International Journal of Pure and Applied Mathematics
Volume 107 No. 2 2016, 423-438
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
doi: 10.12732/ijpam.v107i2.12
ijpam.eu

INITIAL SOFT L-FUZZY QUASI-UNIFORM SPACES

Young Sun Kim[1], Yong Chan Kim[2 §]
[1]Department of Applied Mathematics
Pai Chai University
Dae Jeon, 302-735, KOREA
[2]Department of Mathematics
Gangneung-Wonju University
Gangneung, Gangwondom 210-702, KOREA

Abstract: In this paper, we prove the existences of initial soft L-fuzzy (quasi-)uniformities in a complete residuated lattice. From this fact, we define subspaces and product spaces for soft L-fuzzy (quasi-)uniformities. Moreover, we give their examples.

AMS Subject Classification: 03E72, 06A15, 06F07, 54A05
Key Words: complete residuated lattice, initial soft L-fuzzy quasi-uniformities, uniformly continuous soft maps

1. Introduction

Hájek in [5] introduced a complete residuated lattice which is an algebraic structure for many valued logic. It is an important mathematical tool for algebraic structure of fuzzy contexts, see [2,6,9,10]. Recently, Molodtsov in [13] introduced the soft set as a mathematical tool for dealing information as the uncertainty of data in engineering, physics, computer sciences and many other diverse field. Presently, the soft set theory is making progress rapidly, see [1,4]. Pawlak's rough set (see [14,15]) can be viewed as a special case of soft rough sets, see [4]. The topological structures of soft sets have been developed by many researchers, see [3,9,10,16,19,20].

Received: November 2, 2015
Published: April 9, 2016

§Correspondence author

Kim in [9] introduced a fuzzy soft $F : A \to L^U$ as an extension as the soft $F : A \to P(U)$ where L is a complete residuated lattice. Kim [9,10] introduced the soft topological structures, L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders in complete residuated lattices.

In this paper, we prove the existences of initial soft L-fuzzy (quasi-) uniformities in a complete residuated lattice. From this fact, we define subspaces and product spaces for soft L-fuzzy (quasi-)uniformities. Moreover, we give their examples.

2. Preliminaries

Definition 1. (see [2,6]) An algebra $(L, \wedge, \vee, \odot, \to, 0, 1)$ is called a complete residuated lattice if it satisfies the following conditions:

(C1) $L = (L, \leq, \vee, \wedge, 1, 0)$ is a complete lattice with the greatest element 1 and the least element 0;

(C2) $(L, \odot, 1)$ is a commutative monoid;

(C3) $x \odot y \leq z$ iff $x \leq y \to z$ for $x, y, z \in L$.

In this paper, we assume that $(L, \leq, \odot, \to, \oplus, ^*)$ is a complete residuated lattice with an order reversing involution $x^* = x \to 0$ which is defined by $x \oplus y = (x^* \odot y^*)^*$ unless otherwise specified and we denote $L_0 = L - \{0\}$.

Lemma 2. (see [2,6]) *For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.*

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, *then* $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ *and* $z \to x \leq y \to x$,

(3) $x \odot y \leq x \wedge y \leq x \vee y \leq x \oplus y$,

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \odot (\bigvee_i y_i) = \bigvee_i (x \odot y_i)$,

(6) $x \oplus (\bigwedge_i y_i) = \bigwedge_i (x \oplus y_i)$,

(7) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(8) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(9) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(10) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(11) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(12) $x \odot (x \to y) \leq y$ *and* $x \to y \leq (y \to z) \to (x \to z)$,

(13) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(14) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$,

(15) $x \to y \leq (x \odot z) \to (y \odot z)$ *and* $(x \to y) \odot (y \to z) \leq x \to z$,

(16) $(x \oplus z) \odot y \leq x \oplus (y \odot z)$,

(17) $x \to y = y^* \to x^*$.

Definition 3. (see [9]) Let X be an initial universe of objects and E the set of parameters (attributes) in X. A pair (F, A) is called a *fuzzy soft set* over X, where $A \subset E$ and $F : A \to L^X$ is a mapping. We denote $S(X, A)$ as the family of all fuzzy soft sets under the parameter A.

Definition 4. (see [9,10]) Let (F, A) and (G, A) be two fuzzy soft sets over a common universe X.

(1) (F, A) is a fuzzy soft subset of (G, A), denoted by $(F, A) \leq (G, A)$ if $F(a) \leq G(a)$, for each $a \in A$.

(2) $(F, A) \wedge (G, A) = (F \wedge G, A)$ if $(F \wedge G)(a) = F(a) \wedge G(a)$ for each $a \in A$.

(3) $(F, A) \vee (G, A) = (F \vee G, A)$ if $(F \vee G)(a) = F(a) \vee G(a)$ for each $a \in A$.

(4) $(F, A) \odot (G, A) = (F \odot G, A)$ if $(F \odot G)(a) = F(a) \odot G(a)$ for each $a \in A$.

(5) $(F, A)^* = (F^*, A)$ if $F^*(a) = (F(a))^*$ for each $a \in A$.

(6) $(F, A) \oplus (G, A) = (F \oplus G, A)$ if $(F \oplus G)(a) = (F^*(a) \odot G^*(a))^*$ for each $a \in A$.

Definition 5. (see [9,10]) Let $S(X, A)$ and $S(Y, B)$ be the families of all fuzzy soft sets over X and Y, respectively. The mapping $f_\phi : S(X, A) \to S(Y, B)$ is a soft mapping where $f : X \to Y$ and $\phi : A \to B$ are mappings.

