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GRAPHICAL METHOD FOR INTERVAL BIMATRIX GAMES

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Abstract: This paper deals with two-person non-zero sum games with interval payoffs. Graphical method to find a mixed strategy equilibrium is adapted to interval bimatrix games. In addition, interval bimatrix games Nash equilibrium is attained by graphical method. Numerical examples are also illustrated.

AMS Subject Classification: 91A05, 91A10
Key Words: interval bimatrix game, bimatrix game, interval payoff

1. Introduction

Interval game theory which is special case of fuzzy game theory, is an important content in interval fuzzy mathematics. Interval game has widely played an important role in the field of decision making theory such as economics, management, operation research etc. In this paper, we consider an interval based approach. Two-person non-zero sum games with interval payoff have already been studied in recent years by various researchers [1],[5],[6],[9]. Bimatrix game can be considered as a natural extension of the matrix game. Nash[2] defines the concept of Nash equilibrium solutions in bimatrix games for single pair of payoff matrices. Here, we present graphical method for solving interval bimatrix games by using

$$\ell : \tilde{\mathbb{R}} \to \mathbb{R}, \quad \ell([a, b]) = b - a.$$

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2. Interval Numbers

An interval number $\tilde{a}$ is closed subset of real numbers. Moreover its represented as follows,

$$\tilde{a} = [a_L, a_R] = \{x \in \mathbb{R} : \ a_L \leq x \leq a_R\}$$

in which a_L and a_R are respectively referred to as the lower and upper bound of the interval $\tilde{a}$ and $a_L \leq a_R$. If $a_L = a_R$, then $\tilde{a} = [a, a]$ is a real number. Midpoint and half-width of an interval number $\tilde{a}$ is defined as follows,

$$m(\tilde{a}) = \frac{a_L + a_R}{2}, w(\tilde{a}) = \frac{a_R - a_L}{2}.$$

The set of all interval numbers is represented by $\tilde{\mathbb{R}}$.

2.1. Basic Interval Arithmetic

Let $\tilde{a} = [a_L, a_R]$ and $\tilde{b} = [b_L, b_R]$ be two interval numbers. The arithmetic operations are defined as follows,

1. $\tilde{a} + \tilde{b} = [a_L + b_L, a_R + b_R]$;
2. $\tilde{a} - \tilde{b} = [a_L - b_R, a_R - b_L]$;
3. $\tilde{a}\tilde{b} = [minS, maxS], S = \{a_L b_L, a_L b_R, a_R b_L, a_R b_R\}$;
4. $\frac{\tilde{a}}{\tilde{b}} = \tilde{a}(1/\tilde{b})$, $(0 \notin \tilde{b}$ and $\frac{1}{\tilde{b}} = \{\tilde{b} : \left(\frac{1}{\tilde{b}}\right) \in \tilde{b}\} = \left[\frac{1}{b_R}, \frac{1}{b_L}\right]$ $\frac{\tilde{a}}{\tilde{b}} = \tilde{a}\left(\frac{1}{\tilde{b}}\right) = [a_L, a_R]\left[\frac{1}{b_R}, \frac{1}{b_L}\right] = [\min\left\{\frac{a_L}{b_R}, \frac{a_L}{b_L}, \frac{a_R}{b_R}, \frac{a_R}{b_L}\right\}, \max\left\{\frac{a_L}{b_R}, \frac{a_L}{b_L}, \frac{a_R}{b_R}, \frac{a_R}{b_L}\right\}]$);
5. $\alpha \in \mathbb{R}$ için $\alpha a = \alpha[a_L, a_R] = \{.\alpha[a_L, a_R], \quad \alpha \geq 0 \alpha[a_R, a_L], \quad \alpha < 0;$
6. $\tilde{a}\frac{1}{\tilde{a}} \neq 1$;
7. $\tilde{a} + (-\tilde{a}) \neq 0$.

2.2. Comparasion of Interval Numbers

An extensive research and wide coverage on interval arithmetic and its applications can be found in [4]. A brief comparasion on different interval orders is given in [4] on the basis of decision makers opinion.

Let $\tilde{a} = [a_L, a_R]$ and $\tilde{b} = [b_L, b_R]$ be two disjoint interval numbers. $\tilde{a}$ is less than $\tilde{b}$ if and only if $a_R < a_L$. This is denoted by $\tilde{a} < \tilde{b}$ and the relation is an extension of " $<$ "on the real line. If the closed interval numbers are overlapping, then we use the acceptability index idea suggested by [11].

