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L-FUZZY (K,E)-SOFT TOPOLOGIES AND L-FUZZY (K,E)-SOFT CLOSURE OPERATORS

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Abstract: In this paper, we investigate the properties of fuzzy soft sets and fuzzy soft maps in stsc-quantales. We define a L-fuzzy (K,E)-soft topology and a L-fuzzy (K,E)-soft closure spaces as a Höhle's sense. We study the relations between L-fuzzy (K,E)-soft topologies and L-fuzzy (K,E)-soft closure spaces. We give their examples.

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1. Introduction

In 1999, Molodtsov [13] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In [14], Molodtsov applied successfully in directions such as, smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, probability and theory of measurement. Maji et al. [10,11] gave the first

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practical application of soft sets in decision making problems. In 2003, Maji et al. [11] defined and studied several basic notions of soft set theory. Many researchers have contributed towards the algebraic structure of soft set theory [1-5,7]. In 2011, Shabir and Naz [20] initiated the study of soft topological spaces. They defined soft topology on the collection of soft sets over X and established their several properties. Aygünoglu et.al [2] introduced the concept of (K, E)-soft topology in the sense of Šostak [9]. Cetkin et.al [3] studied (K, E)-soft proximities and discuss their properties.

Hájek [8] introduced a complete residuated lattice which is an algebraic structure for many valued logic and decision rules in complete residuated lattices. Höhle [9] introduced L-fuzzy topologies with algebraic structure L(cqm, quantales, MV-algebra). It has developed in many directions [16-18]. Ramadan et al. [17] define the the concept of L- fuzzy soft topogenous orders, L-fuzzy soft uniform spaces, L- fuzzy soft topological spaces in strictly two sided commutative quantales and investigated the relation between them.

In this paper, we define a L-fuzzy (K, E)-soft topology and a L-fuzzy (K, E)-soft closure spaces as a Aygünoglu et.al [2] in stsc-quantales. We study the relations between L-fuzzy (K, E)-soft topologies and L-fuzzy (K, E)-soft closure spaces. We give their examples.

2. Preliminaries

Let $L = (L, \leq, \vee, \wedge, 0, 1)$ be a completely distributive lattice with the least element 0 and the greatest element 1 in L.

Definition 1. [8,9,17] A complete lattice $(L, \leq, \odot)$ is called a strictly two-sided commutative quantale (stsc-quantale, for short) iff it satisfies the following properties.

(L1) $(L, \odot)$ is a commutative semigroup,

(L2) $x = x \odot 1$, for each $x \in L$ and 1 is the universal upper bound,

(L3) $\odot$ is distributive over arbitrary joins, i.e. $(\bigvee_i x_i) \odot y = \bigvee_i (x_i \odot y)$.

There exists a further binary operation $\rightarrow$ (called the implication operator or residuated) satisfying the following condition

$$x \rightarrow y = \bigvee \{z \in L | x \odot z \leq y\}.$$

Then it satisfies Galois correspondence; i.e, $(x \odot z) \leq y$ iff $z \leq (x \rightarrow y)$.

In this paper, we always assume that $(L, \leq, \odot, \rightarrow, \oplus, ^*)$ is a stsc-quantales with an order reversing involution * which is defined by

$$x \oplus y = (x^* \odot y^*)^*, \quad x^* = x \rightarrow 0$$

unless otherwise specified.

Remark 2. Every completely distributive lattice $(L, \leq, \wedge, \vee, ^*)$ with order reversing involution * is a stsc-quantale $(L, \leq, \odot, \oplus, ^*)$ with a strong negation * where $\odot = \wedge$ and $\oplus = \vee$.

Lemma 3. [8,9,17] For each $x, y, z, x_i, y_i, w \in L$, we have the following properties.

(1) $1 \to x = x$, $0 \odot x = 0$,

(2) If $y \leq z$, then $x \odot y \leq x \odot z$, $x \oplus y \leq x \oplus z$, $x \to y \leq x \to z$ and $z \to x \leq y \to x$,

(3) $x \leq y$ iff $x \to y = 1$.

(4) $(\bigwedge_i y_i)^* = \bigvee_i y_i^*$, $(\bigvee_i y_i)^* = \bigwedge_i y_i^*$,

(5) $x \to (\bigwedge_i y_i) = \bigwedge_i (x \to y_i)$,

(6) $(\bigvee_i x_i) \to y = \bigwedge_i (x_i \to y)$,

(7) $x \to (\bigvee_i y_i) \geq \bigvee_i (x \to y_i)$,

(8) $(\bigwedge_i x_i) \to y \geq \bigvee_i (x_i \to y)$,

(9) $(x \odot y) \to z = x \to (y \to z) = y \to (x \to z)$,

(10) $x \odot y = (x \to y^*)^*$ and $x \oplus y = x^* \to y$,

(11) $(x \to y) \odot (z \to w) \leq (x \odot z) \to (y \odot w)$,

(12) $x \to y \leq (x \odot z) \to (y \odot z)$ and $(x \to y) \odot (y \to z) \leq x \to z$,

(13) $(x \to y) \odot (z \to w) \leq (x \oplus z) \to (y \oplus w)$.

(14) $x \to y = y^* \to x^*$.

(15) $(x \vee y) \odot (z \vee w) \leq (x \vee z) \vee (y \odot w) \leq (x \oplus z) \vee (y \odot w)$.

(16) $\bigvee_{i \in \Gamma} x_i \to \bigvee_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$ and $\bigwedge_{i \in \Gamma} x_i \to \bigwedge_{i \in \Gamma} y_i \geq \bigwedge_{i \in \Gamma} (x_i \to y_i)$,

(17) $(x \odot y) \odot (z \oplus w) \leq (x \odot z) \oplus (y \odot w)$.

Throughout this paper, X refers to an initial universe, E and K are the sets of all parameters for X, and L^X is the set of all L-fuzzy sets on X.

Definition 4. [4] A map f is called an L- fuzzy soft set on X, where f is a mapping from E into L^X, i.e., $f_e := f(e)$ is an L- fuzzy set on X, for each $e \in E$. The family of all L- fuzzy soft sets on X is denoted by $(L^X)^E$. Let f and g be two L- fuzzy soft sets on X.