(1) The image of $(F,A) \in S(X,A)$ under the mapping f_ϕ is denoted by $f_\phi((F,A)) = (f_\phi(F), B)$ where

$$f_\phi(F)(b)(y) = \begin{cases} \bigvee_{a\in\phi^{-1}(\{b\})} f^{\rightarrow}(F(a))(y), & \text{if } \phi^{-1}(\{b\}) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

(2) The inverse image of $(G,B) \in S(Y,B)$ under the mapping f_ϕ is denoted by $f_\phi^{-1}((G,B)) = (f_\phi^{-1}(G), A)$ where

$$f_\phi^{-1}(G)(a)(x) = f^{\leftarrow}(G(\phi(a)))(x),\ \forall a \in A, x \in X.$$

(3) The soft mapping $f_\phi : S(X,A) \to S(Y,B)$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Lemma 6. (see [9,10]) *Let $f_\phi : S(X,A) \to S(Y,B)$ be a soft mapping. Then we have the following properties. For $(F,A), (F_i,A) \in S(X,A)$ and $(G,B), (G_i,B) \in S(Y,B)$,*

(1) $(G,B) \geq f_\phi(f_\phi^{-1}((G,B)))$ with equality if f is surjective,

(2) $(F,A) \leq f_\phi^{-1}(f_\phi((F,A)))$ with equality if f is injective,

(3) $f_\phi^{-1}((G,B)^*) = (f_\phi^{-1}((G,B)))^*$,

(4) $f_\phi^{-1}(\bigvee_{i\in I}(G_i,B)) = \bigvee_{i\in I} f_\phi^{-1}((G_i,B))$,

(5) $f_\phi^{-1}(\bigwedge_{i\in I}(G_i,B)) = \bigwedge_{i\in I} f_\phi^{-1}((G_i,B))$,

(6) $f_\phi(\bigvee_{i\in I}(F_i,A)) = \bigvee_{i\in I} f_\phi((F_i,A))$,

(7) $f_\phi(\bigwedge_{i\in I}(F_i,A)) \leq \bigwedge_{i\in I} f_\phi((F_i,A))$ with equality if f is injective,

(8) $f_\phi^{-1}((G_1,B) \odot (G_2,B)) = f_\phi^{-1}((G_1,B)) \odot f_\phi^{-1}((G_2,B))$,

(9) $f_\phi^{-1}((G_1,B) \oplus (G_2,B)) = f_\phi^{-1}((G_1,B)) \oplus f_\phi^{-1}((G_2,B))$,

(10) $f_\phi((F_1,A) \odot (F_2,A)) \leq f_\phi((F_1,A)) \odot f_\phi((F_2,A))$ with equality if f is injective,

(11) $f_\phi((F_1,A) \oplus (F_2,A)) \leq f_\phi((F_1,A)) \oplus f_\phi((F_2,A))$ with equality if f is injective.

Definition 7. (see [9,10]) A mapping $\mathcal{U} : S(X \times X, A) \to L$ is called a soft L-fuzzy quasi-uniformity on X iff it satisfies the properties.

(SU1) There exists $(U,A) \in S(X \times X, A)$ such that $\mathcal{U}((U,A)) = 1$, (SU2) If $(V,A)\le(U,A)$, then $\mathcal{U}((V,A))\le\mathcal{U}((U,A))$,

(SU3) For every $(U,A),(V,A)\in S(X\times X,A)$,

$$\mathcal{U}((U,A)\odot(U,A))\ge\mathcal{U}((U,A))\odot\mathcal{U}((V,A))$$

(SU4) If $\mathcal{U}((U,A))\neq 0$, then $(1_{\triangle},A)\le(U,A)$, where

$$1_{\Delta}(a)(x,y)=\begin{cases}1 & \text{if } x=y\\ 0 & \text{if } x\neq y,\end{cases}$$

(SU5) $\bigvee\{\mathcal{U}((V,A))\mid (V,A)\circ(V,A)\le(U,A)\}\ge\mathcal{U}((U,A))$

$$\begin{aligned}&((V,A)\circ(V,A))(a)(x,y)=(V(a)\circ V(a))(x,y)\\ &=\textstyle\bigvee_{z\in X}(V(a)(z,x)\odot V(a)(x,y)),\quad \forall\, x,y\in X,a\in A.\end{aligned}$$

The triple $(X,A,\mathcal{U})$ is called a soft L-fuzzy quasi-uniform space.

A soft L-fuzzy quasi-uniform space $(X,A,\mathcal{U})$ is called a soft L-fuzzy uniform space if

(U) $\bigvee\{\mathcal{U}((V,A))\mid (V,A)\le(U^s,A)\}\ge\mathcal{U}((U,A))$ where $U^s(a)(x,y)=U(a)(y,x)$ and $(U,A)^s=(U^s,A)$.

Let $(X,A,\mathcal{U}_X)$ and $(Y,B,\mathcal{U}_Y)$ be soft L-fuzzy quasi-uniform spaces and $f_\phi:(X,A)\to(Y,B)$ be a soft map. Then $f_\phi:(X,A,\mathcal{U}_X)\to(Y,B,\mathcal{U}_Y)$ is called an uniformly continuous soft map if, for all $(V,B)\in S(Y\times Y,B)$,

$$\mathcal{U}_Y((V,B))\le\mathcal{U}_X(((f\times f)^{-1}_\phi((V,B))).$$

Lemma 8. (see [10]) *Let $(X,A,\mathcal{U})$ be a soft L-fuzzy quasi uniform space. For each $(U,A)\in S(X\times X,A)$ and $(F,A)\in S(X,A)$, we define , for all $x\in X,a\in A$,*

$$(U,A)[(F,A)](a)(x)=\bigvee_{y\in X}(F(a)(y)\odot U(a)(y,x)).$$

Then we have the following properties.