Let $\tilde{\mathbb{R}}$ be the set of all closed intervals on the real line $\mathbb{R}$. The function

$$\mathcal{A} : \tilde{\mathbb{R}} x \tilde{\mathbb{R}} \longrightarrow [\infty, 0)$$

such that

$$\mathcal{A}(\tilde{a} \prec \tilde{b}) = \frac{m(\tilde{b}) - m(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}$$

$(w(\tilde{b}) + w(\tilde{a}) \neq 0)$ is called *acceptability function*. Thus, the number $\mathcal{A}(\tilde{a} \prec \tilde{b})$ is called grade of acceptability of the $\tilde{a}$ to be inferior to $\tilde{b}$.

1. $\mathcal{A}(\tilde{a} \prec \tilde{b}) \geq 1$ when $m(\tilde{b}) > m(\tilde{a})$ and $a_R \leq b_L$
2. $0 < \mathcal{A}(\tilde{a} \prec \tilde{b}) < 1$ when $m(\tilde{b}) > m(\tilde{a})$ and $b_L \leq a_R$
3. $\mathcal{A}(\tilde{a} \prec \tilde{b}) = 0$ when $m(\tilde{b}) = m(\tilde{a})$. In this case, an extensive research about comparasion of $\tilde{a}$ and $\tilde{b}$ can be found in [6].

Example1. Let $\tilde{a} = [10, 20]$ and $\tilde{b} = [24, 28]$ be two interval numbers.Then,

$$\mathcal{A}(\tilde{a} \prec \tilde{b}) = \frac{26 - 15}{2 + 5} = \frac{11}{7} > 1.$$

Hence, $\tilde{a}$ is less than $\tilde{b}$ with full satisfaction.

Example 2. Let $\tilde{a} = [1, 5]$ and $\tilde{b} = [3, 17]$ be two interval numbers.Then,

$$\mathcal{A}(\tilde{a} \prec \tilde{b}) = \frac{10 - 3}{7 + 2} = \frac{7}{9} \in (0, 1).$$

Hence, $\tilde{a}$ is less than $\tilde{b}$ with grade of satisfaction $\frac{7}{9}$.

3. Interval Bimatrix Games

A bimatrix game is a two player game, player I and player II, player I has m pure strategies $\{I_1, I_2, \ldots, I_m\}$ while playerII has n pure strategies $\{II_1, II_2, \ldots, II_n\}$. There is no longer a value c, such that

$$H_I(I_i, II_j) + H_{II}(I_i, II_j) = c$$

where $H_I(I_i, II_j), H_{II}(I_i, II_j)$ are expected payoffs of player I and player II, respectively. That is, the pay-offs of player Ior player IIdo not give information about pay-off of the other one. Thus, if player I plays $I_{\dot{I}}$ and player II plays II_J, then the payoff is as follows

$$H(I_i, II_j) = (H_I(I_i, II_j) + H_{II}(I_i, II_j))$$

If player I selects the strategy i and player II selects the strategy j, then $[a_{ijL}, a_{ijR}]$ and $[b_{ijL}, b_{ijR}]$ are payoffs of the player I and player II, respectively. Hence, a bimatrix game is determined by a pair of matrix $(\tilde{A}, \tilde{B})$. When payoff matrix whose entries are interval numbers, is given, the payyoff matris of two-person non-zero sum game $(\tilde{A}, \tilde{B})$ is represented as follows:

$$\begin{bmatrix} ([a_{11L}, a_{11R}], [b_{11L}, b_{11R}]) & ([a_{12L}, a_{12R}], [b_{12L}, b_{12R}]) & \cdots & ([a_{1mL}, a_{1mR}], [b_{1mL}, b_{1mR}]) \\ ([a_{21L}, a_{21R}], [b_{21L}, b_{21R}]) & ([a_{22L}, a_{22R}], [b_{22L}, b_{22R}]) & \cdots & ([a_{2mL}, a_{2mR}], [b_{2mL}, b_{2mR}]) \\ \vdots & \vdots & \vdots & \vdots \\ ([a_{1mL}, a_{1mR}], [b_{1mL}, b_{1mR}]) & ([a_{2mL}, a_{2mR}], [b_{2mL}, b_{2mR}]) & \cdots & ([a_{nmL}, a_{nmR}], [b_{nmL}, b_{nmR}]) \end{bmatrix}.$$