(1) f is an L-fuzzy soft subset of g and we write $f \sqsubseteq g$ if $f_e \leq g_e$, for each $e \in E$. f and g are equal if $f \sqsubseteq g$ and $g \sqsubseteq f$.

(2) The intersection of f and g is an L- fuzzy soft set $h = f \sqcap g$, where $h_e = f_e \wedge g_e$, for each $e \in E$.

(3) The union of f and g is an L- fuzzy soft set $h = f \sqcup g$, where $h_e = f_e \vee g_e$, for each $e \in E$.

(4) An L- fuzzy soft set $h = f \odot g$ is defined as $h_e = f_e \odot g_e$, for each $e \in E$.

(5) An L- fuzzy soft set $h = f \oplus g$ is defined as $h_e = f_e \oplus g_e$, for each $e \in E$.

(6) The complement of an L- fuzzy soft sets on X is denoted by f^*, where $f^* : E \to L^X$ is a mapping given by $f^*_e = (f_e)^*$, for each $e \in E$.

(7) f is called a null L- fuzzy soft set and is denoted by 0_X , if $f_e(x) = 0$, for each $e \in E$, $x \in X$.

(8) f is called an absolute L- fuzzy soft set and is denoted by 1_X , if $f_e(x) = 1$, for each $e \in E$, $x \in X$ and $(1_x)_e(x) = 1$.

Definition 5. Let $\varphi : X \to Y$ and $\psi : E_1 \to E_2$ be two mappings, where E_1 and E_2 are parameters sets for the crisp sets X and Y, respectively. Then $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called a fuzzy soft mapping.

(1) For $f \in (L^X)^{E_1}$, the image of f under the fuzzy soft mapping φ_ψ defined by, $\forall k \in K, \forall y \in Y$,

$$\varphi(f)_{e_2}(y) = \begin{cases} \bigvee_{\varphi(x)=y}(\bigvee_{\psi(e_1)=e_2} f_{e_1}(x)), & \text{if } x \in \varphi^{-1}(y), e_1 \in \psi^{-1}(e_2) \\ 0, & \text{otherwise.} \end{cases}$$

(2) For $f \in (L^X)^{E_1}$, the pre-image of g defined by

$$\varphi_\psi^{-1}(g)_e(x) = g_{\psi(e)}(\varphi(x)), \forall e \in E, \forall x \in X.$$

(3) The soft mapping $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ is called injective (resp. surjective, bijective) if f and ϕ are both injective (resp. surjective, bijective).

Definition 6. [2,15] A mapping $\mathcal{T} : K \to L^{(L^X)^E}$ (where $\mathcal{T}_k := \mathcal{T}(k) : (L^X)^E \to L$ is a mapping for each $k \in K$) is called an L-fuzzy (K, E)-soft topology on X if it satisfies the following conditions for each $k \in K$.

(O1) $\mathcal{T}_k(0_X) = \mathcal{T}_k(1_X) = 1$,

(O2) $\mathcal{T}_k(f \odot g) \geq \mathcal{T}_k(f) \odot \mathcal{T}_k(g) \ \ \forall\ f, g \in (L^X)^E$,

(O3) $\mathcal{T}_k(\bigsqcup_i f_i) \geq \bigwedge_{i \in I} \mathcal{T}_k(f_i)\ \forall\ f_i \in (L^X)^E,\ i \in I$.

The pair $(X, \mathcal{T})$ is called an L-fuzzy (K, E)-soft topological space. Let $(X, \mathcal{T}^1)$ be an L-fuzzy (K_1, E_1)-soft topological space and $(Y, \mathcal{T}^2)$ be an L-fuzzy (K_2, E_2)-soft topological space. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be mappings. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{T}^1)$ into $(Y, \mathcal{T}^2)$ is called L-fuzzy soft continuous if

$$\mathcal{T}^2_{\eta(k)}(f) \leq \mathcal{T}^1_k(\varphi_\psi^{-1}(f)) \ \ \forall\ f \in (L^Y)^{E_2}, k \in K_1.$$

Definition 7. [5] A map $\mathcal{C} : K \times (L^X)^E \times L_0 \to (L^X)^E$ is called an L-fuzzy (K, E)-soft closure operator if it satisfies the following conditions;

(C1) $\mathcal{C}(k, 0_X, r) = 0_X$,

(C2) $\mathcal{C}(k, f, r) \sqsupseteq f$,

(C3) If $f_1 \sqsubseteq f_2$, then $\mathcal{C}(k, f_1, r) \sqsubseteq \mathcal{C}(k, f_2, r))$,

(C4) If $r_1 \leq r_2$, then $\mathcal{C}(k, f, r_1) \sqsubseteq \mathcal{C}(k, f, r_2)$,

(C5) $\mathcal{C}(k, f_1 \oplus f_2, r \odot s) \sqsubseteq \mathcal{C}(k, f_1, r) \oplus \mathcal{C}(k, f_2, s)$.

The pair $(X, \mathcal{C})$ is called an L-fuzzy (K, E)-soft closure space.

An L- fuzzy (K, E)-soft closure operator is called topological if

(T) $\mathcal{C}(k, \mathcal{C}(k, f, r), r) \sqsubseteq \mathcal{C}(k, f, r)$.

Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be L-fuzzy (K, E)-soft closure operators on X. Then $\mathcal{C}_1$ is finer than $\mathcal{C}_2$ if $\mathcal{C}_1(k, f, r) \sqsubseteq \mathcal{C}_2(k, f, r)$, for all $f \in (L^X)^E, r \in L_0$.