(1) $(F,A)\le(U,A)[(F,A)]$ *for each* $\mathcal{U}((U,A))>0$,

(2) $(U,A)\le(U,A)\circ(U,A)$, *for each* $\mathcal{U}((U,A))>0$,

(3) $((V,A)\circ(U,A))[(F,A)]=(V,A)[(U,A)[(F,A)]]$,

(4) $(U,A)[\bigvee_i(F_i,A)]=\bigvee_i(U,A)[(F_i,A)]$,

(5) $((U_1,A)\odot(U_2,A))[(F_1,A)\odot(F_2,A)]\le(U_1,A)[(F_1,A)]\odot(U_2,A)[(F_2,A)]$,

(6) $((U_1,A)\odot((U_2,A),A))[(F_1,A)\oplus(F_2,A)]\le(U_1,A)[(F_1,A)]$
$\oplus((U_2,A),A)[(F_2,A)]$.

3. Initial Soft L-Fuzzy Quasi-Uniform Spaces

Lemma 9. *Let $(X, A, \mathcal{U})$ be a soft L-fuzzy quasi-uniform space. We define a function $\mathcal{U}^s : S(X \times X, A) \to L$ by*

$$\mathcal{U}^s((U, A)) = \mathcal{U}((U^s, A))$$

Then $(X, A, \mathcal{U}^s)$ is a soft L-fuzzy quasi-uniform space.

Proof. We easily proved from $(V, A) \circ (V, A) \leq (U, A)$ iff $(V^s, A) \circ (V^s, A) \leq (U^s, A)$.

Theorem 10. *Let $\{(X_k, B_k, \mathcal{V}_k) \mid k \in \Gamma\}$ be a family of soft L-fuzzy (resp. quasi-)uniform spaces, X a set and for each $k \in \Gamma$, $(f_k)_{\phi_k} : (X, A) \to (X_k, B_k)$ a soft map. We define a function $\mathcal{U} : S(X \times X, A) \to L$ by*

$$\begin{aligned}&\mathcal{U}((U, A)) = \bigvee\{\odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i}))\\ &\mid \odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A)\}.\end{aligned}$$

where the $\bigvee$ is taken over every finite index $K = \{k_1, ..., k_n\} \subset \Gamma$. Then:

(1) The structure $\mathcal{U}$ is the coarsest soft L-fuzzy (resp. quasi-)uniformity on X for which each $(f_k)_{\phi_k}$ is an uniformly continuous soft map.

(2) A map $f_\phi : (Z, C, \mathcal{W}) \to (X, A, \mathcal{U})$ is an uniformly continuous soft map iff for each $k \in \Gamma$, $(f_k)_{\phi_k} \circ f_\phi : (Z, C, \mathcal{W}) \to (X_k, B_k, \mathcal{V}_k)$ is an uniformly continuous soft map.

(3)

$$\begin{aligned}&\bigwedge\{(U, A)[(F, A)] \mid \mathcal{U}((U, A)) \geq r\}\\ &\leq \bigwedge\{\odot_{i=1}^n (f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i}))[(F, A)] \mid \odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \geq r\}\\ &\bigwedge\{(U, A)[(F, A)] \mid \mathcal{U}((U, A)) \geq r\}\\ &\geq \bigwedge\{\odot_{i=1}^n (f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i}))[(F, A)] \mid \odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \geq s\}\end{aligned}$$

where the $\bigwedge$ is taken over every finite index $K = \{k_1, ..., k_n\} \subset \Gamma$, for all $r > s$.

(4) If $\{\mathcal{V}_j = \mathcal{V}_Y, f_j = f, \phi_j = \phi, Y_j = Y, B_j = B \mid i \in \Gamma\}$, then

$$\mathcal{U}((U, A)) = \bigvee\{\mathcal{V}_Y((V, B)) \mid (f \times f)^{-1}_\phi((V, B)) \leq (U, A)\}.$$

Proof. (1) First, we will show that $\mathcal{U}$ is a soft L-fuzzy quasi-uniformity on X.

(SU1) Since each $(X_k, B_k, \mathcal{V}_k)$ is a soft L-fuzzy quasi uniform space, by (SU1), there exists $(V_k, B_k) \in S(X_k \times X_k, B_k)$ such that $\mathcal{V}_k((V_k, B_k)) = 1$. For all finite indices $K = \{k_1, ..., k_n\} \subset \Gamma$, put $(V, A) = \odot_{i=1}^{n} (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i}))$. Then there exists $(V, A) \in S(X \times X, A)$ such that $\mathcal{U}((V, A)) = 1$.

(SU2) It is trivial.

(SU3) Suppose there exists $(U, A), (W, A) \in S(X \times X, A)$ such that

$$\mathcal{U}((U, A) \odot (W, A)) \not\geq \mathcal{U}((U, A)) \odot \mathcal{U}((W, A)).$$

By Lemma 2(5) and the definition of $\mathcal{U}((U, A))$, there exists a finite index set $K = \{k_1, ..., k_n\} \subset \Gamma$ such that

$$\begin{aligned} &\mathcal{U}((U, A) \odot (W, A)) \not\geq \odot_{i=1}^{n} \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \odot \mathcal{U}((W, A)), \\ &\odot_{i=1}^{n} (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A). \end{aligned}$$

Also, by definition of $\mathcal{U}((W, A))$, there exists a finite index set $L = \{p_1, ..., p_m\} \subset \Gamma$ such that

$$\begin{aligned} &\mathcal{U}((U, A) \odot (W, A)) \not\geq \odot_{i=1}^{n} \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \odot (\odot_{j=1}^{m} \mathcal{V}_{p_j}((W_{p_j}, B_{p_j}))), \\ &\odot_{j=1}^{m} (f_{p_j} \times f_{p_j})^{-1}_{\phi_{p_j}}((W_{p_j}, B_{p_j})) \leq (W, A). \end{aligned}$$