A mixed strategy set of player I is

$$S_I = \left\{ x = (x_1, x_2, ..., x_m) \in \mathbb{R}^m \colon x_i \geq 0,\ \sum_{i=1}^{m} x_i = 1\ \forall i = 1, ..., m \right\}$$

Similarly, a mixed strategy set of player II is

$$S_{II} = \left\{ y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n \colon y_i \geq 0,\ \sum_{j=1}^{n} y_i = 1\ \forall i = 1, ..., n \right\}$$

When player I plays mixed strategy of $x \in S_I$ and player II plays mixed strategy of $y \in S_{II}$, player I receives payoff

$$H_I(x, y) = \sum_{j=1}^{n} \sum_{i=1}^{m} x_i [a_{ijL}, a_{ijR}] y_j$$

and player II receives payoff

$$H_{II}(x, y) = \sum_{j=1}^{n} \sum_{i=1}^{m} x_i [b_{ijL}, b_{ijR}] y_j.$$

Let $x^{\square} \in S_I$, $y^{\square} \in S_{II}$ be mixed strategies for player I and player II, respectively. If the following inequalities

$$H_I(x, y^{\square}) \quad \leq \quad H_I(x^{\square}, y^{\square})$$

$$H_{II}(x^{\star}, y) \quad \leq \quad H_{II}(x^{\star}, y^{\star})$$

are satisfied for arbitrary $x \in S_I$, $y \in S_{II}$ then pair of $(x^{\star}, y^{\star})$ is called strategies of equilibrium (or pair of equilibrium) of the game.

4. Graphical Method for Interval Bimatrix Games

In a bimatrix game, we assume that the payoff matrixs of players whose strategies are not dominant. Let each of players have two strategy. The pay-off matrixs for player I and player II are respectively as follows

$$(\tilde{A}, \tilde{B}) = \begin{pmatrix} ([a_{11L}, a_{11R}], [b_{11L}, b_{11R}]) & ([a_{12L}, a_{12R}], [b_{12L}, b_{12R}]) \\ ([a_{21L}, a_{21R}], [b_{21L}, b_{21R}]) & ([a_{22L}, a_{22R}], [b_{22L}, b_{22R}]) \end{pmatrix},$$

$$\tilde{A} = \begin{pmatrix} [a_{11L}, a_{11R}] & [a_{12L}, a_{12R}] \\ [a_{21L}, a_{21R}] & [a_{22L}, a_{22R}] \end{pmatrix} = \begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix},$$

$$\tilde{B} = \begin{pmatrix} [b_{11L}, b_{11R}] & [b_{12L}, b_{12R}] \\ [b_{21L}, b_{21R}] & [b_{22L}, b_{22R}] \end{pmatrix} = \begin{pmatrix} \tilde{b}_{11} & \tilde{b}_{12} \\ \tilde{b}_{21} & \tilde{b}_{22} \end{pmatrix},$$

$$\begin{aligned} h_I(x, y) &= x^T \tilde{A} y \\ &= xy(\tilde{a}_{11} + \tilde{a}_{22} - \tilde{a}_{21} - \tilde{a}_{12}) + x(\tilde{a}_{12} - \tilde{a}_{22}) + y(\tilde{a}_{21} - \tilde{a}_{22}) + \tilde{a}_{22}, \end{aligned} \tag{1}$$

$$\begin{aligned} h_{II}(x, y) &= x^T \widetilde{B} y \\ &= xy(\tilde{b}_{11} + \tilde{b}_{22} - \tilde{b}_{21} - \tilde{b}_{12}) + x(\tilde{b}_{12} - \tilde{b}_{22}) + y(\tilde{b}_{21} - \tilde{b}_{22}) + \tilde{b}_{22}, \end{aligned} \tag{2}$$

$$\tilde{A} y \leq h_I(x, y) \begin{pmatrix} [1, 1] \\ [1, 1] \end{pmatrix}. \tag{3}$$

If (3) is edited, then we obtain (4) as follows

$$\left. \begin{array}{l} \tilde{a}_{11} y + \tilde{a}_{12}(1 - y) \leq h_I \\ \tilde{a}_{21} y + \tilde{a}_{22}(1 - y) \leq h_I \end{array} \right\} \tag{4}$$