Let $(X, \mathcal{C}_X)$ be L-fuzzy (K_1, E_1)-soft closure spaces and $(Y, \mathcal{C}_Y)$ be L-fuzzy (K_2, E_2)-soft closure spaces. Let $\varphi : X \to Y$, $\psi : E_1 \to E_2$ and $\eta : K_1 \to K_2$ be maps. Then $\varphi_{\psi,\eta}$ is called an L-fuzzy soft closed map if, for each $k \in K_1, f \in (L^X)^{E_1}, r \in L_0$,

$$\varphi_{\psi,\eta}(\mathcal{C}_X(k, f, r)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_\psi(f), r).$$

3. L-Fuzzy (K, E)-Soft Topologies and L-fuzzy (K, E)-Soft Closure Operators

Lemma 8. Let $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ be a soft mapping. Then we have the following properties. For $f, f_i \in (L^X)^{E_1}$ and $g, g_i \in (L^Y)^{E_2}$,

(1) $g \sqsupseteq \varphi_\psi(\varphi_\psi^{-1}(g))$ with equality if φ_ψ is surjective,

(2) $f \sqsubseteq \varphi_\psi^{-1}(\varphi_\psi(f))$ with equality if φ_ψ is injective,

(3) if φ_ψ is injective,

$$\varphi(f)_{e_2}(y) = \begin{cases} f_{e_1}(x), & \text{if } x \in \varphi^{-1}(y), e_1 \in \psi^{-1}(e_2) \\ 0, & \text{otherwise,} \end{cases}$$

(4) $\varphi_\psi^{-1}(g^*) = (\varphi_\psi^{-1}(g))^*$,

(5) $\varphi_\psi^{-1}(\bigvee_{i\in I} g_i) = \bigvee_{i\in I} \varphi_\psi^{-1}(g_i)$,

(6) $\varphi_\psi^{-1}(\bigwedge_{i\in I} g_i) = \bigwedge_{i\in I} \varphi_\psi^{-1}(g_i)$,

(7) $\varphi_\psi(\bigvee_{i\in I} f_i) = \bigvee_{i\in I} \varphi_\psi(f_i)$,

(8) $\varphi_\psi(\bigwedge_{i\in I} f_i) \sqsubseteq \bigwedge_{i\in I} \varphi_\psi(f_i)$ with equality if φ_ψ is injective,

(9) $\varphi_\psi^{-1}(g_1 \odot g_2) = \varphi_\psi^{-1}(g_1) \odot \varphi_\psi^{-1}(g_2)$,

(10) $\varphi_\psi^{-1}(g_1 \oplus g_2) = \varphi_\psi^{-1}(g_1) \oplus \varphi_\psi^{-1}(g_2)$,

(11) $\varphi_\psi(f_1 \odot f_2) \sqsubseteq \varphi_\psi(f_1) \odot \varphi_\psi(f_2)$ with equality if φ_ψ is injective,

(12) $\varphi_\psi(f_1 \oplus f_2) \sqsubseteq \varphi_\psi(f_1) \oplus \varphi_\psi(f_2)$ with equality if φ_ψ is injective.

Proof. (1) If $\psi^{-1}(\{e_2\}) \neq \emptyset$ and $\varphi^{-1}(\{y\}) \neq \emptyset$, then

$$\begin{aligned}&\varphi_\psi(\varphi_\psi^{-1}(g))(e_2)(y) = \bigvee_{\varphi(x)=y} \bigvee_{\psi(e_1)=e_2} \varphi_\psi^{-1}(g)(e_1)(x)\\ &= g(\psi(e_1))(\varphi(x)) = g(e_2)(y).\end{aligned}$$

If $\psi^{-1}(\{e_2\}) = \emptyset$ or $\varphi^{-1}(\{y\}) = \emptyset$, then $\varphi_\psi(\varphi_\psi^{-1}(f))(e_2)(y) = 0$. Hence the result holds.

(2)

$$\begin{aligned}&\varphi_\psi^{-1}(\varphi_\psi(f)(e_1)(x) = \varphi_\psi(f)(\psi(e_1))(\varphi(x))\\ &= \bigvee_{\varphi(z)=\varphi(x)} \bigvee_{\psi(e)=\psi(e_1)} \varphi_\psi^{-1}(g)(e)(z) \geq \varphi(e_1)(x).\end{aligned}$$

If φ_ψ is injective, the equality holds.

(3) If $\psi^{-1}(\{e_2\}) \neq \emptyset$ and $\varphi^{-1}(\{y\}) \neq \emptyset$, there exist unique $e_1 \in \psi^{-1}(\{e_2\})$ and $x \in \varphi^{-1}(\{y\})$. Hence the result holds.

(4)

$$\begin{aligned}&\varphi_{\psi}^{-1}(g^*)(e_1)(x) = g^*(\psi(a))(\varphi(x)) = (g(\psi(e_1))(\varphi(x))^* \\ &= (\varphi_{\psi}^{-1}(g)(e_1)(x))^*.\end{aligned}$$

(11)

$$\begin{aligned}&\varphi_{\psi}(f_1)(e_2)(y) \odot \varphi_{\psi}(f_2)(e_2)(y) \\ &= \bigvee_{\varphi(x)=y} \bigvee_{\psi(e_1)=e_2} f_1(e_1)(x) \odot \bigvee_{\varphi(z)=y} \bigvee_{\psi(e_3)=e_2} f_2(e_3)(z) \\ &\geq \bigvee_{\varphi(x)=y} \bigvee_{\psi(e_1)=e_2} (f_1(e_1)(x) \odot f_2(e_1)(x)) \\ &= \varphi_{\psi}((f_1 \odot f_2))(e_2)(y).\end{aligned}$$

If φ_{ψ} is injective, by (3), If $\psi^{-1}(\{e_2\}) \neq \emptyset$ and $\varphi^{-1}(\{y\}) \neq \emptyset$, there exist unique $e_1 \in \psi^{-1}(\{e_2\})$ and $x \in \varphi^{-1}(\{y\})$.

$$\begin{aligned}&\varphi_{\psi}(f_1)(e_2)(y) \odot \varphi_{\psi}(f_2)(e_2)(y) = f_1(e_1)(x) \odot f_2(e_1)(x) \\ &= \varphi_{\psi}((f_1 \odot f_2))(e_2)(y).\end{aligned}$$

If $\psi^{-1}(\{e_2\}) = \emptyset$ and $\varphi^{-1}(\{y\}) = \emptyset$,

$$\varphi_{\psi}(f_1)(e_2)(y) \odot \varphi_{\psi}(f_2)(e_2)(y) = 0 = \varphi_{\psi}((f_1 \odot f_2))(e_2)(y).$$

Other cases are similarly proved.