Since

$$U_m = \begin{cases} V_m, & \text{if } m \in L - L \cap K, \\ V_m \odot W_m, & \text{if } m \in L \cap K, \\ W_m, & \text{if } m \in K - L \cap K, \end{cases}$$

and $\odot_{m \in L \cup K} (f_m \times f_m)^{-1}_{\phi_m}((U_m, B_m)) \leq (U, A) \odot (W, A)$,

$$\begin{aligned} &\mathcal{U}((U, A) \odot (W, A)) \geq \odot_{m \in L \cup K} \mathcal{V}_m((U_m, B_m)) \\ &\geq (\odot_{m \in L - L \cap K} \mathcal{V}_m((U_m, B_m))) \odot (\odot_{m \in L \cap K} \mathcal{V}_m((U_m, B_m))) \\ &\odot (\odot_{m \in K - L \cap K} \mathcal{V}_m((U_m, B_m))) \\ &\geq (\odot_{i=1}^{n} \mathcal{V}_{k_i}((V_{k_i}, B_{k_i}))) \odot (\odot_{j=1}^{m} \mathcal{V}_{p_j}((V_{p_j}, B_{p_j}))). \end{aligned}$$

It is a contradiction. Hence $\mathcal{U}((U, A) \odot (W, A)) \geq \mathcal{U}((U, A)) \odot \mathcal{U}((W, A))$, for all $(U, A), (W, A) \in S(X \times X, A)$.

(SU4) Let $\mathcal{U}((U, A)) \neq 0$. Then there exists a finite index $K = \{k_1, ..., k_n\} \subset \Gamma$ such that

$$\odot_{i=1}^{n} \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \neq 0,$$

$$\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i},B_{k_i}))\leq (U,A)..$$

Hence $\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\neq 0$ for $k_i\in K=\{k_1,...,k_n\}$. Thus $(1_{\triangle_{X_{k_i}}},B_{k_i})\leq (V_{k_i},B_{k_i})$ for $k_i\in K=\{k_1,...,k_n\}$. Therefore,

$$\begin{aligned}&(1_{\triangle_X},A)\leq \odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((1_{\triangle_{X_{k_i}}},B_{k_i}))\\&\leq \odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i},B_{k_i}))\leq (U,A).\end{aligned}$$

(SU5) Suppose there exists $(U,A)\in S(X\times X,A)$ such that

$$\bigvee\{\mathcal{U}((U_1,A))\mid (U_1,A)\circ(U_1,A)\leq (U,A)\}\not\geq \mathcal{U}((U,A)).$$

By the definition of $\mathcal{U}((U,A))$, there exists a finite index $K=\{k_1,...,k_n\}\subset\Gamma$ such that

$$\begin{aligned}&\bigvee\{\mathcal{U}((U_1,A))\mid (U_1,A)\circ(U_1,A)\leq (U,A)\}\not\geq \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i})),\\&\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i},B_{k_i}))\leq (U,A)..\end{aligned}$$

For each $k_i\in K$, since $(X_{k_i},\mathcal{V}_{k_i})$ is a soft L-fuzzy quasi-uniform space, by (SU5),

$$\begin{aligned}&\bigvee\{\mathcal{V}_{k_i}((U_{k_i},B_{k_i}))\mid (U_{k_i},B_{k_i})\circ(U_{k_i},B_{k_i})\leq (V_{k_i},B_{k_i})\}\\&\geq \mathcal{V}_{k_i}((V_{k_i},B_{k_i})).\end{aligned}$$

By Lemma 2(5), for each $k_i\in K$, there exists $(V_{k_i},B_{k_i})\in S(X_{k_i}\times X_{k_i},B_{k_i})$ with $(W_{k_i},B_{k_i})\circ(W_{k_i},B_{k_i})\leq (V_{k_i},B_{k_i})$ such that

$$\begin{aligned}&\bigvee\{\mathcal{U}((U_1,A))\mid (U_1,A)\circ(U_1,A)\leq (U,A)\}\\&\not\geq \odot_{i=1}^{n}\mathcal{V}_{k_i}((W_{k_i},B_{k_i})).\end{aligned}$$

Put $(W,A)=\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i}))$. For each $k_i\in K$, we have

$$\begin{aligned}&(W,A)\circ(W,A)\\&\leq \odot_{i=1}^{n}((f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i}))\circ(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i})))\\&\leq \odot_{i=1}^{n}((f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i})\circ(W_{k_i},B_{k_i})))\\&\leq \odot_{i=1}^{n}(f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i},B_{k_i}))\leq (U,A).\end{aligned}$$

Then we have $(W,A)\circ(W,A)\leq (U,A)$ and

$$\mathcal{U}((W,A))\geq \odot_{i=1}^{n}\mathcal{V}_{k_i}((W_{k_i},B_{k_i})).$$

It is a contradiction. Hence $\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\circ(U_1,A) \leq (U,A)\} \geq \mathcal{U}((U,A))$, for all $(U,A)\in S(X\times X,A)$.

(U) Let $\{(X_k,\mathcal{U}_k) \mid k\in\Gamma\}$ be a family of soft L-fuzzy uniform spaces. Suppose that there exists $(U,A)\in S(X\times X,A)$ such that

$$\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\leq (U,A)^s\} \not\geq \mathcal{U}((U,A)).$$

By the definition of $\mathcal{U}$, there exists a finite index $K=\{k_1,...,k_n\}\subset\Gamma$ such that

$$\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\leq (U,A)^s\} \not\geq \odot_{i=1}^n \mathcal{V}_{k_i}((U_{k_i},B_{k_i})),$$
$$\odot_{i=1}^n (f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((U_{k_i},B_{k_i}))\leq (U,A).$$

For each $k_i\in K$, since $(X_{k_i},\mathcal{V}_{k_i})$ is a soft L-fuzzy uniform space, by (U),

$$\bigvee\{\mathcal{V}_{k_i}((W,A)) \mid (W,A)\leq (U_{k_i},B_{k_i})^s\}\geq \mathcal{V}_{k_i}((U_{k_i},B_{k_i})).$$

For each $k_i\in K$, there exists $(W_{k_i},B_{k_i})\in L^{X_{k_i}\times X_{k_i}}$ with $(W_{k_i},B_{k_i})\leq (U_{k_i},B_{k_i})^s$ such that

$$\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\leq (U,A)^s\} \not\geq \odot_{i=1}^n \mathcal{V}_{k_i}((W_{k_i},B_{k_i})).$$