If (1) is written in (3), then (5) is obtained as follows

$$\left. \begin{array}{c} (\tilde{a}_{11} + \tilde{a}_{22} - \tilde{a}_{21} - \tilde{a}_{12})(1 - x) y + (\tilde{a}_{12} - \tilde{a}_{22})(1 - x) \leq 0 \\ (\tilde{a}_{11} + \tilde{a}_{22} - \tilde{a}_{21} - \tilde{a}_{12}) xy + (\tilde{a}_{12} - \tilde{a}_{22}) x \geq 0 \end{array} \right\} \tag{5}$$

Let $\tilde{p}$ and $\tilde{q}$ be as follows

$$\tilde{p} \quad = \quad \tilde{a}_{11} + \tilde{a}_{22} - \tilde{a}_{21} - \tilde{a}_{12},$$

$$\tilde{q} \;=\; \tilde{a}_{12} - \tilde{a}_{22}.$$

Moreover, we consider the length of interval $I(= [a, b])$ as $\ell(I) = b - a$. Let $\widetilde{\mathbb{R}}$ be set of all closed interval;

$$\ell : \widetilde{\mathbb{R}} \longrightarrow \mathbb{R}, \quad \ell([a, b]) = b - a$$

The length is sample of set function. Therefore, domain of I is set of interval and range of I is also set of generalized real numbers. If we consider the above change of variable and the function ℓ, then (5) can be written as follows,

$$\left.\begin{array}{c} \ell(\tilde{p})(1-x)y - \ell(\ \tilde{q})(1-x) \le 0 \\ \ell(\tilde{p})xy - \ell(\ \tilde{q})x \ge 0 \end{array}\right\} \tag{6}$$

Let's investigate the following conditions to determine x according to definition of mixed strategy:

(i) $x = 0 \Longrightarrow \ell(\tilde{p})y - \ell(\tilde{q}) \le 0;$

(ii) $x = 1 \Longrightarrow \ell(\tilde{p})y - \ell(\tilde{q}) \ge 0;$

(iii) $0 < x < 1 \Longrightarrow \ell(\tilde{p})y - \ell(\tilde{q}) = 0.$

Let K be solution set of (6), so K consists of the following conditions,

$$\left.\begin{array}{ccc} (0, y), & \ell(\tilde{p})y - \ell(\tilde{q}) \le 0 & y \in [0, 1] \\ (1, y), & \ell(\tilde{p})y - \ell(\tilde{q}) \ge 0 & y \in [0, 1] \\ (x, y), & \ell(\tilde{p})y - \ell(\tilde{q}) \ge 0 & y \in [0, 1],\ x \in (0, 1) \end{array}\right\} \tag{7}$$

If $\ell(\tilde{p}) = \ell(\tilde{q}) = 0$, then (7) is satisfied for all x and y. Therefore, the solution is all points $x \in [0, 1], y \in [0, 1]$ of unit square. If $\ell(\tilde{p}) = 0$, $\ell(\tilde{q}) \neq 0$, then $x = 0$ or $x = 1$. If $\ell(\tilde{p}) > 0$, then the solutions from (7) are obtained as follows,

$$\left.\begin{array}{cc} (0, y) & y \le \alpha = \frac{\ell(\tilde{p})}{\ell(\tilde{q})} \\ (1, y) & y \ge \alpha \\ (x, y) & x \in (0, 1),\ y = \alpha \end{array}\right\}$$

Hence, the curve of solutions set K for player I can be represented in the plain:

Similarly, for player II:

$$\tilde{B}x \le h_{II}(x, y) \begin{pmatrix} [1, 1] \\ [1, 1] \end{pmatrix}. \tag{8}$$

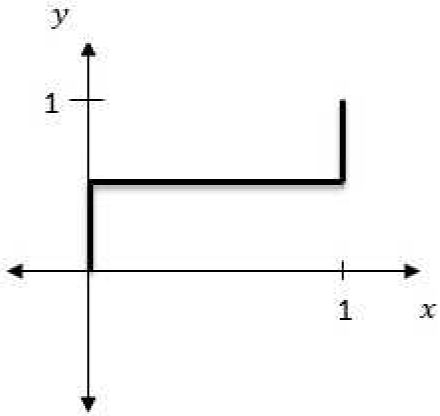

Figure 1

If value of $h_{II}(x,y)$ is written in (8), then (9) is obtained as follows,

$$\begin{aligned}(\tilde{b}_{11}+\tilde{b}_{22}-\tilde{b}_{21}-\tilde{b}_{12})(1-x)y+(\tilde{b}_{12}-\tilde{b}_{22})(1-x) &\leq 0,\\ (\tilde{b}_{11}+\tilde{b}_{22}-\tilde{b}_{21}-\tilde{b}_{12})xy+(\tilde{b}_{12}-\tilde{b}_{22})x &\geq 0.\end{aligned} \tag{3}$$

