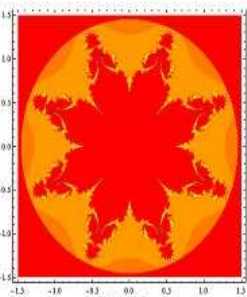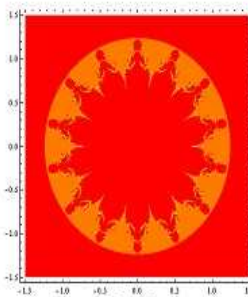
Fig.-32
Mandelbrot set for
α =0.1, β =0.9 , γ =0.1 for n=9

Fig.-33
Mandelbrot set for
α=0.1, β =0.9 , γ=0.1 for n=15

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