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A CONJECTURE ON N1L CONFIGURATIONS

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Abstract: N1L configurations occur in linear double error correcting codes. Bounding their length is a question of interest. A conjecture is given which would yield a lower bound.

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1. Introduction

As in [1], an N_{1L} configuration is a set S of weight 5 vectors, where there is a weight 5 "anchor" vector v and a position $i \in v$, such that for $w \in S - \{v\}$, $i \in w$ and $|v \cap w| = 2$; and further the linear span has minimum weight 5.

An N'_{1L} configuration is a configuration of weight 3 vectors, which may be obtained from an N_{1L} configuration by deleting v, and the positions of v in the remaining vectors. Together with the incidence matrix M, a partition of the rows into 4 parts must be given. Note that in each part, the rows are disjoint (i.e., the column sets of the 1 entries are).

Let r denote the number of rows and c the number of columns in M. The minimum value of c for a given value of r is a value of interest. Result of an exhaustive search [1] showed that for $r = 4, 8, 12$ the minimum value of c is 8,12,16 respectively. A further search showed that for $r = 16$ there are configurations with $c = 19$.

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Here the output of these computations is examined further, and a conjecture made. Some partial results and computational results are given.

In an array, number rows from 0 at the top, and columns from 0 on the left. The weight $|v|$ of a 0-1 vector v is the number of 1's.

2. TM Configurations

Let $s \geq 3$ be an integer, intended to be the part size of an N_{1L} configuration with constant part size. A T matrix is defined to be a $s \times 4$ matrix of 0's and 1's, where each row or column has at most one 1, and there are three 1's altogether. A TM configuration is a matrix M comprised of a $4 \times (s-2)$ array of T matrices, with the following properties.

1. Let T_{ij} be the T matrix at position ij in the array thereof. Let z_{ij} be the position of the 0 column of T_{ij}. Then for fixed j the four z_{ij} are distinct. In particular each column of M has weight 3.
2. If $3 \leq s \leq 6$ each row has weight 1 or 2, and if $6 \leq s$ each row has weight 2 or 3.

Conjecture 1. *For $r = 4s$ where $s \geq 3$ there are N_{1L} configurations M which are the concatenation of a TM configuration M_1 and a second matrix M_2.*

Note that the total weight of M_2 is 24. In particular, if the conjecture is true $c \leq r + 16$ may be achieved. The computations of [1] produce examples verifying the conjecture for $s = 3, 4$.

3. Proof of Conjecture 1 for $s \leq 6$

Define a two-part restriction to be a restriction on an incidence matrix M, which is necessary for M to be an N'_{1L} configuration. The following two restrictions are such:

1. If r_1, r_2 are rows from different parts them $|r_1 \cap r_2| \leq 1$.
2. If r_{11}, r_{12} are rows from part 1, and r_{21}, r_{22} are rows from part 2, then $|(r_{11} \cup r_{12}) \cap (r_{21} \cup r_{22})| \leq 3$.

Say that an extension M of a TM configuration to an N'_{1L} configuration is trivial if M_2 consists of 24 weight 1 columns. Note that for such, if M_1 satisfies the above two restrictions then M does.

Theorem 2. *If M_1 is a TM configuration satisfying restrictions 1 and 2 above, and $s \leq 6$, then a trivial extension M of M_1 is an N'_{1L} configuration.*

Proof. If $s \leq 6$ then every row of M_2 is nonempty, so it suffices to verify that no linear combination v of at most 4 rows of the N_{1L} configuration has $0 < |v| < 5$. The linear combinations may be assumed to involve at least 2 parts. Let w_0 denote the weight in the anchor columns, and w_2 the weight in the M_2 columns.

The following cases suffice, where the case label gives the number of rows per part.

11. This follows by restriction 1.

111. $w_0 = 4$, $w_2 \geq 3$.

211. $w_0 = 2$, $w_2 \geq 4$.

22. This follows by restriction 2.

1111. $w_0 = 4$, $w_2 = 4$. □

Theorem 3. *Conjecture 1 holds if $s \leq 6$.*

Proof. By theorem 2 it suffices to show that there is a TM configuration satisfying restrictions 1 and 2. Here, we merely note that such are readily found by a computer program. □

4. Further Computations

The computations of [1] produce examples for $s = 3, 4$ where M_2 may be taken as having 12 weight 2 columns, and $c = r + 4$ may be achieved. For $s = 4$ there is an M_2 with 11 columns, and $c = r + 3$ may be achieved.

Searches over TM configurations can be conducted. This provides exact values for $s = 3, 4$ for TM based N_{1L} configurations, and examples for $s = 5$.

Define a T_- matrix to be an $s \times 3$ matrix of weight 3 with disjoint rows and columns, i.e., a T matrix with the 0 column removed. A T_-A matrix is defined to be the concatenation of $s - 2$ T_- matrices, which satisfies requirement 2 for a TM configuration. These can be canonicalized up to row permutation. (This and other canonicalizations descried here are readily accomplished using the Nauty [2] library).

For a selection of 4 T_-A's consider the TM configuration, where in part i the T_-'s of the T_-A matrix have a 0 column inserted at position i. The resulting TM matrix need only be considered if it satisfies the two-part restrictions 1 and 2 given above. The remaining configurations can be canonicalized, where the row permutations act on the parts.

Given such a possible M_1, a search can be conducted through an additional 12 columns, looking for N'_{1L} configurations.

When $s = 4$ there are 536 TM matrices of interest. For 1 of them, the minimum length of an M_2 is 19, and there are M_2's with 10 weight 2 columns and a weight 4 column. For an additional 5, the minimum length of an M_2 is 12, and there are M_2's with 12 weight 2 columns.

When $s = 5$ there are 4919449 TM matrices of interest. Let M_1 be a random TM configuration. Starting with a trivial M_2, weight 1 columns may be combined randomly until the resulting row space is no longer weight 5. This random search finds an N'_{1L} configuration with $r = 20$ and $c = 24$ quite rapidly.

For a remark on using Nauty, the graphs which need to be canonicalized are bipartite. After canonicalization the $n_1 \times n_2$ submatrix which is the 0-1 matrix where the i, j entry is 1 if there is an edge between vertex i of part 1 and vertex j of part 2 may be extracted. The entire matrix may be reconstructed from this. In the case $s = 5$ above, where $n_1 = 20$ and $n_2 = 16$, the size was reduced from 288 bytes to 40 bytes, allowing the canonicalized graphs to fit in memory. The time required to generate the TM's was 325 minutes.

References

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