Let $\tilde{p}_1$and $\tilde{q}_1$ be as follows,

$$\begin{aligned}\tilde{p}_1 &= \tilde{b}_{11}+\tilde{b}_{22}-\tilde{b}_{21}-\tilde{b}_{12}\\ \tilde{q}_1 &= \tilde{b}_{22}-\tilde{b}_{21}\end{aligned}$$

If we consider the above change of variable and the function ℓ, then solution set L for player II consists of the following conditions,

$$\left.\begin{array}{lll}(x,0), & \ell(\tilde{p}_1)y-\ell(\tilde{q}_1)\leq 0 & x\in[0,1]\\ (x,1), & \ell(\tilde{p}_1)y-\ell(\tilde{q}_1)\geq 0 & x\in[0,1]\\ (x,y), & \ell(\tilde{p}_1)y-\ell(\tilde{q}_1)=0 & x\in[0,1], y\in(0,1)\end{array}\right\} \tag{10}$$

If $\ell(\tilde{p}_1)=\ell(\tilde{q}_1)=0$, then (10) is satisfied for all x and y. Therefore, the solution is all points $x\in[0,1]$, $y\in[0,1]$ of unit square. If $\ell(\tilde{p}_1)=0$, $\ell(\tilde{q}_1)\neq 0$, then $y=0$ or $y=1$. If $\ell(\tilde{p}_1)>0$, then the solutions from (10) are obtained as follows,

$$\left.\begin{array}{ll}(x,0), & x\leq\beta=\frac{\ell(\tilde{q}_1)}{\ell(\tilde{p}_1)}\\ (x,1), & x\geq\beta\\ (x,y), & y\in(0,1), x=\beta\end{array}\right\} \tag{11}$$

Hence, the curve of solutions set L for player II can be represented in the plain:

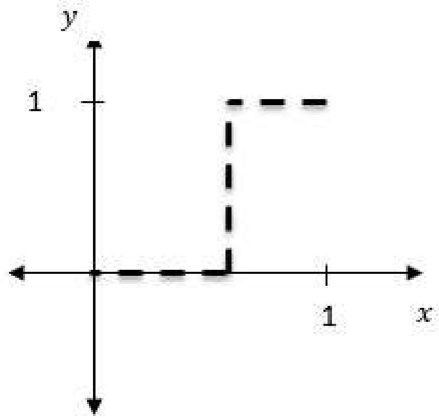

Figure 2

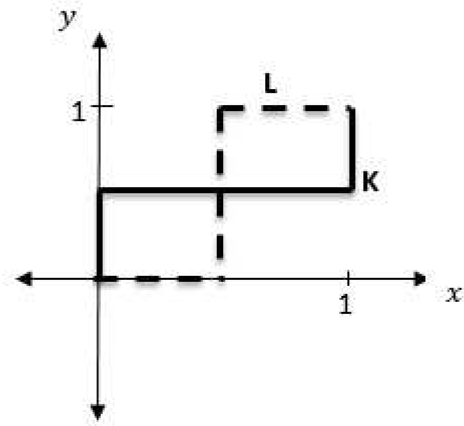

Figure 3

Solution of the game is x and y which are intersection of sets K and L. Thus, the curve of bimatrix game can be represented in the plain:

Example. Let pay-off matrix be given as follows in a bimatrix game, determine game-value and equilibrium strategies

$$(\tilde{A}, \tilde{B}) = \begin{pmatrix} ([1,3],[2,11]) & ([-2,12],[-1,3]) \\ ([-5,8],[2,5]) & ([3,14],[1,6]) \end{pmatrix}.$$

Solution.