On the other hand, we have

$$\begin{aligned}&\odot_{i=1}^n (f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i}))\leq \odot_{i=1}^n (f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((U_{k_i},B_{k_i})^s)\\ &=\odot_{i=1}^n ((f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((U_{k_i},B_{k_i})))^s=(\odot_{i=1}^n (f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((U_{k_i},B_{k_i})))^s\\ &\leq (U,A)^s.\end{aligned}$$

Put $(W,A)=\odot_{i=1}^n (f_{k_i}\times f_{k_i})^{-1}_{\phi_{k_i}}((W_{k_i},B_{k_i}))$. Thus there exists $(W,A)\in S(X\times X,A)$, such that

$$(W,A)\leq (U,A)^s,\quad \mathcal{U}((W,A))\geq \odot_{i=1}^n \mathcal{V}_{k_i}((W_{k_i},B_{k_i})).$$

Hence

$$\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\leq (U,A)^s\}\geq \mathcal{U}((W,A))\geq \odot_{i=1}^n \mathcal{V}_{k_i}((W_{k_i},B_{k_i})).$$

It is a contradiction. So, $\bigvee\{\mathcal{U}((U_1,A)) \mid (U_1,A)\leq (U,A)^s\}\geq \mathcal{U}((U,A))$, for all $(U,A)\in S(X\times X,A)$.

Second, by the definition of $\mathcal{U}$, for all $k\in\Gamma$, $(V_k,B_k)\in S(X_k\times X_k,B_k)$,

$$\mathcal{U}((f_k\times f_k)^{-1}_{\phi_k}((V_k,B_k)))\geq \mathcal{V}_k((V_k,B_k)).$$

Hence each $(f_k)_{\phi_k} : (X, A, \mathcal{U}) \to (X_k, B_k, \mathcal{V}_k)$ is an uniformly continuous soft map.

Finally, if $(f_k)_{\phi_k} : (X, A, \mathcal{U}') \to (X_k, B_k, \mathcal{V}_k)$ is an uniformly continuous soft map ,that is, $\mathcal{U}'((f_k \times f_k)^{-1}_{\phi_k}((U, A))) \geq \mathcal{V}_k((U, A))$ for all $k \in \Gamma$, then it is proved that $\mathcal{U}' \geq \mathcal{U}$ from the following:

$$\begin{aligned}
&\mathcal{U}((U, A))\\
&= \bigvee\{\odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \mid \odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A)\}\\
&\leq \bigvee\{\odot_{i=1}^n \mathcal{U}'((f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})))\\
&\mid \odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A)\}\\
&\leq \bigvee\{\mathcal{U}'(\odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})))\\
&\mid \odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A)\}\\
&\leq \mathcal{U}'((U, A)), \quad \text{for all } (U, A) \in S(X \times X, A).
\end{aligned}$$

(2) Necessity of the composition condition is clear since the composition of an uniformly continuous soft maps is an uniformly continuous soft map.

Conversely, suppose that $f_\phi : (Z, C, \mathcal{W}) \to (X, A, \mathcal{U})$ is not an uniformly continuous soft map. There exists $(U, A) \in S(X \times X, A)$ such that

$$\mathcal{W}((f \times f)^{-1}_\phi((U, A))) \not\geq \mathcal{U}((U, A)).$$

By the definition of $\mathcal{U}$, there exists a finite index set $K = \{k_1, ..., k_n\} \subset \Gamma$ such that

$$\begin{gathered}
\mathcal{W}((f \times f)^{-1}_\phi((U, A))) \not\geq \odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})),\\
\odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})) \leq (U, A).
\end{gathered}$$

On the other hand, for each $k_i \in K$, since $(f_{k_i} \circ f)_{\phi_{k_i} \circ \phi}$ is an uniformly continuous soft map, we have

$$\begin{aligned}
&\mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \leq \mathcal{W}(((f_{k_i} \circ f) \times (f_{k_i} \circ f))^{-1}_{\phi_{k_i} \circ \phi}((V_{k_i}, B_{k_i})))\\
&= \mathcal{W}((f \times f)^{-1}_\phi((f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})))).
\end{aligned}$$

It follows that

$$\begin{aligned}
&\odot_{i=1}^n \mathcal{V}_{k_i}((V_{k_i}, B_{k_i})) \leq \odot_{i=1}^n \mathcal{W}((f \times f)^{-1}_\phi \circ (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i})))\\
&\leq \mathcal{W}(\odot_{i=1}^n (f \times f)^{-1}_\phi((f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i}))))\\
&= \mathcal{W}((f \times f)^{-1}_\phi(\odot_{i=1}^n (f_{k_i} \times f_{k_i})^{-1}_{\phi_{k_i}}((V_{k_i}, B_{k_i}))))\\
&\leq \mathcal{W}((f \times f)^{-1}_\phi((U, A))).
\end{aligned}$$

It is a contradiction.

(3) Since

$$\begin{aligned}&\mathcal{U}(\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i})))\\&\geq \odot_{i=1}^{n}\mathcal{U}((f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i})))\geq \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq r,\end{aligned}$$

we have

$$\begin{aligned}&\bigwedge\{(U,A)[(F,A)] \mid \mathcal{U}((U,A))\geq r\}\\&\leq \bigwedge\{\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))[(F,A)] \mid \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq r\}\end{aligned}$$

Conversely, for $s<r$, suppose that

$$\begin{aligned}&\bigwedge\{(U,A)[(F,A)] \mid \mathcal{U}((U,A))\geq r\}\\&\not\geq \bigwedge\{\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))[(F,A)] \mid \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq s\}\end{aligned}$$

Then there exists $(U,A)\in S(X\times X,A)$ with $\mathcal{U}((U,A))\geq r$ such that

$$\begin{aligned}&(U,A)[(F,A)]\\&\not\geq \bigwedge\{\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))[(F,A)] \mid \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq s\}.\end{aligned}$$

On the other hand, since $\mathcal{U}((U,A))\geq r$, for $r>s$, there exists a finite index $K=\{k_1,...,k_n\}\subset\Gamma$ such that

$$\mathcal{U}((U,A))\geq \odot_{i=1}^{n}\mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq s,$$

$$\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))\leq (U,A)..$$

It implies

$$\begin{aligned}&\bigwedge\{\odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))[(F,A)] \mid \mathcal{V}_{k_i}((V_{k_i},B_{k_i}))\geq s\}\\&\leq \odot_{i=1}^{n}(f_{k_i}\times f_{k_i})_{\phi_{k_i}}^{-1}((V_{k_i},B_{k_i}))[(F,A)]\\&\leq (U,A)[(F,A)].\end{aligned}$$

It is a contradiction.