Example 9. Let $X = \{h_i \mid i = \{1, ..., 5\}\}$ with h_i=house and $E_X = \{e, b, w, c\}$ with e=expensive,b= beautiful, w=wooden, c= creative.

Define a binary operation $\odot$ on $[0, 1]$ by

$$x \odot y = \max\{0, x + y - 1\},\ x \to y = \min\{1 - x + y, 1\}$$

$$x \oplus y = \min\{1, x + y\},\ x^* = 1 - x.$$

Then $([0, 1], \odot, \to, 0, 1)$ is a stsc-quantale (ref.[8,9,18]). Let $E_1 = \{e, b, w\} \subset E_X$, $f_1, f_2 \in ([0, 1]^X)^{E_1}$ as follows:

$$(f_1)_e = (0.8, 0.1, 0.5, 0.9, 0.6),\ (f_1)_b = (0.7, 0.9, 0.4, 0.5, 0.7)$$
$$(f_1)_w = (0.4, 0.7, 0.5, 0.6, 0.5)$$

$$(f_2)_e = (0.5, 0.9, 0.4, 0.8, 0.4),\ (f_2)_b = (0.3, 1.0, 0.2, 0.4, 0.5)$$
$$(f_2)_w = (0.5, 0.4, 0.8, 0.5, 0.1)$$

$$(f_1 \odot f_2)_e = (0.3, 0, 0, 0.7, 0), (f_1 \odot f_2)_b = (0, 0.9, 0, 0, 0.2)$$
$$(f_1 \odot f_2)_w = (0, 0.1, 0.3, 0.1, 0)$$

$$(f_1 \oplus f_2)_e = (1, 1, 0.9, 1, 1), (f_1 \oplus f_2)_b = (1, 1, 0.6, 0.9, 1)$$
$$(f_1 \oplus f_2)_w = (0.9, 1, 1, 1, 0.6)$$

(1) Let $Y = \{y_1, y_2, y_3\}$ and $E_2 = \{b_1, b_2\}$. Define $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ as follows:

$$\varphi(h_1) = \varphi(h_2) = y_1, \varphi(h_3) = \varphi(h_4) = y_2, \varphi(h_5) = y_3$$

$$\psi(e) = \psi(b) = b_1, \psi(w) = b_2.$$

Since

$$\begin{aligned} \varphi_\psi(f_1)(b_1)(y_1) &= \bigvee_{\varphi(x)=y_1} \bigvee_{\psi(a)=b_1} f_1(a)(x) \\ &= f(e)(h_1) \vee f(b)(h_1) \vee f(e)(h_2) \vee f(b)(h_2) \\ &= 0.8 \vee 0.7 \vee 0.1 \vee 0.9. \end{aligned}$$

$$\varphi_\psi(f_1)(b_1) = (0.9, 0.9, 0.7),\ \varphi_\psi(f_1)(b_2) = (0.7, 0.6, 0.5)$$
$$\varphi_\psi(f_2)(b_1) = (1.0, 0.8, 0.5),\ \varphi_\psi(f_2)(b_2) = (0.5, 0.8, 0.1)$$

$$\varphi_\psi(f_1 \odot f_2)(b_1) = (0.9, 0.7, 0.2), \varphi_\psi(f_1 \odot f_2)(b_2) = (0.1, 0.3, 0)$$

$$\begin{aligned} (\varphi_\psi(f_1) \odot \varphi_\psi(f_2))(b_1) &= (0.9, 0.7, 0.2) \\ (\varphi_\psi(f_1) \odot \varphi_\psi(f_2))(b_2) &= (0.2, 0.4, 0) \end{aligned}$$

$$\varphi_\psi(f_1 \oplus f_2)(b_1) = (1, 1, 1), \varphi_\psi(f_1 \oplus f_2)(b_2) = (1, 1, 0.6)$$

$$\begin{aligned} (\varphi_\psi(f_1) \odot \varphi_\psi(f_2))(b_1) &= (0.9, 0.7, 0.2) \\ (\varphi_\psi(f_1) \odot \varphi_\psi(f_2))(b_2) &= (0.2, 0.4, 0) \end{aligned}$$

$$\begin{aligned} \varphi_\psi^{-1}(\varphi_\psi(f_1))(e) &= (0.9, 0.9, 0.9, 0.9, 0.7) \\ \varphi_\psi^{-1}(\varphi_\psi(f_1))(b) &= (0.9, 0.9, 0.9, 0.9, 0.7) \\ \varphi_\psi^{-1}(\varphi_\psi(f_1))(w) &= (0.7, 0.7, 0.6, 0.6, 0.5) \end{aligned}$$

Since f is not injective,

$$\varphi_\psi(f_1) \odot \varphi_\psi(f_2) \neq \varphi_\psi(f_1 \odot f_2), \quad \varphi_\psi^{-1}(\varphi_\psi(f_1)) \neq f_1.$$

But $\varphi_\psi(f_1) \oplus \varphi_\psi(f_2) = \varphi_\psi(f_1 \oplus f_2)$ and φ_ψ is not injective.

Let $g \in ([0,1]^Y)^{E_2}$ as follows:

$$g_{b_1} = (0.5, 0.9, 0.6),\ g_{b_1} = (0.7, 0.4, 0.3)$$

$$\begin{aligned}
\varphi_\psi^{-1}(g)_e &= (0.5, 0.5, 0.9, 0.9, 0.6)\\
\varphi_\psi^{-1}(g)_b &= (0.5, 0.5, 0.9, 0.9, 0.6)\\
\varphi_\psi^{-1}(g)_w &= (0.7, 0.7, 0.4, 0.4, 0.3)
\end{aligned}$$

Since φ_ψ is surjective, we have $\varphi_\psi(\varphi_\psi^{-1}(g)) = g$.