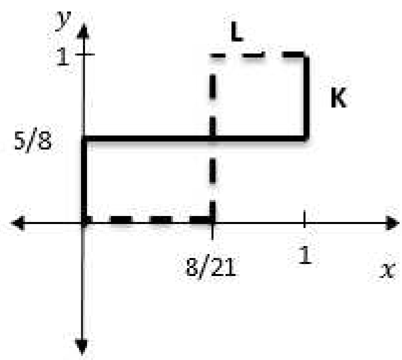

Figure 4

$$\tilde{A} = \begin{pmatrix} [1,3] & [-2,12] \\ [-5,8] & [3,14] \end{pmatrix}$$

$$\tilde{p} = \tilde{a}_{11} + \tilde{a}_{22} - \tilde{a}_{21} - \tilde{a}_{12} = [-3,37] \qquad \Rightarrow \ell(\tilde{p}) = 40, \ell(\ \tilde{q}) = 25$$

$$\tilde{q} = \tilde{a}_{12} - \tilde{a}_{22} = [-9,16] \qquad \Rightarrow \ \alpha = \frac{\ell(\ \tilde{q})}{\ell(\tilde{p})} = \frac{25}{40} = \frac{5}{8} > 0$$

Therefore,

$$\begin{cases} (0,y), & y \le \alpha = \frac{5}{8} \\ (1,y), & y \ge \frac{5}{8} \\ (x,y), & 0 < x < 1, \ y = \frac{5}{8} \end{cases}$$

Similarly, we obtain solutions set for player II.

$$\tilde{B} = \begin{pmatrix} [2,11] & [-1,3] \\ [2,5] & [1,6] \end{pmatrix}$$

$$\tilde{p}_1 = \tilde{b}_{11} + \tilde{b}_{22} - \tilde{b}_{21} - \tilde{b}_{12} = [-5,16] \qquad \Rightarrow \ell(\tilde{p}_1) = 21, \ell(\tilde{q}_1) = 8$$

$$\tilde{q}_1 = \tilde{b}_{22} - \tilde{b}_{21} = [-4,4] \qquad \Rightarrow \beta = \frac{\ell(\tilde{q}_1)}{\ell(\tilde{p}_1)} = \frac{8}{21} > 0$$

the solutions for player II are obtained as follows,

$$\begin{cases} (x,0), & x \le \beta = \frac{8}{21} \\ (x,1), & x \ge \frac{8}{21} \\ (x,y), & 0 < y < 1, \ x = \frac{8}{21} \end{cases}$$

Hence, intersection point of K and L can be represented in the plain:

Hence, we have three intersection point in the plain $(x_1, y_1) = (0,0)$, $(x_2, y_2) = (\frac{8}{21}, \frac{5}{8})$, $(x_3, y_3) = (1,1)$.

For point $(x_1, y_1) = (0,0)$, the point $x_1 = 0$ $(y_1 = 0)$ corresponds to mixed strategy $x_1^{\square} = (0,1)(y_1^{\square} = (0,1))$ of player I (player II). Thus, $(x_1^{\square}, y_1^{\square})$ is a pair of equilibrium of the game. For point $(x_2, y_2) = (\frac{8}{21}, \frac{5}{8})$, the point $x_2 = \frac{8}{21}$ $(y_2 = \frac{5}{8})$ corresponds to mixed strategy $x_2^{\square} = \left(\frac{8}{21}, \frac{5}{8}\right)$ $(y_2^{\square} = \left(\frac{8}{21}, \frac{5}{8}\right))$ of player I

(player II). Thus, (x_2^{*}, y_2^{*}) is a pair of equilibrium of the game. Similarly, for point $(x_3, y_3) = (1, 1)$, the point $x_3 = 1$ $(y_3 = 1)$ corresponds to mixed strategy $x_3^{*} = (1, 0)$ $(y_3^{*} = (1, 0))$ of player I (player II). Thus, (x_3^{*}, y_3^{*}) is a pair of equilibrium of the game. If players choose (x_1^{*}, y_1^{*}) equilibrium strategy then the expected payoffs are as follows,

$$(H_I(x_1^{*}, y_1^{*}), H_{II}(x_1^{*}, y_1^{*})) = ([3, 14], [1, 6]).$$

If players choose (x_2^{*}, y_2^{*}) equilibrium strategy then the expected payoffs are as follows,

$$(H_I(x_2^{*}, y_2^{*}), H_{II}(x_2^{*}, y_2^{*})) = \left(\left[\frac{-217}{168}, \frac{737}{84} \right], \left[\frac{75}{56}, \frac{357}{56} \right] \right).$$

If players choose (x_3^{*}, y_3^{*}) equilibrium strategy then the expected payoffs are as follows,

$$(H_I(x_3^{*}, y_3^{*}), H_{II}(x_3^{*}, y_3^{*})) = ([1, 3], [2, 11]).$$

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