(4) From the definition of $\mathcal{U}((U,A))$,

$$\mathcal{U}((U,A))\geq \bigvee\{\mathcal{V}_Y((V,B)) \mid (f_\phi)^{-1}((V,B))\leq (U,A)\}.$$

Suppose

$$\mathcal{U}((U,A))\not\leq \bigvee\{\mathcal{V}_Y((V,B)) \mid (f_\phi)^{-1}((V,B))\leq (U,A)\}.$$

Then there exists a finite subsets $K = \{i_1, ..., i_n\}$ with $\odot_{k=1}^n (f_\phi)^{-1}((V_{i_k}, B)) \leq (U, A)$ such that

$$\odot_{k=1}^n \mathcal{V}_Y((V_{i_k}, B)) \not\leq \bigvee\{\mathcal{V}_Y((V, B)) \mid (f_\phi)^{-1}((V, B)) \leq (U, A)\}.$$

On the other hand, since $\odot_{k=1}^n (f_\phi)^{-1}((V_{i_k}, B)) = (f_\phi)^{-1}(\odot_{k=1}^n (V_{i_k}, B)) \leq (U, A)$, we have

$$\begin{aligned} &\bigvee\{\mathcal{V}_Y((V, B)) \mid (f_\phi)^{-1}((V, B)) \leq (U, A)\} \\ &\geq \mathcal{V}_Y(\odot_{k=1}^n (V_{i_k}, B)) \geq \odot_{k=1}^n \mathcal{V}_Y((V_{i_k}, B)). \end{aligned}$$

It is a contradiction. Hence the result follows. □

From Theorem 10, we define the following definition.

Definition 11. Let $\mathcal{U}_i$ be soft L-fuzzy quasi-uniformities on X_i for $i \in \Gamma$. Let X be a set and, for each $i \in \Gamma$, $(f_i)_{\phi_i} : (X, A) \to (X_i, B_i)$ a soft map. The *initial soft L-fuzzy quasi-uniformity* on X induced by $\{(f_i)_{\phi_i} \mid i \in \Gamma\}$ is the coarsest soft L-fuzzy quasi-uniformity on X for which $(f_i)_{\phi_i}$ is an uniformly continuous soft map.

The *product soft L-fuzzy quasi-uniformity* on $X = \Pi_{i\in\Gamma} X_i$ is the initial soft L-fuzzy quasi-uniformity induced by $\{(\pi_i)_{\phi_i} : (X, A) \to (X_i, B_i) \mid i \in \Gamma\}$ of projection soft maps.

The pair $(Y, B, \mathcal{U}_Y)$ is a subspace of $(X, A, \mathcal{U})$ where $\mathcal{U}_Y$ is the initial soft L-fuzzy quasi-uniformity induced by an inclusion soft map $i_\phi : (Y, B) \to (X, A)$.

From Theorem 10, we obtain the following corollary.

Corollary 12. *Let $\mathcal{U}_i$ be soft L-fuzzy quasi-uniformity on X for $i \in \Gamma$. Let X be a set and, for each $i \in \Gamma$, $(id_i)_{id} : (X, A) \to (X, A, \mathcal{U}_i)$ an identity soft map. Define the map $\mathcal{U} : S(X \times X, A) \to L$ on X by*

$$\begin{aligned} &\mathcal{U}((U, A)) \\ &= \bigvee\{\odot_{i=1}^n \mathcal{U}_{k_i}((U_{k_i}, B_{k_i})) \mid \odot_{i=1}^n (U_{k_i}, B_{k_i}) \leq (U, A)\} \end{aligned}$$

where the $\bigvee$ is taken over every finite index $K = \{k_1, ..., k_n\} \subset \Gamma$. Then $\mathcal{U}$ is the coarsest soft L-fuzzy quasi-uniformity on X which $\mathcal{U}$ is finer than $\mathcal{U}_i$ for each $i \in \Gamma$.

Example 13. Let $X = \{h^i \mid i = \{1, ..., 4\}\}$ with h^i=house and $E_X = \{e, b, w, c, i\}$ with e=expensive,b= beautiful, w=wooden, c= creative, i=in the green surroundings.

Define operations $\odot$, $\to$ and * on [0,1](called Lukasierwicz structure) by

$$x \odot y = \max\{0, x + y - 1\}, x^* = 1 - x$$

$$x \to y = \min\{1 - x + y, 1\}.$$

Then $([0,1], \odot, \to, ^* 0, 1)$ is a complete residuated lattice (ref.[2,6]). Let $A = \{e, b\} \subset E_X$, $Y = \{y^i \mid i = \{1,2,3\}\}$, $Z = \{z^i \mid i = \{1,2,3\}\}$, $B = \{b_i \mid i = \{1,2\}\}$ and $C = \{c_i \mid i = \{1,2\}\}$ be given.