(2) Let $Z = \{z_1, z_2, ..., z_6\}$ and $E_3 = \{c_1, c_2, c_3, c_4\}$. Define $\pi_\psi : ([0,1]^X)^{E_1} \to ([0,1]^Z)^{E_3}$ as follows:

$$\pi(h_i) = z_i, \psi(e) = c_1, \psi(b) = c_2, \psi(w) = c_3.$$

$$\begin{aligned}
\pi_\psi(f_1)(c_1) &= (0.8, 0.1, 0.5, 0.9, 0.6, 0)\\
\pi_\psi(f_1)(c_2) &= (0.7, 0.9, 0.4, 0.5, 0.7, 0)\\
\pi_\psi(f_1)(c_3) &= (0.4, 0.7, 0.5, 0.6, 0.5, 0)\\
\pi_\psi(f_1)(c_4) &= (0, 0, 0, 0, 0, 0)
\end{aligned}$$

$$\begin{aligned}
\pi_\psi(f_2)(c_1) &= (0.5, 0.9, 0.4, 0.8, 0.4, 0)\\
\pi_\psi(f_2)(c_2) &= (0.3, 1.0, 0.2, 0.4, 0.5, 0)\\
\pi_\psi(f_2)(c_3) &= (0.5, 0.4, 0.8, 0.5, 0.1, 0)\\
\pi_\psi(f_2)(c_4) &= (0, 0, 0, 0, 0, 0)
\end{aligned}$$

$$\begin{aligned}
\pi_\psi(f_1 \odot f_2)(c_1) &= (0.3, 0, 0, 0.7, 0, 0)\\
\pi_\psi(f_1 \odot f_2)(c_2) &= (0, 0.9, 0, 0, 0.2, 0)\\
\pi_\psi(f_1 \odot f_2)(c_3) &= (0, 0.1, 0.3, 0.1, 0, 0)\\
\pi_\psi(f_1 \odot f_2)(c_4) &= (0, 0, 0, 0, 0, 0)
\end{aligned}$$

Since π is injective,

$$\pi_\psi f_1 \odot \pi_\psi f_2 = \pi_\psi(f_1 \odot f_2)$$

$$\pi_\psi f_1 \oplus \pi_\psi f_2 = \pi_\psi(f_1 \oplus f_2),\ \pi_\phi^{-1}(\pi_\phi(f_1)) = f_1.$$

Let $p \in ([0,1]^Z)^{E_3}$ as follows:

$$\begin{aligned}
p_{c_1} &= (0.5, 0.3, 0.9, 0.1, 0.6, 0.2)\\
p_{c_2} &= (0, 0.2, 0.7, 0.5.0.4, 0.3, 0.1)\\
p_{c_3} &= (0.3, 0.4, 0.2, 0.7, 0.6, 0.9)\\
p_{c_4} &= (0.8, 0.2, 0.4, 0.5.0.6, 0.1.0.3)
\end{aligned}$$

$$\begin{aligned}
\pi_\psi^{-1}(p)_e &= (0.5, 0.3, 0.9, 0.1, 0.6)\\
\pi_\psi^{-1}(p)_b &= (0, 0.2, 0.7, 0.5.0.4, 0.3)\\
\pi_\psi^{-1}(p)_w &= (0.3, 0.4, 0.2, 0.7, 0.6)
\end{aligned}$$

$$\begin{array}{ll} \pi_\psi(\pi_\psi^{-1}(p))(c_1) & = (0.5, 0.3, 0.9, 0.1, 0.6, 0) \\ \pi_\psi(\pi_\psi^{-1}(p))(c_2) & = (0, 0.2, 0.7, 0.5.0.4, 0.3, 0) \\ \pi_\psi(\pi_\psi^{-1}(p))(c_3) & = (0.3, 0.4, 0.2, 0.7, 0.6, 0) \\ \pi_\psi(\pi_\psi^{-1}(p))(c_4) & = (0, 0, 0, 0, 0, 0) \end{array}$$

Since φ_ψ is injective, we have $\varphi_\psi(\varphi_\psi^{-1}(p)) \sqsubseteq p$. □

Theorem 10. (1) Let $(X, \mathcal{T})$ be an L-fuzzy (K, E)-soft topological space. Define $\mathcal{C}_\mathcal{T} : K \times (L^X)^E \times L_0 \to (L^X)^E$ as

$$\mathcal{C}_\mathcal{T}(k, f, r) = \bigwedge\{g \in (L^X)^E \mid g \sqsupseteq f, \mathcal{T}_k(g^*) \geq r\}.$$

Then (1) $\mathcal{C}_\mathcal{T}$ is a topological L-fuzzy (K, E)-soft closure operator.

(2) Let $(X, \mathcal{C})$ be an L-fuzzy (K, E)-soft closure space. Define $\mathcal{T}_\mathcal{C} : (L^X)^E \to L$ as

$$(\mathcal{T}_\mathcal{C})_k(f) = \bigvee\{r \in L \mid \mathcal{C}(k, f^*, r) \sqsubseteq f^*\}.$$

Then $\mathcal{T}_\mathcal{C}$ is an L-fuzzy (K, E)-soft topology on X.

(3) Let $(X, \mathcal{T})$ be an L-fuzzy (K, E)-soft topological space. Then $\mathcal{T} = \mathcal{T}_{\mathcal{C}_\mathcal{T}}$.

Proof. (1) (C1), (C2), (C3) and (C4) are easily proved.

(C5)

$$\begin{array}{l} \mathcal{C}_\mathcal{T}(k, f, r) \oplus \mathcal{C}_\mathcal{T}(k, g, s) = \bigwedge\{f_1 \in (L^X)^E \mid f_1 \sqsupseteq f, \mathcal{T}_k(f_1^*) \geq r\} \\ \oplus \bigwedge\{g_1 \in (L^X)^E \mid g_1 \sqsupseteq g, \mathcal{T}_k(g_1^*) \geq s\} \\ \sqsupseteq \bigwedge\{f_1 \oplus g_1 \in (L^X)^E \mid f_1 \oplus g_1 \sqsupseteq f \oplus g, \mathcal{T}_k(f_1^* \odot g_1^*) \geq r \odot s\} \\ \sqsupseteq \mathcal{C}_\mathcal{T}(k, f \oplus g, r \odot s) \end{array}$$

(T) Suppose there exist $k \in K, f \in (L^X)^E$ and $r \in L_0$ such that

$$\mathcal{C}_\mathcal{T}(k, \mathcal{C}_\mathcal{T}(k, f, r), r) \not\sqsubseteq \mathcal{C}_\mathcal{T}(k, f, r).$$