(1) Put $V, V \odot V \in S(Y \times Y, B)$ as

$V(b_1)$	y^1	y^2	y^3	$V(b_2)$	y^1	y^2	y^3
y^1	1	0.6	0.5	y^1	1	0.5	0.3
$y2$	0.1	1	0.5	y^2	0.7	1	0.5
y^3	0.4	0.6	1	y^3	0.6	0.6	1

$(V \odot V)(b_1)$	y^1	y^2	y^3	$(V \odot V)(b_2)$	y^1	y^2	y^3
y^1	1	0.2	0	y^1	1	0	0
y^2	0	1	0	y^2	0.4	1	0
y^3	0	0.2	1	y^3	0.2	0.2	1

We define $\mathcal{V}_Y : S(Y \times Y, B) \to [0,1]$ as follows:

$$\mathcal{V}_Y((U,A)) = \begin{cases} 1, & \text{if } (U,A) = (1_{Y\times Y}, B) \\ 0.6, & \text{if} (U,A) \geq (V,B), \\ 0.3, & \text{if} (U,A) \geq (V \odot V, B), (U,A) \not\geq (V,B), \\ 0, & \text{otherwise.} \end{cases}$$

Since $(V,B) \circ (V,B) = (V,B)$ and $(V \odot V, B) \circ (V \odot V, B) = (V \odot V, B)$, $\mathcal{V}_Y$ is a soft L-fuzzy quasi-uniformity on Y, but not a soft L-fuzzy quasi-uniformity on Y because

$$\begin{aligned} &\bigvee\{\mathcal{V}_Y((U,B)) \mid (U,B) \leq (V,B)^s\} = \mathcal{V}_Y((V \odot V, B)) = 0.3 \\ &\not\geq \mathcal{V}_Y((V,B)) = 0.6. \end{aligned}$$

(2) Put $W \in S(Z \times Z, C)$ as

$W(c_1)$	z^1	z^2	z^3	$W(c_2)$	z^1	z^2	z^3
z^1	1	0.5	0.5	z^1	1	0.3	0.3
z^2	0	1	0.5	z^2	0.5	1	0.4
z^3	0.3	0.6	1	z^3	0.6	0.5	1

We define $\mathcal{W}_Z : S(Z \times Z, C) \to [0,1]$ as follows:

$$\mathcal{W}_Z((U,C)) = \begin{cases} 1, & \text{if } (U,C) = (1_{Z\times Z}, B) \\ 0.5, & \text{if} (U,C) \geq (W,C), \\ 0, & \text{otherwise.} \end{cases}$$

Since $(W,C)\circ(W,C)=(W,C)$, $\mathcal{W}_Z$ is a soft L-fuzzy quasi-uniformity on Z, but not a soft L-fuzzy quasi-uniformity on Z because

$$\begin{aligned}&\bigvee\{\mathcal{W}_Z((U,C))\mid (U,C)\le (W,C)^s\}=0\\ &\not\geq \mathcal{W}_Z((W,C))=0.5.\end{aligned}$$

(3) Put $f_1 : X=\{h^1,h^3,h^4\}\to\{y^1,y^2,y^3\}$ and $\phi_1 : A=\{e,b\}\to\{b^1,b^2\}$ as follows

$$f_1(h^1)=y^1, f_1(h^3)=y^2, f_1(h^4)=y^3,$$

$$\phi_1(e)=b_1, \phi_1(b)=b_2$$

$(f_1)^{-1}_{\phi_1}(V)(e)$	h^1	h^3	h^4
h^1	1	0.6	0.5
h^3	0.1	1	0.5
h^4	0.4	0.6	1

$(f_1)^{-1}_{\phi_1}(V)(b)$	h^1	h^3	h^4
h^1	1	0.5	0.3
h^3	0.7	1	0.5
h^4	0.6	0.6	1

$(f_1)^{-1}_{\phi_1}(V\odot V)(e)$	h^1	h^3	h^4
h^1	1	0.2	0
h^3	0	1	0
h^4	0	0.2	1

$(f_1)^{-1}_{\phi_1}(V\odot V)(b)$	h^1	h^3	h^4
h^1	1	0	0
h^3	0.4	1	0
h^4	0.2	0.2	1

By Theorem 10(4), we can obtain a soft L-fuzzy quasi-uniformity $(f_1)^{-1}_{\phi_1}(\mathcal{V}_Y) : S(X\times X,A)\to[0,1]$ as follows:

$$(f_1)^{-1}_{\phi_1}(\mathcal{V}_Y)((U,A))=\begin{cases}1, & \text{if } (U,A)=(1_{X\times X},A)\\ 0.6, & \text{if}(U,A)\ge((f_1)^{-1}_{\phi_1}(V),A),\\ & (U,A)\not\geq((f_1)^{-1}_{\phi_1}(V\odot V),A),\\ 0.3, & \text{if}(U,A)\ge((f_1)^{-1}_{\phi_1}(V\odot V),A),\\ 0, & \text{otherwise.}\end{cases}$$

(4) Put $f_2 : X=\{h^1,h^3,h^4\}\to\{z^1,z^2,z^3\}$ and $\phi_2 : A=\{e,b\}\to\{c^1,c^2\}$ as follows

$$f_2(h^1)=z^1, f_2(h^3)=z^2, f_2(h^4)=z^3,$$

$$\phi_2(e)=c_1, \phi_2(b)=c_2$$

$(f_2)^{-1}_{\phi_2}(W)(e)$	h^1	h^3	h^4	$(f_2)^{-1}_{\phi_2}(W)(b)$	h^1	h^3	h^4
h^1	1	0.5	0.5	h^1	1	0.3	0.3
h^3	0	1	0.5	h^3	0.5	1	0.4
h^4	0.3	0.6	1	h^4	0.6	0.5	1

By Theorem 10(4), we can obtain a soft L-fuzzy quasi-uniformity $(f_2)^{-1}_{\phi_2}(\mathcal{W}_Z) : S(X \times X, A) \to [0,1]$ as follows:

$$(f_2)^{-1}_{\phi_1}(\mathcal{W}_Z)((U,A)) = \begin{cases} 1, & \text{if } (U,A) = (1_{X\times X}, A) \\ 0.5, & \text{if}(U,A) \geq ((f_2)^{-1}_{\phi_2}(W), A), \\ 0, & \text{otherwise.} \end{cases}$$