By the definition of $\mathcal{C}_\mathcal{T}(k, f, r)$, there exists $g \in (L^X)^E$ with $f \sqsubseteq g$ and $\mathcal{T}_k(g^*) \geq r$ such that

$$\mathcal{C}_\mathcal{T}(k, \mathcal{C}_\mathcal{T}(k, f, r), r) \not\sqsubseteq g.$$

On the other hand, since $f \sqsubseteq g$ and $\mathcal{T}(g^*) \geq r$, $\mathcal{C}_\mathcal{T}(k, f, r) \sqsubseteq \mathcal{C}_\mathcal{T}(k, g, r) = g$. Thus,

$$\mathcal{C}_\mathcal{T}(k, \mathcal{C}_\mathcal{T}(k, f, r), r) \sqsubseteq g.$$

It is a contradiction. Hence $\mathcal{C}_\mathcal{T}(k, \mathcal{C}_\mathcal{T}(k, f, r), r) \sqsubseteq \mathcal{C}_\mathcal{T}(k, f, r)$.

(2)

$$\begin{aligned}
&(\mathcal{T}_{\mathcal{C}})_k(f) \odot (\mathcal{T}_{\mathcal{C}})_k(g) \\
&= \bigvee\{r \in L \mid \mathcal{C}(k, f^*, r) \sqsubseteq f^*\} \odot \bigvee\{s \in L \mid \mathcal{C}(k, g^*, s) \sqsubseteq g^*\} \\
&\leq \bigvee\{r \odot s \mid \mathcal{C}(k, f^*, r) \oplus \mathcal{C}(k, g^*, s) \sqsubseteq f^* \oplus g^*\} \\
&\leq \bigvee\{r \odot s \mid \mathcal{C}(k, f^* \oplus g^*, r \odot s) \sqsubseteq f^* \oplus g^*\} \\
&\leq (\mathcal{T}_{\mathcal{C}})_k(f \odot g).
\end{aligned}$$

Let $\bigwedge_{i\in I}(\mathcal{T}_{\mathcal{C}})_k(f_i) \geq r$;i.e. $(\mathcal{T}_{\mathcal{C}})_k(f_i) \geq r$ for all $i \in I$. For each $s < r$, by the definition of $(\mathcal{T}_{\mathcal{C}})_k$, $\mathcal{C}(k, f_i^*, s) \sqsubseteq f_i^*$ for all $i \in I$. Hence

$$\mathcal{C}(k, \bigwedge_{i\in I} f_i^*, s) \sqsubseteq \bigwedge_{i\in I} \mathcal{C}(k, f_i^*, s) \sqsubseteq \bigwedge_{i\in I} f_i^*.$$

Thus, $(\mathcal{T}_{\mathcal{C}})_k(\bigvee_{i\in I} f_i) \geq s$. It implies

$$(\mathcal{T}_{\mathcal{C}})_k(\bigvee_{i\in I} f_i) \geq \bigwedge_{i\in I}(\mathcal{T}_{\mathcal{C}})_k(f_i).$$

Thus $\mathcal{T}_{\mathcal{C}}$ is an L-fuzzy (K, E)-soft topology on X. We only show that $\mathcal{T}_{\mathcal{C}_{\mathcal{T}}} = \mathcal{T}$.

Suppose that there exists $f \in (L^X)^E$ such that

$$(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k(f) \not\leq \mathcal{T}_k(f).$$

Then there exists $r \in L_0$ with $\mathcal{C}_{\mathcal{T}}(k, f^*, r) = f^*$ such that

$$r \not\leq \mathcal{T}_k(f).$$

On the other hand, since $\mathcal{C}_{\mathcal{T}}(k, f^*, r) = f^*$, we have $\mathcal{T}_k(f) \geq r$ from the definition of $\mathcal{C}_{\mathcal{T}}$. It is a contradiction. Hence $(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k \leq \mathcal{T}_k$.

Suppose that there exists $g \in (L^X)^E$ such that

$$(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k(g) \not\geq \mathcal{T}_k(g).$$

Then there exists $r \in L_0$ such that

$$(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k(g) \not\geq \mathcal{T}_k(g) = r.$$

On the other hand, since $\mathcal{T}_k(g) \geq r$, we have

$$\mathcal{C}_{\mathcal{T}}(k, g^*, r) = \bigwedge\{f \mid f \sqsupseteq g^* \mathcal{T}_k(f^*) \geq r\} = g^*.$$

Thus, $(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k(g) \geq r$. It is a contradiction. Hence $(\mathcal{T}_{\mathcal{C}_{\mathcal{T}}})_k \geq \mathcal{T}_k$. □

Theorem 11. Let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be L-fuzzy (K_1, E_1)-soft and L-fuzzy (K_2, E_2)-soft topological spaces, respectively. Let $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ be a soft map. If $\varphi_{\psi,\eta} : (X, \mathcal{T}_X) \to (Y, \mathcal{T}_Y)$ is an L-fuzzy soft continuous map, then $\varphi_{\psi,\eta} : (X, \mathcal{C}_{\mathcal{T}_X}) \to (Y, \mathcal{C}_{\mathcal{T}_Y})$ is an L-fuzzy soft closure map.

Proof. Let $(\mathcal{T}_Y)_{\eta(k)}(g) \leq (\mathcal{T}_X)_k(\varphi_\psi^{-1}(g))$ for all $g \in (L^Y)^{E_2}$. Then

$$\begin{aligned}
&\mathcal{C}_{\mathcal{T}_Y}(\eta(k), \varphi_\psi(f), r)) \\
&= \bigwedge\{g_1 \in (L^Y)^{E_2} \mid \varphi_\psi(f) \sqsubseteq g_1,\ (\mathcal{T}_Y)_{\eta(k)}(g_1^*) \geq r\} \\
&\sqsupseteq \bigwedge\{\varphi_\psi(\varphi_\psi^{-1}(g_1)) \in (L^Y)^{E_2} \mid \varphi_\psi^{-1}(\varphi_\psi(f)) \sqsubseteq \varphi_\psi^{-1}(g_1), \\
&\ (\mathcal{T}_X)_k(\varphi_\psi^{-1}(g_1^*)) \geq r\} \ \text{(by Lemma 8 (1))} \\
&\sqsupseteq \varphi_\psi(\bigwedge\{\varphi_\psi^{-1}(g_1) \in (L^X)^E \mid \varphi_\psi^{-1}(\varphi_\psi(f)) \sqsubseteq \varphi_\psi^{-1}(g_1), \\
&\ (\mathcal{T}_X)_k(\varphi_\psi^{-1}(g_1^*)) \geq r\} \ \text{(by Lemma 8 (8))} \\
&\sqsupseteq \varphi_\psi(\bigwedge\{\varphi_\psi^{-1}(g_1) \in (L^X)^E \mid f \sqsubseteq \varphi_\psi^{-1}(g_1), \\
&\ (\mathcal{T}_X)_k(\varphi_\psi^{-1}(g_1^*)) \geq r\} \ \text{(by Lemma 8 (2))} \\
&\sqsupseteq \varphi_\psi(\mathcal{C}_{\mathcal{T}_X}(k, f, r)).
\end{aligned}$$