$(f_1)^{-1}_{\phi_1}(V) \odot (f_2)^{-1}_{\phi_2}(W)(e)$	h^1	h^3	h^4
h^1	1	0.1	0
h^3	0	1	0
h^4	0	0.2	1

$(f_1)^{-1}_{\phi_1}(V) \odot (f_2)^{-1}_{\phi_2}(W)(b)$	h^1	h^3	h^4
h^1	1	0	0
h^3	0.2	1	0
h^4	0.2	0.1	1

From Theorem 10, we obtain a soft L-fuzzy quasi-uniformity $(f_1)^{-1}_{\phi_1}(\mathcal{V}_Y) \odot (f_2)^{-1}_{\phi_2}(\mathcal{W}_Z) : S(X \times X, A) \to [0,1]$ as follows:

$$(f_1)^{-1}_{\phi_1}(\mathcal{V}_Y) \odot (f_2)^{-1}_{\phi_2}(\mathcal{W}_Z)((U,A)) =$$
$$\begin{cases} 1, & \text{if } (U,A) = (1_{X\times X}, A) \\ 0.6, & \text{if}(U,A) \geq ((f_1)^{-1}_{\phi_1}(V), A), \\ 0.5, & \text{if}(U,A) \geq ((f_2)^{-1}_{\phi_2}(W), A), \\ & (U,A) \not\geq ((f_1)^{-1}_{\phi_1}(V), A), \\ 0.3, & \text{if}(U,A) \geq ((f_1)^{-1}_{\phi_1}(V \odot V), A), \\ & (U,A) \not\geq ((f_2)^{-1}_{\phi_2}(W), A), \\ 0.1, & \text{if}(U,A) \geq ((f_1)^{-1}_{\phi_1}(V) \odot (f_2)^{-1}_{\phi_2}(W), A), \\ & (U,A) \not\geq ((f_1)^{-1}_{\phi_1}(V \odot V), A), \\ 0, & \text{otherwise.} \end{cases}$$

References

[1] K.V. Babitha, J.J. Sunil, Soft set relations and functions, *Compu. Math. Appl.*, **60**(2010), 1840-1849, **doi:** 10.1016/j.camwa.2010.07.014.

[2] R. Bělohlávek,*Fuzzy Relational Systems*, Kluwer Academic Publishers, New York , (2002), **doi:** 10.1007/978-1-4615-0633-1.

[3] N. Cağman, S. Karatas and S. Enginoglu, Soft topology, *Comput. Math. Appl.*, **62**(1) (2011), 351-358. **doi:** 10.1016/j.camwa.2011.05.016.

[4] F. Feng, X. Liu, V.L. Fotea, Y.B. Jun, Soft sets and soft rough sets, *Information Sciences*, **181** (2011), 1125-1137, **doi:** 10.1016/j.ins.2010.11.004.

[5] P. Hájek, *Metamathematices of Fuzzy Logic*, Kluwer Academic Publishers, Dordrecht (1998), **doi:** 10.1007/978-94-011-5300-3.

[6] U. Höhle, S.E. Rodabaugh, *Mathematics of Fuzzy Sets: Logic, Topology, and Measure Theory*, The Handbooks of Fuzzy Sets Series 3, Kluwer Academic Publishers, Boston, 1999, **doi:** 10.1007/978-1-4615-5079-2.

[7] B. Hutton, Uniformities in fuzzy topological spaces, *J. Math. Anal. Appl.*, **58** (1977), 74-79. **doi:** 10.1016/0022-247x(77)90192-5.

[8] A.K. Katsaras, On fuzzy uniform spaces, *J. Math. Anal. Appl.*, **101**, 1984, 97-113. **doi:** 10.1016/0022-247x(84)90060-x.

[9] Y.C. Kim and J.M. Ko, Soft L-topologies and soft L-neighborhood systems, (accepted to) J. Math. Comput. Sci.

[10] Y.C. Kim and J.M. Ko, Soft L-fuzzy quasi-uniformities and soft L-fuzzy topogenous orders, (submit to) J. Intelligent and Fuzzy Systems.

[11] R. Lowen, Fuzzy uniform spaces, *J. Math. Anal. Appl.*, **82** (1981), 370-385, **doi:** 10.1016/0022-247x(81)90202-x.

[12] R. Lowen, Fuzzy neighborhood spaces, *Fuzzy Sets and Systems*, **7** (1982), 165-189.

[13] D. Molodtsov, Soft set theory, *Comput. Math. Appl.*, **37**(1999), 19-31.

[14] Z. Pawlak, Rough sets, *Int. J. Comput. Inf. Sci.*, **11** (1982), 341-356.

[15] Z. Pawlak, Rough probability, *Bull. Pol. Acad. Sci. Math.*, **32**(1984), 607-615.

[16] M. Shabir and M. Naz, On soft topological spaces, *Comput. Math. Appl.*,**61** (2011), 1786-1799, **doi:** 10.1016/j.camwa.2011.02.006.

[17] B. Tanay and M. B. Kandemir, Topological structure of fuzzy soft sets, *Comput. Math. Appl.*, **61**(10) (2011), 2952-2957, **doi:** 10.1016/j.camwa.2011.03.056.

[18] W.Z. Wu and W.X. Zhang, Neighborhood operator systems and approximations , *Information Sciences*, **144** (2002), 201-217.

[19] Hu Zhao and Sheng-Gang Li, L-fuzzifying soft topological spaces and L-fuzzifying soft interior operators, *Ann. Fuzzy Math. Inform.*, **5**(3) (2013), 493-503.

[20] Í. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, Remarks on soft topological spaces, *Ann. Fuzzy Math. Inform.*, **3**(2) (2012), 171-185.