□

Theorem 12. Let $(X, \mathcal{C}_X)$ and $(Y, \mathcal{C}_Y)$ be L-fuzzy (K_1, E_1)-soft and L-fuzzy (K_2, E_2)-soft closure spaces, respectively. Let $\varphi_\psi : (L^X)^{E_1} \to (L^Y)^{E_2}$ be a soft map. Then $\varphi_{\psi,\eta} : (X, \mathcal{C}_X) \to (Y, \mathcal{C}_Y)$ is an L-fuzzy soft closure map iff $\mathcal{C}_X(k, \varphi_\psi^{-1}(g), r) \sqsubseteq \varphi_\psi^{-1}(\mathcal{C}_Y(\eta(k), g, r))$.

Proof. Let $\varphi_{\psi,\eta}(\mathcal{C}_X(k, f, r)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_\psi(f), r)$. Put $f = \varphi_\psi^{-1}(g)$. Then

$$\begin{aligned}
&\varphi_{\psi,\eta}(\mathcal{C}_X(k, \varphi_\psi^{-1}(g), r)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_\psi(\varphi_\psi^{-1}(g)), r) \\
&\sqsubseteq \mathcal{C}_Y(\eta(k), g, r)
\end{aligned}$$

Hence $\mathcal{C}_X(k, \varphi_\psi^{-1}(g), r)) \sqsubseteq \varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k), g, r))$.

Let $\mathcal{C}_X(k, \varphi_\psi^{-1}(g), r)) \sqsubseteq \varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k), g, r))$. Put $g = \varphi_\psi(f)$.

$$\begin{aligned}
&\mathcal{C}_X(k, f, r) \sqsubseteq \mathcal{C}_X(k, \varphi_\psi^{-1}(\varphi_\psi(f)), r) \\
&\sqsubseteq \varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k), \varphi_\psi(f), r))
\end{aligned}$$

Hence $\varphi_{\psi,\eta}(\mathcal{C}_X(k, f, r)) \sqsubseteq \mathcal{C}_Y(\eta(k), \varphi_\psi(f), r)$. □

Theorem 13. Let $(X, \mathcal{C}_X)$ and $(Y, \mathcal{C}_Y)$ be L-fuzzy (K_1, E_1)-soft and L-fuzzy (K_2, E_2)-soft closure spaces, respectively. Let $\varphi_\psi : (L^X)^E \to (L^Y)^{E_2}$ be a soft map. Then the following properties;

(1) If $\varphi_{\psi,\eta} : (X, \mathcal{C}_X) \to (Y, \mathcal{C}_Y)$ is an L-fuzzy soft closure map, then $\varphi_{\psi,\eta} : (X, \mathcal{T}_{\mathcal{C}_X}) \to (Y, \mathcal{T}_{\mathcal{C}_Y})$ is an L-fuzzy soft continuous map.

(2) $\varphi_{\psi,\eta} : (X, A, \mathcal{T}_X) \to (Y, B, \mathcal{T}_Y)$ is an L-fuzzy soft continuous map iff $\varphi_{\psi,\eta} : (X, A, \mathcal{C}_{\mathcal{T}_X}) \to (Y, B, \mathcal{C}_{\mathcal{T}_Y})$ is an L-fuzzy soft closure map.

Proof. (1) Let $\varphi_{\psi,\eta}$ be a closed soft map. By Theorem 11,

$$\varphi_\psi^{-1}(\mathcal{C}_Y(\eta(k), g, r)) \sqsupseteq \mathcal{C}_X(k, \varphi_\psi^{-1}(g), r)$$

for all $g \in (L^Y)^{E_2}$. Then

$$\begin{aligned}(\mathcal{T}_{\mathcal{C}_Y})_k(g) &= \bigvee\{r \in L \mid g^* \sqsupseteq \mathcal{C}_Y(\eta(k), g^*, r)\} \\ &\leq \bigvee\{r \in L \mid \varphi_\psi^{-1}(g^*) \sqsupseteq \varphi_{\psi,\eta}^{-1}(\mathcal{C}_Y(\eta(k), g^*, r))\} \\ &\leq \bigvee\{r \in L \mid \varphi_\psi^{-1}(g^*) \sqsupseteq \mathcal{C}_X(k, \varphi_\psi^{-1}(g^*), r)\} \\ &\leq (\mathcal{T}_{\mathcal{C}_X})_k(\varphi_\psi^{-1}(g)).\end{aligned}$$

(2) Let $\varphi_{\psi,\eta} : (X, \mathcal{C}_{\mathcal{T}_X}) \to (Y, \mathcal{C}_{\mathcal{T}_Y})$ be a closure soft map. Since $\mathcal{T}_{\mathcal{C}_\mathcal{T}} = \mathcal{T}$ and $\mathcal{T}_{\mathcal{C}_\mathcal{T}} = \mathcal{T}$ from Theorem 10(3), by (1), $\varphi_\psi : (X, \mathcal{T}_X) \to (Y, \mathcal{T}_Y)$ is an L-fuzzy soft continuous map. □

Example 14. Let X and E_X be given as Example 9. Define a binary operation $\wedge$ on $[0,1]$ by

$$x \wedge y = \min\{x, y\}, x^* = 1 - x, x \vee y = (x^* \wedge y^*)^*$$

$$x \to y = \begin{cases} 1, & \text{if } x \leq y, \\ y, & \text{otherwise.} \end{cases}$$

Then $([0,1], \wedge, \to, 0, 1)$ is a stsc-quantale (ref.[8,9,18]). Let f_1 and f_2 as Example 9. Then we obtain:

$$\begin{aligned}(f_1 \vee f_2)_e &= (0.8, 0.9, 0.5, 0.9, 0.6) \\ (f_1 \vee f_2)_b &= (0.7, 1.0, 0.4, 0.5, 0.7) \\ (f_1 \vee f_2)_w &= (0.5, 0.7, 0.8, 0.6, 0.5)\end{aligned}$$

$$\begin{aligned}(f_1 \wedge f_2)_e &= (0.5, 0.1, 0.4, 0.8, 0.4) \\ (f_1 \wedge f_2)_b &= (0.3, 0.9, 0.2, 0.4, 0.5) \\ (f_1 \wedge f_2)_w &= (0.4, 0.4, 0.5, 0.5, 0.1)\end{aligned}$$

For $K = \{k_1, k_2\}$, we define a $[0,1]$-fuzzy (K,E)-soft topology $\mathcal{T}_X : K \to [0,1]^{([0,1]^X)^{E_1}}$ as follows:

$$(\mathcal{T}_X)_{k_1}(f) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } 1_X, \\ 0.7, & \text{if } f = f_1 \\ 0.4, & \text{if } f = f_2 \\ 0.5, & \text{if } f = f_1 \vee f_2 \\ 0.6, & \text{if } f = f_1 \wedge f_2 \\ 0, & \text{otherwise.} \end{cases}$$

$$(\mathcal{T}_X)_{k_2}(f) = \begin{cases} 1, & \text{if } f = 0_X \text{ or } 1_X, \\ 0.5, & \text{if } f = f_1 \\ 0, & \text{otherwise.} \end{cases}$$

From Theorem 10(1), we obtain a $[0,1]$-fuzzy (K,E)-soft closure operator $\mathcal{C}_{\mathcal{T}_X} : K \times ([0,1]^X)^{E_1} \times (0,1] \to ([0,1]^X)^{E_1}$ as follows:

$$\mathcal{C}_{\mathcal{T}_X}(k_1, f, r) = \begin{cases} 1_X, & \text{if } f = 1_X, r \in (0,1], \\ f_1^*, & \text{if } f \sqsubseteq f_1^*, f \not\sqsubseteq f_1^* \wedge f_2^*,\ r \leq 0.7, \\ f_2^*, & \text{if } f \sqsubseteq f_2^*, f \not\sqsubseteq f_1 \wedge f_2^*,\ r \leq 0.4, \\ f_1^* \wedge f_2^* & \text{if } f \sqsubseteq f_1^* \wedge f_2^*,\ r \leq 0.5 \\ f_1^* \vee f_2^* & \text{if } f \sqsubseteq f_1^* \vee f_2^*, \\ & \quad f \not\sqsubseteq f_1^*,\ f \not\sqsubseteq f_2^*, r \leq 0.6, \\ 0_X, & \text{otherwise.} \end{cases}$$

$$\mathcal{C}_{\mathcal{T}_X}(k_2, f, r) = \begin{cases} 1_X, & \text{if } f = 1_X, r \in (0,1], \\ f_1^*, & \text{if } f \sqsubseteq f_1^*,\ r \leq 0.5, \\ 0_X, & \text{otherwise.} \end{cases}$$

□

Example 15. Let X and E_X be given as Example 14. Let $([0,1], \odot, \oplus, \to, ^*, 0, 1)$ be a complete residuated lattice as Example 9. Let $E = \{b, w, c\} \subset E_X$, $f \in ([0,1]^X)^E$ be a fuzzy soft set as follow:

$$\begin{aligned} f_b &= (0.5, 0.3, 0.5, 0.6, 0.2) \\ f_c &= (0.1, 0.2, 0.6, 0.5, 0.5) \\ f_w &= (0.4, 0.4, 0.5, 0.6, 0.6) \end{aligned}$$

$$\begin{aligned} (f \odot f)_b &= (0.0, 0.0, 0.0, 0.2, 0.0) \\ (f \odot f)_c &= (0.0, 0.0, 0.2, 0.0, 0.0) \\ (f \odot f)_w &= (0.0, 0.0, 0.0, 0.2, 0.2) \end{aligned}$$

Let $K=\{k_1,k_2\}$ be given. Define a $[0,1]$-fuzzy (K,E)-soft topology $\mathcal{T}_X : K \to [0,1]^{([0,1]^X)^E}$ as follows:

$$(\mathcal{T}_X)_{k_1}(h)=\begin{cases} 1, & \text{if } h=0_X \text{ or } h=1_X,\\ 0.6, & \text{if } h=f\\ 0.3, & \text{if } h=f\odot f\\ 0, & \text{otherwise.}\end{cases}$$

$$(\mathcal{T}_X)_{k_2}(h)=\begin{cases} 1, & \text{if } h=0_X \text{ or } h=1_X,\\ 0.4, & \text{if } h=f\\ 0, & \text{otherwise.}\end{cases}$$

We obtain a $[0,1]$-fuzzy (K,E)-soft closure operator $\mathcal{C}_{\mathcal{T}_X} : K\times([0,1]^X)^E\times(0,1]\to([0,1]^X)^E$ as follows:

$$\mathcal{C}_{\mathcal{T}_X}(k_1,h,r)=\begin{cases} 1_X, & \text{if } h=1_X, r\in(0,1],\\ f^*, & \text{if } h\sqsubseteq f^*, r\le 0.6\\ f^*\oplus f^* & \text{if } h\sqsubseteq f^*\oplus f^*\\ & \ h\not\sqsubseteq f^*, r\le 0.3\\ 0_X, & \ \text{otherwise.}\end{cases}$$

$$\mathcal{C}_{\mathcal{T}_X}(k_2,h,r)=\begin{cases} 1_X, & \text{if } h=1_X, r\in(0,1],\\ f^*, & \text{if } h\sqsubseteq f^*, r\le 0.4\\ 0_X, & \text{otherwise.}\end{cases}$$

□

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